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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

TOPOLOGICAL DIAGRAM ANALYSIS OF BOTTOM MESON DECAYS EMITTING TWO PSEUDOSCALAR MESONS

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We investigate weak nonleptonic decays of B mesons emitting two pseudoscalar (P) mesons for the Cabibbo–Kobayashi–Maskawa enhanced as well as suppressed modes. We employ the quark diagram approach at $SU(3)$ symmetry for various weak quark-level processes responsible for these decays. Several relations are obtained among their decay amplitudes and the corresponding branching fractions, for which some experimental data exist.

В работе исследуются слабые нелептонные распады B-мезонов, излучающих два псевдоскалярных (P) мезона, в рамках как расширенных, так и ограниченных мод Кабиббо–Кобаяши–Маскавы. Процесс рассматривается в приближении кварковых диаграмм в симметрии $SU(3)$ для различных слабых на кварковом уровне процессов, ответственных за рассматриваемые распады. Получено несколько отношений амплитуд распадов и соответствующие отношения мод распада, для которых имеются экспериментальные данные.

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INTRODUCTION

Understanding nonleptonic weak decays is crucial for testing the Standard Model, and on the potential effects of physics beyond it. Particularly, nonleptonic weak decays of B mesons are interesting because their studies are vital for extracting information about the strong interaction interplay with the weak processes [1–8]. However, due to the nonperturbative strong interactions involved in these decays, the task is hampered for the computation of matrix elements between the initial and the final hadron states. In order to deal with these complicated matrix elements, usually the naïve and QCD factorization schemes [9–25], including probable final state interactions (FSI) [26–33], have been employed to predict branching fractions of

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nonleptonic decays of B mesons. For some channels of B decays, however, the factorization calculations appear to be in clear disagreement with current measurement [34–38]. The semi-phenomenological analyses of two-body decays of heavy flavor mesons have indicated the presence of large nonfactorizable contributions [39–48], especially for the color suppressed decays. These decays have also been studied using flavor $SU(3)$ symmetry [49–51], where various dynamical factors get lumped into a few reduced matrix elements, which are generally determined using some experimental results. Fortunately, the experimental progress for weak semileptonic and nonleptonic decays of the bottom mesons during the last years has been really astounding, due to which a good amount of experimental data now exist [52], which have inspired several theoretical works [53–60] on the weak decays of B mesons.

In the present work, we have studied $B \to PP$ weak decays investigating contributions arising from various quark-level weak interaction processes. Due to the strong interaction interference, like FSI and nonfactorizable contributions, on these processes, it is not possible to calculate their contributions reliably. For instance, weak annihilation and W-exchange contributions, which are naively expected to be suppressed in comparison with the W-emission terms, may become significant due to possible nonfactorizable effects arising through soft-gluon exchange around the weak vertex. Since such effects are not calculable from the first principles, we employ the model-independent topological diagram approach, naively called Topological Quark Diagram Scheme (hereafter referred to as TQDS) [61–73]; wherein decay amplitudes (referred to as topological amplitudes) can be expressed independently in terms of the topologies of possible quark flavor diagrams like: a) the external W-emission diagram, b) the internal W-emission diagram, c) the W-exchange diagram, d) the W-annihilation, and e) the W-loop diagram, and parameterize their contributions to B-meson decays. The QDS has already been shown [71, 72] to be a useful technique for heavy-flavor weak decays.

In Sec. 1, we construct the weak Hamiltonian responsible for the $B \to PP$ decays. Choosing appropriate components of the weak quark-level processes, we then obtain several straightforward relations among their decay amplitudes in the Cabibbo–Kobayashi–Maskawa (CKM) enhanced as well as suppressed modes in Sec. 2. In Sec. 3, we proceed to derive corresponding relations among their branching fractions in the TQDS using $SU(2)$-isospin, $SU(2){-}U$ spin, flavor $SU(3)$ frameworks. In this analysis, we have considered only those decays for which some experimental data exist to establish the applicability of the TQDS. Consequently, predictions of some of the decay branching fractions are also made, which can provide further tests of the scheme. Summary and discussion are given in the last section.

1. WEAK HAMILTONIAN

To the lowest order in weak interaction, the nonleptonic Hamiltonian has the usual current $\otimes$ current form

$$H_w = \frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu + \text{h.c.}, \tag{1}$$

where the weak current J_μ is given by

$$J_\mu = \left(\bar{u}\,\bar{c}\,\bar{t}\right)\gamma_\mu(1-\gamma_5)\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}. \tag{2}$$

Weak eigenstates (d', s' and b') are related to the mass eigenstates (d, s and b) through the Cabibbo–Kobayashi–Maskawa mixing. The weak Hamiltonian generating the b-quark decays is thus given by

$$H_w^{\Delta b=1} = \frac{G_F}{\sqrt{2}}[V_{ub}V_{cd}^*(\bar{u}b)(\bar{d}c) + V_{ub}V_{cs}^*(\bar{u}b)(\bar{s}c) + V_{ub}V_{ud}^*(\bar{u}b)(\bar{d}u) + V_{ub}V_{us}^*(\bar{u}b)(\bar{s}u)+ \\ + V_{cb}V_{ud}^*(\bar{c}b)(\bar{d}u) + V_{cb}V_{us}^*(\bar{c}b)(\bar{s}u) + V_{cb}V_{cs}^*(\bar{c}b)(\bar{s}c) + V_{cb}V_{cd}^*(\bar{c}b)(\bar{d}c)]. \quad (3)$$

The color and space-time structure is omitted. Selection rules for various decay modes generated by the Hamiltonian are given below:

(i) CKM enhanced modes: $\Delta C = 1$, $\Delta S = 0$; $\Delta C = 0$, $\Delta S = -1$;

(ii) CKM suppressed modes: $\Delta C = 1$, $\Delta S = -1$; $\Delta C = 0$, $\Delta S = 0$;

(iii) CKM doubly suppressed modes: $\Delta C = \Delta S = -1$; $\Delta C = -1$, $\Delta S = 0$.

Since only quark fields appear in the Hamiltonian, the B-meson decays are seriously affected by the strong interactions. One usually identifies the two scales in these decays: short-distance scale, at which W-exchange takes place, and long-distance scale, where final state hadrons are formed. The short-distance effects are calculable using the perturbative QCD, which are expressed in terms of certain QCD coefficients. The long-distance effects being nonperturbative are the source of major problems in obtaining the decay amplitudes from the Hamiltonian, even after including the short-distance modifications.

There are many ways that the quarks produced in a weak nonleptonic process can arrange themselves into final state hadrons. All B-meson decays can be expressed topologically in terms of a few quark-level diagrams [71–73]: a) the external W-emission diagram, b) the internal W-emission diagram, c) the W-exchange diagram, d) the W-annihilation, and e) the W-loop penguin diagram. Initially, it was expected that W-exchange and W-annihilation diagrams are suppressed due to the helicity and color considerations, and the penguin diagrams, involving W-loop, contribute to only two out of six decay modes. Thus, the dominant quark-level process apparently seems to involve W-emission, in which light quark in the B meson behaves like spectator. However, measurements of some of B-meson decays have challenged this naïve and simple picture, and it is now established that the nonspectator contributions [39–48] may play a significant role in understanding the weak decays of heavy-flavor hadrons. In fact, exchange of the soft gluons around the weak vertex also enhances such nonspectator contributions from the W-exchange, W-annihilation and W-loop diagrams. Unfortunately, these effects, being nonperturbative, cannot be determined unambiguously from the first principles. The QCD sum rules approach [74] has been used to estimate them, but so far it has not given reliable results.

In the absence of the exact dynamical calculations, we have employed the quark diagram scheme [61–73] to investigate contributions from different weak quark-level diagrams. Such a scheme gives a model-independent way to analyze data to test the mechanism of the various quark-level processes, and to make useful predictions for the meson decays. The decay amplitudes are obtained using the valence quark structure of the particles involved in the B-meson decays. Using the tensorial notation, the decay amplitudes are then obtained from the following contractions:

$$H_w^{\Delta b=1} = [a(B^m P_m^i P_n^k) + d(B^i P_n^m P_m^k)]H_{[i,k]}^n + [a'(B^m P_m^i P_n^k)+ \\ + d'(B^i P_n^m P_m^k)]H_{(i,k)}^n + [c(B^n P_n^m P_m^i)]H_i. \quad (4)$$

The brackets $[,]$ and $(,)$, respectively, denote antisymmetrization and symmetrization among the indices i, k. In the flavor $SU(4)$, the b quark behaves like singlet, and u, d, s, and c quarks form a quartet. Thus, the Hamiltonian for $\Delta b = 1$ weak process belongs to the representations appearing in

$$4^* \otimes 1 \otimes 4^* \otimes 4 = 4^* \oplus 4^* \oplus 20' \oplus 36^*. \tag{5}$$

The weak spurion H_i, $H^n_{[i,k]}$ and $H^n_{(i,k)}$ belong to the 4^*, $20'$, and 36^* representations, respectively [75]. However, we do not use the complete $SU(4)$-TQDS due to $SU(4)$ being badly broken, and exploit the TQDS at the $SU(3)$ level through the following $SU(4) \to SU(3)$ decomposition:

$$\begin{aligned} 4^* &\supset 3^* \oplus 1, \\ 20' &\supset 8 \oplus (6 \oplus 3^*) \oplus 3, \\ 36^* &\supset 6^* \oplus (15 \oplus 3^*) \oplus (8 \oplus 1) \oplus 3. \end{aligned} \tag{6}$$

The $SU(3)$-TQDS relates $\Delta C = 1$, $\Delta S = 0$ mode with $\Delta C = 1$, $\Delta S = -1$; $\Delta C = 0$, $\Delta S = -1$ mode with $\Delta C = 0$, $\Delta S = 0$; and $\Delta C = \Delta S = -1$ mode with $\Delta C = -1$, $\Delta S = 0$. The tensor B^i denotes the parent B mesons:

$$B^1 = B^-(\bar{u}b), \quad B^2 = \bar{B}^0(\bar{d}b), \quad B^3 = \bar{B}^0_s(\bar{s}b), \quad B^4 = B^-_c(\bar{c}b). \tag{7}$$

The P^i_j denotes bottomless pseudoscalar mesons with the quark content $(\bar{q}^i q_j)$. Using $SU(3)$ nonet (or $SU(4)$ sixteenplet) symmetry, the diagonal states are taken to be:

$$\frac{\pi^0 + \eta \sin\theta_p + \eta' \cos\theta_p}{\sqrt{2}}, \quad \frac{-\pi^0 + \eta \sin\theta_p + \eta' \cos\theta_p}{\sqrt{2}}, \quad -\eta\cos\theta_p + \eta' \sin\theta_p, \eta_c, \tag{8}$$

where the mixing angle $\theta_p = \theta_{\text{ideal}} - \phi_P$; $\phi_P = -15.4°$ follows from the radiative decay widths [76–78], and $\eta_c(\bar{c}c)$ is taken to be charmonium singlet (ideal mixing). The physical mesons η, η' are defined as follows [53–60]:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi_P & -\sin\phi_P \\ \sin\phi_P & \cos\phi_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \tag{9}$$

where the flavor wave functions of η_8 and η_1 are given by

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \tag{10}$$

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}). \tag{11}$$

There exists a straight correspondence between the terms appearing in (4) and various topological quark-level diagrams. The terms with coefficients $(a + a')$ represent external W-emission, $(a - a')$ represent internal W-emission, the terms with coefficients $(d - d')$ represent W-exchange, and $(d + d')$ — for W-annihilation processes. The last term, having coefficient c, represents the W-loop penguin diagram contributions. In addition, all the QCD effects have been absorbed in these parameters, the following contractions may also be constructed in the light of nonet (or sixteenplet) symmetry:

$$+[h\,(B^i P^k_n P^m_m)]H^n_{[i,k]} + [h'(B^i P^k_n P^m_m)]H^n_{(i,k)}, \tag{12}$$

$$+[f(B^i P^m_n P^n_m) + f'(B^i P^m_m P^n_n) + f''(B^n P^i_n P^m_m)]H_i. \tag{13}$$

However, these terms correspond to the OZI violating diagrams, which are expected to be suppressed, and hence are ignored in the present scheme.

2. DECAY AMPLITUDES RELATIONS

Choosing the relevant components of the Hamiltonian given in (3), we obtain the decay amplitudes in the $SU(3)$ quark-diagram scheme for $B \to PP$, for which some experimental results are available. Depending upon the weak quark-level processes involved in these decays, we have categorized their relations given below in three different ways:

1) Only single weak process (W-emission or W-exchange or W-annihilation) contributes, (A, B, C and D).

2) Combination of two (or more) weak processes (other than penguin diagram) contributes, (E, F and G).

3) Combination of penguin diagram with other weak processes contributes, (H and I).

A. W-External Emission:

$$A(\bar{B}_s^0 \to \pi^- D_s^+) = V_{ud}/V_{us} A(\bar{B}^0 \to K^- D^+), \tag{14}$$

$$\sqrt{2} A(B^- \to \pi^0 D_s^-) = A(\bar{B}^0 \to D_s^- \pi^+), \tag{15}$$

$$A(\bar{B}_s^0 \to K^+ D^-) = V_{cd}/V_{cs} A(\bar{B}^0 \to \pi^+ D_s^-). \tag{16}$$

B. W-Internal Emission:

$$A(\bar{B}_s^0 \to K^0 \eta_c) = V_{cd}/V_{cs} A(B^- \to K^- \eta_c) = V_{cd}/V_{cs} A(\bar{B}^0 \to \bar{K}^0 \eta_c), \tag{17}$$

$$A(\bar{B}_s^0 \to \eta_c \eta) = -\cos\theta_p A(B^- \to \eta_c K^-), \tag{18}$$

$$A(\bar{B}_s^0 \to K^0 D^0) = V_{ud}/V_{us} A(\bar{B}^0 \to \bar{K}^0 D^0). \tag{19}$$

C. W-Emission (Both):

$$A(B^- \to \pi^- D^0) = V_{ud}/V_{us} A(B^- \to K^- D^0). \tag{20}$$

D. W-Exchange Only:

$$A(\bar{B}^0 \to K^- K^+) = V_{du}/V_{us} A(\bar{B}_s^0 \to \pi^+ \pi^-), \tag{21}$$

$$A(\bar{B}_s^0 \to \pi^0 \pi^0) = A(\bar{B}_s^0 \to \pi^+ \pi^-), \tag{22}$$

$$A(\bar{B}^0 \to \bar{D}^0 D^0) = A(\bar{B}^0 \to D_s^- D_s^+) = V_{dc}/V_{sc} A(\bar{B}_s^0 \to D^- D^+), \tag{23}$$

$$\sqrt{2} A(\bar{B}_s^0 \to \pi^0 D^0) = V_{us}/V_{ud} A(\bar{B}^0 \to K^- D_s^+). \tag{24}$$

E. W-External Emission and W-Exchange:

$$A(\bar{B}_s^0 \to D_s^+ K^-) = V_{us}/V_{ud} A(\bar{B}^0 \to \pi^- D^+), \tag{25}$$

$$A(\bar{B}_s^0 \to D_s^- K^+) = V_{cs}/V_{cd} A(\bar{B}^0 \to \pi^+ D^-). \tag{26}$$

F. W-Internal Emission and W-Exchange:

$$A(\bar{B}^0 \to \eta D^0) = \tan\theta_p A(\bar{B}^0 \to \eta' D^0). \tag{27}$$

G. W-Emission Both and W-Annihilation:

$$A(B^- \to \pi^- \eta) = \tan\theta_p A(B^- \to \pi^- \eta'). \tag{28}$$

H. W-External Emission and Penguin:

$$A(\bar{B}^0 \to D^+ D_s^-) = A(B^- \to D_s^- D^0), \quad (29)$$

$$A(\bar{B}_s^0 \to D^- D_s^+) = A(B^- \to D^- D^0). \quad (30)$$

I. W-Exchange Only and Penguin:

$$A(\bar{B}^0 \to \pi^0 \eta') = \cot\theta_p A(\bar{B}^0 \to \pi^0 \eta). \quad (31)$$

Note, that the relations (15), (22) and (29) follow from the $SU(2)$ isospin framework, and (14), (16), (19)–(21), (25), (26) from the $SU(2){-}U$ spin for TQDS.

3. RELATIONS AND PREDICTIONS FOR BRANCHING FRACTIONS

The decay rate formula for $B \to PP$ has the generic form [61–70]:

$$\Gamma(B \to PP) = (\text{nonkinematic factors})^2 \left(\frac{k}{8\pi m_B^2}\right) |A|^2, \quad (32)$$

where k is the 3-momentum of the final states and is given by

$$k = |p_1| = |p_2| = \frac{1}{2m_B}\left\{(m_B^2 - (m_1 + m_2)^2)(m_B^2 - (m_1 - m_2)^2)\right\}^{1/2}. \quad (33)$$

Several relations are obtained between branching fractions of the decays of $B^-, \bar{B}^0$, and $\bar{B}_s^0$ mesons, corresponding to the decay amplitude relations given in the previous section. We have used the available experimental values to check the consistency of the relations obtained and to predict the branching fractions of some of the decay not observed so far. We give our values just below the branching relations. These values are obtained by multiplying the known experimental value by the factor (given on RHS). For instance, in relation (35), branching fraction of $B(B^- \to \pi^0 D_s^-)$ is obtained by multiplying the experimental value of branching fraction $B(\bar{B}^0 \to D_s^- \pi^+) = (2.16 \pm 0.26) \cdot 10^{-5}$ by the factor 0.53 given in (35). Similar to the decay amplitude relations, we have categorized relations among the branching fractions according to the contributions arising from one or more of the weak quark diagrams. We have distinguished the $b \to s$ penguin process from that of $b \to d$.

A. W-External Emission:

$$B(\bar{B}_s^0 \to \pi^- D_s^+) = 18.65 B(\bar{B}^0 \to K^- D^+), \quad (3.04 \pm 0.23) \cdot 10^{-3}, \quad (3.67 \pm 0.39) \cdot 10^{-3}, \quad (34)$$

$$B(B^- \to \pi^0 D_s^-) = 0.53 B(\bar{B}^0 \to D_s^- \pi^+), \quad (1.6 \pm 0.5) \cdot 10^{-5}, \quad (1.14 \pm 0.13) \cdot 10^{-5}, \quad (35)$$

$$B(\bar{B}_s^0 \to K^+ D^-) = 0.051 B(\bar{B}^0 \to \pi^+ D_s^-), \quad (0.11 \pm 0.01) \cdot 10^{-5}. \quad (36)$$

B. W-Internal Emission:

$$B(\bar{B}_s^0 \to K^0 \eta_c) = 17.91 B(B^- \to K^- \eta_c) = 18.94 B(\bar{B}^0 \to \bar{K}^0 \eta_c), \quad (1.7 \pm 0.2) \cdot 10^{-2}, \quad (1.5 \pm 0.2) \cdot 10^{-2}, \quad (37)$$

$$B(\bar{B}_s^0 \to \eta_c \eta) = 0.37 B(B^- \to \eta_c K^-), \quad (3.5 \pm 0.4) \cdot 10^{-4}, \quad (38)$$

$$B(\bar{B}_s^0 \to K^0 D^0) = 18.73 B(\bar{B}^0 \to \bar{K}^0 D^0), \quad (9.7 \pm 1.3) \cdot 10^{-4}. \quad (39)$$

C. W-Emission (Both):

$$B(B^- \to \pi^- D^0) = 19.42 B(B^- \to K^- D^0), \quad (4.81 \pm 0.15) \cdot 10^{-3}, \quad (7.18 \pm 0.41) \cdot 10^{-3}. \tag{40}$$

D. W-Exchange Only:

$$B(\bar{B}^0 \to K^- K^+) = 19.45 B(\bar{B}_s^0 \to \pi^+ \pi^-), \quad (1.3 \pm 0.5) \cdot 10^{-7}, \quad (14.8 \pm 3.7) \cdot 10^{-6}, \tag{41}$$

$$B(\bar{B}_s^0 \to \pi^0 \pi^0) = 0.50 B(\bar{B}_s^0 \to \pi^+ \pi^-), \quad < 2.1 \cdot 10^{-4}, \quad (3.8 \pm 0.9) \cdot 10^{-6}, \tag{42}$$

$$B(\bar{B}^0 \to \bar{D}^0 D^0) = 1.06 B(\bar{B}^0 \to D_s^- D_s^+) = 19.5 B(\bar{B}_s^0 \to D^- D^+), \quad < 4.3 \cdot 10^{-5}, \quad < 3.8 \cdot 10^{-5}, \tag{43}$$

$$B(\bar{B}_s^0 \to \pi^0 D^0) = 0.026 B(\bar{B}^0 \to K^- D_s^+), \quad (0.05 \pm 0.01) \cdot 10^{-5}. \tag{44}$$

E. W-External Emission and W-Exchange:

$$B(\bar{B}_s^0 \to D_s^+ K^-) = 0.05 B(\bar{B}^0 \to \pi^- D^+) \tag{45}$$

and

$$B(\bar{B}^0 \to \pi^+ D^-) = 0.06 B(\bar{B}_s^0 \to D_s^- K^+). \tag{46}$$

Experimentally $B(\bar{B}_s^0 \to D_s^{\pm} K^{\pm}) = (2.03 \pm 0.28) \cdot 10^{-4}$ is available, for which, combining the above two relations, we obtain

$$B(\bar{B}_s^0 \to D_s^{\pm} K^{\pm}) = 0.05 B(\bar{B}^0 \to \pi^- D^+) + 16.67 B(\bar{B}^0 \to \pi^+ D^-), \quad (2.03 \pm 0.28) \cdot 10^{-4}, \quad (1.47 \pm 0.03) \cdot 10^{-4}. \tag{47}$$

F. W-Internal Emission and W-Exchange:

$$B(\bar{B}^0 \to \eta D^0) = 1.54 B(\bar{B}^0 \to \eta' D^0), \quad (2.36 \pm 0.32) \cdot 10^{-4}, \quad (2.13 \pm 0.25) \cdot 10^{-4}. \tag{48}$$

G. W-Emission Both and W-Annihilation:

$$B(B^- \to \pi^- \eta) = 1.53 B(B^- \to \pi^- \eta'), \quad (4.02 \pm 0.27) \cdot 10^{-6}, \quad (4.1 \pm 1.4) \cdot 10^{-6}. \tag{49}$$

H. W-External Emission and Penguin:

$$B(\bar{B}^0 \to D^+ D_s^-) = 0.94 B(B^- \to D_s^- D^0), \quad (7.2 \pm 0.8) \cdot 10^{-3}, \quad (8.8 \pm 1.7) \cdot 10^{-3}, \tag{50}$$

$$B(\bar{B}_s^0 \to D^- D_s^+) = 0.91 B(B^- \to D^- D^0), \quad (3.4 \pm 0.4) \cdot 10^{-4}. \tag{51}$$

I. W-Exchange Only and Penguin:

$$B(\bar{B}^0 \to \pi^0 \eta) = 1.53 B(\bar{B}^0 \to \pi^0 \eta'), \quad < 1.5 \cdot 10^{-6}, \quad (1.8 \pm 0.9) \cdot 10^{-6}. \tag{52}$$

We note that except for the relation (41), the relations obtained here are generally found to be consistent within experimental values. It may be remarked that the relations (34) to (44) among the decays involving only one weak process remain unaffected by any change of phase of the decay amplitudes, which may arise due to elastic FSI. However, other kind of relations, (40), (45) to (52), where two or more weak processes contribute, may be affected by the relative phase factor [49–51]. Further, it may also be noted that the relations (35), (42) and (50) follow from the TQDS at isospin level, and hence are found to be more reliable.

For branching fraction relation (41), which occurs only through the W-exchange diagram, we show large deviation from the experimental values. One may assign this discrepancy to the possible $SU(3)$ breaking, indicating that the probability for $\bar{s}s$-pair production through soft gluons is probably less as compared to that of $\bar{d}d$ pair. However, seeing the large discrepancy, we suggest a new measurement of branching fractions of these decays to resolve the issue.

4. SUMMARY AND CONCLUSION

Reasonably good amount of experimental data is now available for B meson decaying to two pseudoscalar mesons. Theoretically, these decays are usually studied using the factorization scheme, which expresses the decay amplitudes in terms of certain meson decay constants and meson–meson form factors. However, this scheme is unable to explain the experimental results, even after including hard QCD effects and possible phase differences. This may happen because of possible soft gluon exchange effects around the weak vertex, which may enhance the contributions of W-exchange, W-annihilation and W-loop processes. Since these effects are not extractable from the first principles, we have investigated the B-meson decays employing the framework of Topological Quark Diagram Scheme. Firstly, we have obtained decay amplitude relations among B to PP decays using $SU(2)$ isospin, $SU(2){-}U$ spin, and $SU(3)$ for the TQDS. Afterwards, relations among their corresponding branching fractions have been derived in Sec. 3, giving experimental results wherever available. We make the following observations:

• The relations (34), (35), (37), (48), (49), and (50) are consistent with the experimental data within the errors. The relations based upon $SU(2)$ are found to be in better agreement, whereas those based on the $SU(2){-}U$ spin or flavor $SU(3)$ framework seem to show little deviation. So, TQDS seems to hold good validity for these hadronic weak decays of B mesons.

• Using the available branching fraction of the observed decays, we also predict the branching fraction of several decays in (36)–(39), (42), (44), (51), and (52). Explicitly, $B(\bar{B}_s^0 \to K^+D^-) = (0.11 \pm 0.01) \cdot 10^{-5}$, $B(\bar{B}_s^0 \to K^0\eta_c) = (1.7 \pm 0.2) \cdot 10^{-2}$, $B(\bar{B}_s^0 \to \eta_c\eta) = (3.5 \pm 0.4) \cdot 10^{-4}$, $B(\bar{B}_s^0 \to K^0D^0) = (9.7 \pm 1.3) \cdot 10^{-4}$, $B(\bar{B}_s^0 \to \pi^0\pi^0) = (3.8 \pm 0.9) \cdot 10^{-6}$, $B(\bar{B}_s^0 \to \pi^0D^0) = (0.05 \pm 0.01) \cdot 10^{-5}$, $B(\bar{B}_s^0 \to D^-D_s^+) = (3.4 \pm 0.4) \cdot 10^{-4}$, $B(\bar{B}^0 \to \pi^0\eta) = (1.8 \pm 0.9) \cdot 10^{-6}$. Measurement of branching fractions of these decays would help to ascertain the validity of TQDS.

• It may further be noted that the experimental branching fractions for the relations (40) and (47) have the same order, though different magnitudes. This may happen due to the possible phase differences between the two weak processes involved in the decays.

• Observed discrepancy in (41) requires special consideration. One may be tempted to hold $SU(3)$ breaking responsible for this gap. However, looking at the large discrepancy, we suggest new measurements for the decays involved in these relations to resolve the issue.

• Generally, W-exchange and W-annihilation diagrams are expected to be suppressed due to the helicity and color considerations. In the $SU(3)$-based analysis of charmless decays of B mesons, these diagrams are expected to be suppressed because of the factor $f_B/m_B \sim 5\%$. In some cases, their contributions may not be negligible, particularly, when nonfactorizable soft gluon exchanges occur. Available measurements of $B(\bar{B}^0 \to K^- K^+)$, $B(\bar{B}_s^0 \to \pi^+\pi^-)$, and $B(\bar{B}^0 \to K^- D_s^+)$ clearly establish the presence of these diagrams, though their branching fractions are smaller than that of the decays arising from the W-emission diagrams.

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