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ABOUT ABSENCE OF OSCILLATIONS AT CP VIOLATION AND PRESENCE OF INTERFERENCE BETWEEN K_S-, K_L-MESON STATES IN THE SYSTEM OF K^0 MESONS

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Two approaches to the description of K^0-, $\bar{K}^0$-meson transitions into K_1^0 mesons at CP violation in weak interactions are considered. The first approach uses the standard theory of oscillations and the second approach supposes that (K_S, K_L) states which arise at CP violation are normalized but not orthogonal state functions, then there arise interferences between these states but not oscillations. It is necessary to remark that the available experimental data are in good agreement with the second approach. So, we come to the conclusion that oscillations do not arise at CP violation in weak interactions in the system of K^0 mesons. Only interference between K_S and K_L states takes place here.

Для описания перехода K^0-, $\bar{K}^0$-мезонов в K_1^0-, K_S-мезоны при CP-нарушении в слабых взаимодействиях рассматриваются два подхода. В первом подходе используется стандартная теория осцилляций, а во втором подходе предполагается, что K_S-, K_L-состояния, которые возникают при CP-нарушении, являются нормированными, но не ортогональными функциями состояния, тогда возникают не осцилляции, а интерференции между этими состояниями. Отмечено, что существующие экспериментальные данные находятся в хорошем согласии со вторым подходом при $\sin^2\beta = 2{,}23 \cdot 10^{-3}$. Из этого можно сделать вывод, что при нарушении CP-четности в системе K^0-мезонов осцилляции не возникают.

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INTRODUCTION

Oscillations of K^0 mesons (i. e., $K^0 \leftrightarrow \bar{K}^0$) were theoretically [1] and experimentally [2] investigated in the 1950s and 1960s. Recently an understanding has been achieved that these processes go as a double-stadium process [3–6]. A detailed study of K^0-meson mixing and oscillations is very important since the theory of neutrino oscillations is built by analogy with the theory of K^0-meson oscillations.

Previously it was supposed that P parity is a well number; however, after theoretical [7] and experimental [8] works it has become clear that in weak interactions P parity is violated. Then in [9] there was an advanced supposition that in weak interactions CP parity is conserved, but not P parity. In [10] it has been reported that in K_L decays with a probability of about 0.2% there is a two-π decay mode that is a detection of CP violation.

A phenomenological analysis of K^0-meson processes was done in [11] (see also [12]). There nonunitary transformation and nonorthogonal states were used in obtaining K_S, K_L states. It was supposed that these states arise at CP violation. In [13] the same process was considered in the framework of the standard scheme (theory) of K^0-meson oscillations.

The present work is a continuation of the pervious one [13]. Here we will consider elements of the theory of K^0-meson oscillations at strangeness (S) and CP violations and then the case of CP violation in the absence of oscillations. At the same time, we will perform a comparative analysis of the obtained results at CP violation in the above two approaches and also compare these results with the available experimental data.

1. K_1^0-, K_2^0-MESON VACUUM OSCILLATIONS AT INDIRECT VIOLATION OF CP INVARIANCE WITH TAKING INTO ACCOUNT WIDTH DECAYS

The process of K_1^0-, K_2^0-meson vacuum oscillations at indirect violation of CP invariance with taking into account width decays was considered in detail in [13]. Therefore, we are considering the main elements of these oscillations.

It is clear that we have to take into account CP phase δ. We can do it by using the parametrization of Kobayashi–Maskawa matrix [15] proposed by L. Maiani [16]. The expressions for U, U^{-1} will then have the following form:

$$U = \begin{pmatrix} \cos\beta & -\sin\beta\,\mathrm{e}^{-i\delta} \\ \sin\beta\,\mathrm{e}^{i\delta} & \cos\beta \end{pmatrix}, \quad U^{-1} = \begin{pmatrix} \cos\beta & \sin\beta\,\mathrm{e}^{-i\delta} \\ -\sin\beta\,\mathrm{e}^{i\delta} & \cos\beta \end{pmatrix}. \tag{1}$$

Then at CP violation K_1^0, K_2^0 mesons have to transform into superposition states of K_S and K_L mesons:

$$\begin{aligned} K_S &= \cos\beta K_1^0 - \sin\beta K_2^0\,\mathrm{e}^{-i\delta}, \\ K_L &= \sin\beta\,\mathrm{e}^{i\delta} K_1^0 + \cos\beta K_2^0, \end{aligned} \tag{2}$$

and at inverse transformation we get

$$\begin{aligned} K_1^0 &= \cos\beta K_S + \sin\beta\,\mathrm{e}^{-i\delta} K_L, \\ K_2^0 &= -\sin\beta\,\mathrm{e}^{i\delta} K_S + \cos\beta K_L. \end{aligned} \tag{3}$$

In [13] it was shown that

$$m_2 - m_1 \simeq m_L - m_S. \tag{4}$$

If we take into account that K_S, K_L decay and have the decay widths Γ_S, Γ_L, then K_S, K_L mesons with masses m_S and m_L evolve in dependence on time according to the following formulas:

$$\begin{aligned} K_S(t) &= \exp\left(-iE_S t - \frac{\Gamma_S t}{2}\right) K_S(0), \\ K_L(t) &= \exp\left(-iE_L t - \frac{\Gamma_L t}{2}\right) K_L(0), \end{aligned} \tag{5}$$

where

$$E_k^2 = (p^2 + m_k^2), \quad k = S, L.$$

If these mesons are moving without interactions, then

$$\begin{aligned} K_1^0(t) &= \cos\beta \exp\left(-iE_S t - \frac{\Gamma_S t}{2}\right) K_S(0) + \\ &\qquad + \sin\beta\, \mathrm{e}^{-i\delta} \exp\left(-iE_L t - \frac{\Gamma_L t}{2}\right) K_L(0), \\ K_2^0(t) &= -\sin\beta\, \mathrm{e}^{i\delta} \exp\left(-iE_S t - \frac{\Gamma_S t}{2}\right) K_S(0) + \\ &\qquad + \cos\beta \exp\left(-iE_L t - \frac{\Gamma_L t}{2}\right) K_L(0). \end{aligned} \tag{6}$$

Then, putting expressions for K_S, K_L from (2) into expression (6), we get

$$\begin{aligned} K_1^0(t) &= \left[\exp(-iE_S t)\cos^2\beta + \exp(-iE_L t)\sin^2\beta\right] K_1^0(0) + \\ &\qquad + \mathrm{e}^{-i\delta}\left[-\exp(-iE_S t) + \exp(-iE_L t)\right] \sin\beta\, \cos\beta K_2^0(0), \\ K_2^0(t) &= \left[\exp(-iE_S t)\sin^2\beta + \exp(-iE_L t)\cos^2\beta\right] K_1^0(0) + \\ &\qquad + \mathrm{e}^{i\delta}\left[-\exp(-iE_S t) + \exp(-iE_L t)\right] \sin\beta\, \cos\beta K_2^0(0). \end{aligned} \tag{6'}$$

Then, using expression (6′), we get the probability that the meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson given by the following expression:

$$\begin{aligned} P(K_2^0 \to K_1^0, t) = \frac{1}{4}\cos^2\beta \sin^2 2\beta \Bigg[&\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - \\ &- 2\exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right) \cos\left((E_L - E_S)t\right)\Bigg]. \end{aligned} \tag{7}$$

If we suppose that $\cos^2\beta \simeq 1$ and $\sin^2\beta \simeq \varepsilon$, then

$$P(K_2^0 \to K_1^0, t) \simeq \varepsilon\left[\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t\right)\right] \tag{8}$$

and $P(K_2^0 \to K_1^0, t) = P(K_1^0 \to K_2^0, t)$.

Then the probability that meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_1^0 meson and back are given by the following expressions:

$$\begin{aligned} P(K_1^0 \to K_1^0) = \Bigg[&\cos^4\beta\, \mathrm{e}^{-\Gamma_S t} + \sin^4\beta\, \mathrm{e}^{-\Gamma_L t} + \\ &+ 2\sin^2\beta\cos^2\beta \exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t\right)\Bigg], \end{aligned} \tag{9}$$

further

$$P(K_1^0 \to K_1^0) \simeq \left[\mathrm{e}^{-\Gamma_S t} \varepsilon^2 \, \mathrm{e}^{-\Gamma_L t} + 2\varepsilon \exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right) \cos\left((E_L - E_S)t\right) \right], \quad (10)$$

and the probability $P(K_2^0 \to K_2^0)$ is

$$P(K_2^0 \to K_2^0) = \left[\sin^4\beta \, \mathrm{e}^{-\Gamma_S t} + \cos^4\beta \, \mathrm{e}^{-\Gamma_L t} + \right. \\ \left. + 2\sin^2\beta\cos^2\beta \exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right) \cos\left((E_L - E_S)t\right) \right], \quad (11)$$

further

$$P(K_2^0 \to K_2^0) \simeq \left[\varepsilon^2 \, \mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\varepsilon \exp\left(-\frac{(\Gamma_S + \Gamma_L)t}{2}\right) \cos\left((E_L - E_S)t\right) \right]. \quad (11')$$

In all the above expressions we have to add factor $1/2$ since it arises from the primary $K^0, \bar{K}^0$ mesons ($K^0 = (K_1^0 + K_2^0)/\sqrt{2}$, $\bar{K}^0 = (K_1^0 - K_2^0)/\sqrt{2}$).

So, from the above expressions we see that when matrix transformation is unitary the CP phase in the expressions for transition probabilities is absent. In expression (1) matrix U is unitary, i. e., $UU^{-1} = 1$. In principle, we can use the nonunitary matrix, i. e., use matrix U and for back transformation use matrix U^T instead of U^{-1} ($\det U = \det U^T = 1$), then

$$U = \begin{pmatrix} \cos\beta & -\sin\beta \, \mathrm{e}^{-i\delta} \\ \sin\beta \, \mathrm{e}^{i\delta} & \cos\beta \end{pmatrix}, \quad U^T = \begin{pmatrix} \cos\beta & \sin\beta \, \mathrm{e}^{i\delta} \\ -\sin\beta \, \mathrm{e}^{-i\delta} & \cos\beta \end{pmatrix}. \quad (12)$$

Now instead of expressions (2) and (3) we get

$$\begin{aligned} K_S &= \cos\beta K_1^0 - \sin\beta K_2^0 \, \mathrm{e}^{i\delta}, \\ K^L &= \sin\beta \, \mathrm{e}^{-i\delta} K_1^0 + \cos\beta K_2^0, \end{aligned} \quad (13)$$

$$\begin{aligned} K_1^0 &= \cos\beta K_S + \sin\beta \, \mathrm{e}^{-i\delta} K_L, \\ K_2^0 &= -\sin\beta \, \mathrm{e}^{i\delta} K_S + \cos\beta K_L. \end{aligned} \quad (14)$$

Now if mesons are moving without interactions, then

$$\begin{aligned} K_1^0(t) &= \cos\beta \, \exp\left(-iE_S t - \frac{\Gamma_S t}{2}\right) K_S(0) + \\ &\quad + \sin\beta \, \mathrm{e}^{-i\delta} \exp\left(-iE_L t - \frac{\Gamma_L t}{2}\right) K_L(0), \\ K_2^0(t) &= -\sin\beta \, \mathrm{e}^{i\delta} \exp\left(-iE_S t - \frac{\Gamma_S t}{2}\right) K_S(0) + \\ &\quad + \cos\beta \, \exp\left(-iE_L t - \frac{\Gamma_L t}{2}\right) K_L(0). \end{aligned} \quad (15)$$

Then, using expressions (15) and (13) for the probability that the meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson, we get the following expression:

$$P(K_1^0 \to K_1^0) = \Bigg[\cos^4\beta\,\mathrm{e}^{-\Gamma_S t} + \sin^4\beta\,\mathrm{e}^{-\Gamma_L t} + \\ + 2\sin^2\beta\cos^2\beta\,\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\Bigg], \quad (16)$$

or $\sin^2\beta = \varepsilon$, then

$$P(K_1^0 \to K_1^0) \simeq \left[\mathrm{e}^{-\Gamma_S t} + \varepsilon^2\,\mathrm{e}^{-\Gamma_L t} + 2\varepsilon\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\right], \quad (17)$$

and the probability of $P(K_2^0 \to K_2^0)$ transition is

$$P(K_2^0 \to K_2^0) = \Bigg[\sin^4\beta\,\mathrm{e}^{-\Gamma_S t} + \cos^4\beta\,\mathrm{e}^{-\Gamma_L t} + \\ + 2\sin^2\beta\cos^2\beta\,\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\Bigg], \quad (18)$$

or

$$P(K_2^0 \to K_2^0) \simeq \left[\varepsilon^2\,\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\varepsilon\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\right]. \quad (19)$$

Then the probability that the meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson is given by the following expression:

$$P(K_2^0 \to K_1^0, t) = \frac{1}{4}\sin^2 2\beta\Bigg[\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - \\ - 2\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\Bigg] \simeq \\ \simeq \varepsilon\left[\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\exp\left(-\frac{(\Gamma_S+\Gamma_L)t}{2}\right)\cos\left((E_L - E_S)t + 2\delta\right)\right], \quad (20)$$

and $P(K_2^0 \to K_1^0, t) = P(K_1^0 \to K_2^0, t)$ (the above expression has taken into account that $\cos^2\beta \simeq 1$, $\sin^2\beta \simeq \varepsilon$).

The length of oscillations in this case is

$$R_{LS} \cong \frac{\gamma}{2\Delta} \equiv \frac{2\pi hc\gamma}{2\Delta}, \quad (21)$$

where $\Delta = m_L - m_S$ and γ is usual relativistic factor. Expressions (12)–(20) were obtained using the standard technique of oscillations and they are analogous to the expression obtained in [11, 12] at violation of orthogonality of K_S, K_L states.

The plots of transition probabilities $K_1^0 \to K_1^0$ (expression (10) — $P(K^0, K_1^0 \to K_1^0, t) \simeq \mathrm{e}^{-t} + (0.00223)^2\,\mathrm{e}^{-t/580} + 2 \cdot 0.00223\,(\cos(0.477t - 0.752))\,\mathrm{e}^{-t(581/1160)}$) and $K_2^0 \to K_1^0$ (expression (8) — $P(K^0, K_2^0 \to K_1^0, t) \simeq 0.00223(\mathrm{e}^{-t} + \mathrm{e}^{-t/580} - 2 \cdot (\cos(0.477t - 0.752))\,\mathrm{e}^{-t(581/1160)}))$ in dependence on $t_S = t/\tau_S$ (τ_S is K_S lifetime) are given in Fig. 1 (where $\varepsilon = 0.00223$ [14]). The summary plot of expressions (8) and (10) (line) normalized to the experimental data from [14] together with experimental data from [14] (open circles) is given in Fig. 2 (for primary K^0 mesons). From this figure we see that the total transition probability to K_1^0 obtained in the framework of oscillations theory are placed very far from

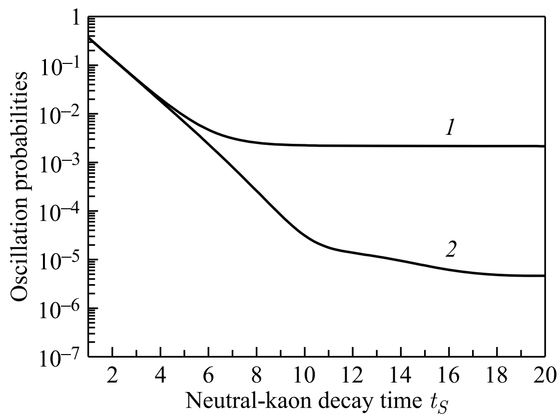

Fig. 1. $K_2^0 \to K_1^0$ transition probability (line *1*, expression (8)) and $K_1^0 \to K_1^0$ transition probability (line *2*, expression (10)) in the presence of oscillations at CP violation in weak interactions ($\varepsilon = 0.00223$) in dependence on t_S for $t_S = t/\tau_S = 1{-}20$

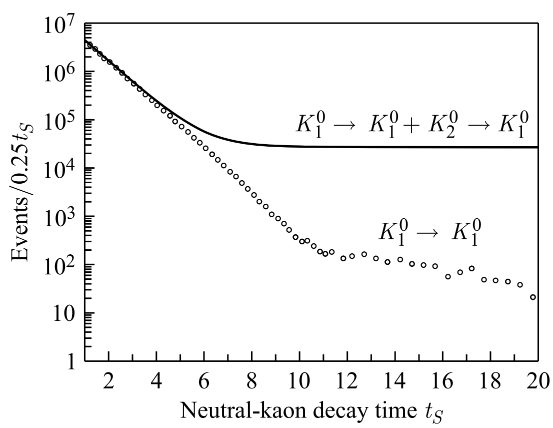

Fig. 2. Summary transition probabilities $(K_1^0 \to K_1^0)+(K_2^0 \to K_1^0)$ (line) when oscillations take place (exprsessions (8)+(10)) normalized to experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data (open circles) from [14] for $t_S = 1{-}20$

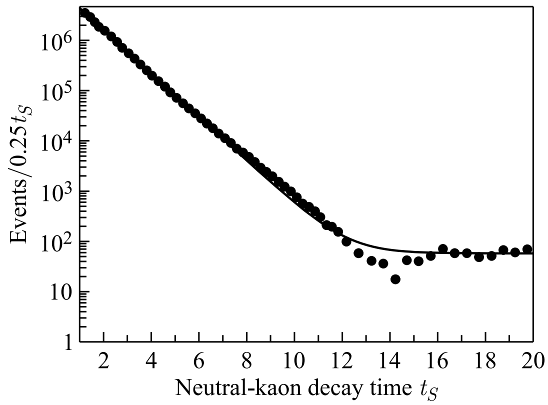

Fig. 3. Summary transition probabilities $(K_1^0 \to K_1^0)+(K_2^0 \to K_1^0)$ (line) when oscillations take place (expressions (8)+(10)) normalized to experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 4.97 \cdot 10^{-6}$) and experimental data (solid circles) from [14] for $t_S = 1{-}20$

experimental data from [14]. Then we can come to the conclusion that at CP violation in weak interactions oscillations do not arise. In reality, when drawing Figs. 1 and 2 it was taken into account that there is phase $\delta = 44°$ (i.e., we used expressions (17) and (20)).

Now we can consider the case when $\varepsilon' = \varepsilon^2 = 4.97 \cdot 10^{-6}$, then

$$P(K^0, \bar{K}^0, K_1^0 \to K_1^0, t) = \exp(-t) + 0.00000497(\exp(-t)+ \\ + \exp(-t/580) \pm 2(\cos(0.477t - 0.752))\exp(-0.500862t)). \quad (22)$$

Figure 3 presents the line obtained by using the above expression which is normalized to the experimental data from [14] at $t_S = 1.22$ and experimental data from [14] for $P(\bar{K}^0, K_1^0 \to K_1^0, t \equiv t_S)$. We see that in this case the interference term which is present in the experimental data is absent. We can make the conclusion that oscillations in this case do not occur either.

We now come to the consideration of the case when oscillations between K_1^0-, K_2^0-meson states do not arise at CP violation.

2. THE CASE WHEN OSCILLATIONS BETWEEN K_1^0-, K_2^0-MESON STATES DO NOT ARISE AT CP VIOLATION

Above we considered the case when at CP violation there can arise oscillations. Now we are considering the case when superposition states arise but there are no oscillations. It arises when the condition for realization of K-meson oscillations cannot be realized. Here an analogue with Cabibbo [17] mixing matrix takes place with one exclusion, namely, since masses of π and K mesons differ very much, the interference between these states in contrast to K_S-, K_L-meson states cannot arise (by the way, in full analogy with the Cabibbo case we could use below the old K_1^0-, K_2^0-meson states instead of using the new K_S, K_L states).

We know that the parameter of CP violation is very small. Then new states $K_1' = \cos\beta K_S + \sin\beta K_L$ and $K_2' = -\sin\beta K_S + \cos\beta K_L$ are equivalent to K_1^0, K_2^0 states ($\cos^2\beta +$

$\sin^2\beta = 1$), where K_S, K_L states are states which arise at small violation of CP parity. They are not orthogonal but normalized quantum mechanic functions of state ($K_S(0) = 1, K_L(0) = 1$, $|K_1^0(0)|^2 + |K_2^0(0)|^2 = |K_S(0)|^2 + |K_L(0)|^2$). Then

$$\begin{gathered} |K_1^0|^2 \equiv |K_1'|^2 = |\cos\beta K_S + \sin\beta K_L|^2, \\ |K_2^0|^2 \equiv |K_2'|^2 = |-\sin\beta K_S + \cos\beta K_L|^2, \\ |K_1' K_2'| \simeq 0. \end{gathered} \tag{23}$$

As we see, in this case instead of oscillations we get interferences between K_S and K_L states. It is of interest to rewrite the above expressions taking into account time dependence. Then taking into account that the standard expressions for $K_S(t)$ and $K_L(t)$ have the following form:

$$K_S(t) = \exp\left(-iE_S t - \frac{1}{2}\Gamma_S t\right), \quad K_L(t) = \exp\left(-iE_L t - \frac{1}{2}\Gamma_L t\right), \tag{24}$$

and putting expressions (24) into (23) for a primary K^0 meson, we get expressions for probabilities $P(K_1^0 \to K_1^0, t)$ and $P(K_2^0 \to K_2^0, t)$:

$$\begin{gathered} P(K_1^0 \to K_1^0, t) = |K_1^0(t)|^2 = \cos^2\beta \exp(-\Gamma_S t) + \sin^2\beta \exp(-\Gamma_L t) + \\ + 2\sin\beta\cos\beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t, \\ P(K_2^0 \to K_2^0, t) = |K_2^0|^2 = \sin^2\beta \exp(-\Gamma_S t) + \cos^2\beta \exp(-\Gamma_L t) - \\ - 2\sin\beta\cos\beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t, \\ |K_1^0 K_2^0| \simeq 0. \end{gathered} \tag{25}$$

Since $K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0)$, for the case of a K^0 meson the expressions (25) in normalized form get the following form:

$$\begin{gathered} P(K^0, K_1^0 \to K_1^0, t) = |K_1^0(t)|^2 = \frac{1}{2}\Bigg[\cos^2\beta \exp(-\Gamma_S t) + \sin^2\beta \exp(-\Gamma_L t) + \\ + 2\sin\beta\cos\beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t\Bigg], \\ |K_2^0|^2 = \frac{1}{2}\Bigg[\sin^2\beta \exp(-\Gamma_S t) + \cos^2\beta \exp(-\Gamma_L t) - \\ - 2\sin\beta\cos\beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t\Bigg], \\ |K_1^0 K_2^0| \simeq 0. \end{gathered} \tag{26}$$

For the case of a $\bar{K}^0$ meson we have

$$\begin{gathered} |K_1^0|^2 = |\cos\beta K_S - \sin\beta K_L|^2, \\ |K_2^0|^2 = |\sin\beta K_S + \cos\beta K_L|^2, \\ |K_1^0 K_2^0| \simeq 0. \end{gathered} \tag{27}$$

Using expressions (24) for normalized case, we then get

$$
\begin{gathered}
P(K^0, K_1^0 \to K_1^0, t) = |K_1^0(t)|^2 = \frac{1}{2}\Bigg[\cos^2\beta\,\exp(-\Gamma_S t) + \sin^2\beta\,\exp(-\Gamma_L t) - \\
- 2\sin\beta\cos\beta\,\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t\Bigg], \\
P(\bar{K}^0, K_2^0 \to K_2^0, t) = |K_2^0|^2 = \frac{1}{2}\Bigg[\sin^2\beta\,\exp(-\Gamma_S t) + \cos^2\beta\,\exp(-\Gamma_L t) + \\
+ 2\sin\beta\cos\beta\,\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos(E_L - E_S)t\Bigg], \\
|K_1^0 K_2^0| \simeq 0.
\end{gathered} \tag{28}
$$

So, we have obtained the above expressions without the renormalization of states by hand and without using nonunitary matrix for transformation, in contrast to [11].

Of interest is the case when in expressions (23) a supplementary CP phase is present. If this phase appears in the unitary form as is in [15] in the form of [16]

$$
U = \begin{pmatrix} \cos\beta & \sin\beta\,\mathrm{e}^{-i\delta} \\ -\sin\beta\,\mathrm{e}^{i\delta} & \cos\beta \end{pmatrix}, \tag{29}
$$

then in the case of K^0 meson instead of expressions (25) in the case of K^0 meson we obtain

$$
\begin{gathered}
P(K^0, K_1^0 \to K_1^0, t) = |K_1^0(t)|^2 = \frac{1}{2}\Bigg[\cos^2\beta\exp(-\Gamma_S t) + \sin^2\beta\exp(-\Gamma_L t) + \\
+ 2\sin\beta\cos\beta\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) + \delta)t\Bigg], \\
P(K^0, K_2^0 \to K_2^0, t) = |K_2^0|^2 = \frac{1}{2}\Bigg[\sin^2\beta\exp(-\Gamma_S t) + \cos^2\beta\exp(-\Gamma_L t) - \\
- 2\sin\beta\cos\beta\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) - \delta)t\Bigg],
\end{gathered} \tag{30}
$$

$$
\begin{gathered}
P(K^0, K_2^0 \to K_2^0, t) = |K_1^0(t)|^2 \simeq \frac{1}{2}\Bigg[\exp(-\Gamma_S t) + \varepsilon^2\exp(-\Gamma_L t) + \\
+ 2\varepsilon\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) - \delta)t\Bigg],
\end{gathered} \tag{31}
$$

and in the case of $\bar{K}^0$ meson instead of expressions (26) we obtain

$$
\begin{gathered}
P(\bar{K}^0, K_1^0 \to K_1^0, t) = |K_1^0(t)|^2 = \frac{1}{2}\Bigg[\cos^2\beta\exp(-\Gamma_S t) + \sin^2\beta\exp(-\Gamma_L t) - \\
- 2\sin\beta\cos\beta\exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) + \delta)t\Bigg],
\end{gathered}
$$

$$P(\bar{K}^0, K_2^0 \to K_2^0, t) = |K_2^0|^2 = \frac{1}{2}\Bigg[\sin^2\beta \exp(-\Gamma_S t) + \cos^2\beta \exp(-\Gamma_L t) + \\ + 2\sin\beta\cos\beta \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) - \delta)t\Bigg], \qquad (32)$$

$$|K_1^0(t)|^2 \simeq \frac{1}{2}\Bigg[\exp(-\Gamma_S t) + \varepsilon^2 \exp(-\Gamma_L t) - \\ - 2\varepsilon \exp\left(\frac{1}{2}(\Gamma_S + \Gamma_L)t\right)\cos((E_L - E_S) - \delta)t\Bigg], \qquad (33)$$

where, using the existing experimental data [14], we can write that the value for $\sin\beta$ is about $\sin\beta = \varepsilon \cong 2.23 \cdot 10^{-3}$.

Figure 4 gives a plot of functions (31) — $P(K^0, K_1^0 \to K_1^0, t) \simeq \mathrm{e}^{-t} + (0.00223)^2\,\mathrm{e}^{-t/580} + 2 \cdot 0.00223(\cos(0.477t - 0.752))\,\mathrm{e}^{-t(581/1160)}$ normalized to the experimental data from [14] at $t_S = 1.22$ together with experimental data from [14] for $t_S = 1{-}20$ ($t_S = t/\tau_S$, τ_S is K_S-meson lifetime).

Figure 5 gives a plot of functions (33) — $P(\bar{K}^0, K_1^0 \to K_1^0, t) \simeq \mathrm{e}^{-t} + (0.00223)^2\,\mathrm{e}^{-t/580} - 2 \cdot 0.00223(\cos(0.477t - 0.752))\,\mathrm{e}^{-t(581/1160)}$ normalized to the experimental data from [14] at $t_S = 1.22$ together with experimental data from [14] for $t_S = 1{-}20$ ($t_S = t/\tau_S$, τ_S is K_S-meson lifetime).

We see that the curves from expressions (31) and (33) are in quite satisfactory agreement with the experimental data obtained in [14] at $\varepsilon \cong 2.23 \cdot 10^{-3}$.

By the way, the signs of the additional CP phase in our approach are different for K_1^0 and K_2^0 mesons, in contrast to [11] where there was used nonunitary matrix transformation in the case of CP violation. The question now arises: what mechanism works at CP violation? If it is possible to determine this sign in experiment for a K_2^0 meson, then we can obtain the

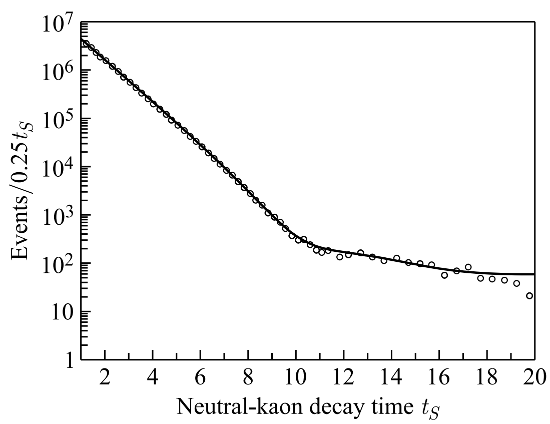

Fig. 4. Transition probabilities of primary K^0 mesons into K_S ($P(K^0, K_1^0 \to K_S, t)$, expression (31)) normalized to the experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data (open circles) from [14] for $t_S = 1{-}20$

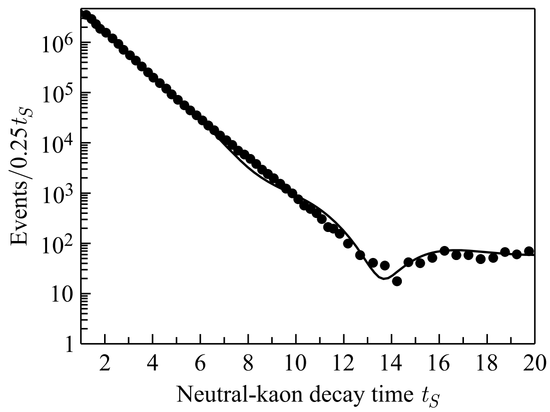

Fig. 5. Transition probabilities of primary K^0 mesons into K_S ($P(\bar{K}^0, K_1^0 \to K_S, t)$, expression (33)) normalized to the experimental data from [14] at $t_S = 1.22$ ($\varepsilon = 0.00223$) and experimental data (solid circles) from [14] for $t_S = 1-20$

answer to this question. If we use nonunitary matrix instead of unitary matrix (29)

$$U = \begin{pmatrix} \cos\beta & \sin\beta\, e^{-i\delta} \\ -\sin\beta\, e^{-i\delta} & \cos\beta \end{pmatrix}, \tag{34}$$

then for K^0 and $\bar{K}^0$ transition probabilities we obtain the same expressions as in [11].

So, as stressed above, the expressions for transition probabilities (31), (33) are in good agreement with the experimental data from [14]. From expressions (31), (33) and Figs. 3, 4 we can then come to the conclusion that at CP violation in weak interactions the standard theory of oscillations is not realized. There takes place only interference between K_S- and K_L-meson states.

At CP violation in weak interactions the mixing states of K_S, K_L mesons arise with very small angle mixing. These states are not orthogonal states. That is, there is an analogy with Cabibbo matrix mixing [17] at π-, K-meson mixings with one distinction: there arises interference between these states since the masses of these states are very close. Then we can in principle not introduce new K_S, K_L states and use the old K_1^0-, K_2^0-meson states, as was done in the case of π, K mesons (or for d, s quarks).

CONCLUSIONS

In this work, we have considered two approaches for description of K^0-, $\bar{K}^0$-meson transitions into K_1^0 mesons at CP violation in weak interactions. The first approach uses the standard theory of oscillations and the second approach supposes that (K_S, K_L) states which arise at CP violation are normalized but not orthogonal state functions, then between these states there arise interferences but not oscillations.

In the presence of oscillations the probability of K^0-, $\bar{K}^0$-meson transition into K_1^0 mesons is proportional to $\sin^2\beta = \varepsilon = 2.23 \cdot 10^{-3}$ and at long distances oscillations occur. In the second case there arises an interference term between K_S- and K_L-meson states. This term is proportional to $\sin\beta = 2.23 \cdot 10^{-3}$ and it disappears at big distances. And at big distances there is a term which is proportional to $\sin^2\beta = \varepsilon^2$. As stressed above, the available experimental data [14] are in good agreement with the second approach. So, we have come to the conclusion that at CP violation in weak interaction in the system of K^0 mesons oscillations do not arise. There takes place only interference between K_S- and K_L-meson states.

Why do oscillations not arise at CP violation? As we can see from Figs. 4 and 5, CP violation becomes apparent at $t_S > 8$. Then short-lived states K_1^0 have time to decay and mainly long-lived K_2^0 states remain which transform into K_S, K_L superposition. And further we see interference between these states.

REFERENCES

1. *Gell-Mann M., Pais A.* Behavior of Neutral Particles under Charge Conjugation // Phys. Rev. 1955. V. 97. P. 1387;
Pais A., Piccioni O. How to Verify Experimentally a Recent Theoretical Suggestion that the K^0 Meson is a "Particle Mixture" // Ibid. V. 100. P. 1487;
Okun L. B. Weak Intreactions of Elementary Particles. M.: Fizmatizdat, 1963.
2. *Treiman S. B., Sachs R. S.* Alternate Modes of Decay of Neutral K Mesons // Phys. Rev. 1956. V. 103. P. 1545.
3. *Beshtoev Kh. M.* Some Remarks on the Problem of Neutrino Oscillation in Vacuum and Matter // Nuovo Cim. A. 1995. V. 168. P. 275.
4. *Beshtoev Kh. M.* To the Theory of Neutrino Oscillation in Matter // JINR Rapid Commun. 1995. No. 3[71]; The Intern. Symp. on Weak and Electromagnetic Interactions in Nuclei, Osaka, Japan, 1995. P. 15.
5. *Beshtoev Kh. M.* Enhancement of the Oscillations of Neutrinos Different Masses in Matter // Proc. of the 24th Intern. Cosmic Ray Conf., Rome, 1995. V. 4. P. 1237.
6. *Beshtoev Kh. M.* Enhancement of the Oscillations of Neutrinos of Different Masses in Matter // Proc. of the 4th Intern. School "Particles and Cosmology", Baksan, 1995. Singapore: World Sci., 1995. P. 290.
7. *Lee T. D., Yang C. N.* Question of Parity Conservation in Weak Interactions // Phys. Rev. 1956. V. 104. P. 254.
8. *Wu C. S. et al.* Experimental Test of Parity Conservation in Beta Decay // Phys. Rev. 1957. V. 105. P. 1413; V. 106. P. 1361.
9. *Landau L. D.* About Conserving Laws at Weak Interactions // Sov. Phys. JETP. 1957. V. 32. P. 405.
10. *Christenson J. H. et al.* Evidence for Constructive Interference between Coherently Regenerated and CP-Nonconserving Amplitudes // Phys. Rev. Lett. 1964. V. 13. P. 138.
11. *Wu T. T., Yang C. N.* Phenomenological Analysis of Violation of CP Invariance in Decay of K and $\bar{K}$ // Ibid. P. 380.
12. *Commins E. D., Bucksbaum P. H.* Weak Interactions of Leptons and Quarks. Cambridge, 1983.
13. *Beshtoev Kh. M.* About Oscillations in the System of K^0 Mesons. JINR Commun. E2-2011-48. Dubna, 2011.
14. *Apostolakis A. et al.* A Determination of the CP Violation Parameter η_{+-} from the Decay of Strangeness-Tagged Neutral Kaons // Phys. Lett. B. 1999. V. 458. P. 545; Phys. Lett. B. Rev. Part. Phys. 2008. V. 667. P. 44.
15. *Kobayashi M., Maskawa K.* CP Violation in the Renormalizable Theory of Weak Interaction // Prog. Theor. Phys. 1973. V. 49. P. 652.
16. *Maiani L.* Weak Mixing, CP Violation and the Standard Model // Proc. of the Intern. Symp. on Lepton–Photon Interaction, Hamburg, DESY, 1977. P. 867; Phys. Lett. B. 1976. V. 62. P. 183.
17. *Cabibbo N.* Unitary Symmetry and Leptonic Decays // Phys. Rev. Lett. 1963. V. 10. P. 531.

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