Письма в ЭЧАЯ. 2016. Т. 13, № 2(200). С. 242–254

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ULTRALIGHT GLUEBALLS IN QUARK–GLUON PLASMA

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We consider the dynamics of the scalar and pseudoscalar glueballs in quark–gluon plasma (QGP). By using the instanton model for the QCD vacuum, we give the arguments that the nonperturbative gluon–gluon interaction is qualitatively different in the confinement and deconfinement phases. Based on this observation it is shown that above T_c the values of the scalar and pseudoscalar glueball masses might be very small. The estimation of the temperature of scale invariance restoration, at which the scalar glueballs become massless, is given. We also discuss the Bose–Einstein condensation of the glueballs and the superfluidity of the glueball matter in QGP.

Рассматривается динамика скалярных и псевдоскалярных глюболов в кварк-глюонной плазме (КГП). С использованием инстантонной модели вакуума КХД показано, что непертурбативное глюон-глюонное взаимодействие количественно различается в фазах конфайнмента и деконфайнмента. На основании этого показывается, что при температурах выше T_c разница масс скалярных и псевдоскалярных глюболов может быть мала. Сделана оценка температуры восстановления киральной симметрии, при которой скалярные глюболы становятся безмассовыми. Также обсуждается конденсация Бозе–Эйнштейна для глюболов и сверхтекучесть глюбольной материи в КГП.

PACS: 12.38.Mh

INTRODUCTION

In spite of the tremendous experimental and theoretical efforts in the investigation of the properties of quark–gluon plasma (QGP) at large temperatures and densities, it is not clear so far what fundamental QCD mechanism leads to the unusual behavior of the matter produced in heavy-ion collisions at high energies at the RHIC and LHC. Indeed, there is strong evidence that even above T_c nonperturbative QCD effects are very important and, in particular, they are responsible for the phenomenon of the so-called strongly-interacting quark–gluon plasma (sQGP) [1] observed at the RHIC and LHC. One of the important outputs of the lattice QCD calculation at finite T is the fact that the values of the deconfinement and chiral restoration temperatures are approximately equal. That means that above T_c one cannot expect the pion to be the Goldstone boson with zero mass in the chiral limit. Instead, a rather massive ($M_\pi \approx 2M_q(T_c) \approx 6\,T_c \sim 1$ GeV) pion state appears above $T_c \approx 150$ MeV. Therefore, this lowest mass quark–antiquark state cannot give a significant contribution to the Equation of State (EoS) of QGP.

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One of the fundamental issues of QCD as the theory of strong interactions is the understanding of the role of gluonic degrees of freedom in the confinement and deconfinement regimes. Thus, it is known that at low temperatures gluons play an important role not only in the dynamics of usual hadrons. In particular, they can form bound states called "glueballs" (see review [2]). However, one cannot expect a significant contribution of glueballs at $T < T_c$ to EoS of the hadron gas due to their large masses $M_G \gg T$. On the other hand, it was found by lattice calculation [3,4] that unlike the quark condensate, the gluon condensate does not vanish at $T > T_c$. We should stress that this condensate plays a fundamental role in the formation of the bound glueball states. This role is similar to that of the quark condensate in the appearance of the massive mesons and baryons built of the light u, d, and s quarks. Therefore, a nonzero value of the gluon condensate above T_c is a strong signal of the existence of the glueballs in the deconfinement phase.

However, it is evident that the properties of the glueballs, in particular, their masses and sizes should change in QGP due to the temperature dependence of the gluon condensate [4,5] and the change of the topological structure of the QCD vacuum at $T > T_c$ [5, 6]. The attempts to estimate the value of the glueball masses at finite T were done in different models in [16–20] and on the lattice [21, 22].

In this paper, we consider the lowest scalar and pseudoscalar glueball states at finite T in the QGP environment within the effective model based on the instanton picture for the QCD vacuum. It is shown that their masses strongly decrease above T_c and become very small at the temperature of the scale invariance restoration $T_{\text{scale}} \approx 1$ GeV. The possibility of the Bose–Einstein condensation and superfluidity of the glueball matter is under discussion.

1. NONPERTURBATIVE QUARK–QUARK, QUARK–GLUON, AND GLUON–GLUON INTERACTIONS BELOW AND ABOVE T_c INDUCED BY INSTANTONS

It is expected that the instantons, a strong vacuum fluctuation of gluon fields, play a very important role in the dynamics of the glueballs below T_c (see review [23]). The effective interaction induced by the instanton between quarks at $T = 0$ is well-known. This is a famous 't Hooft interaction, which for $N_f = 3$ (Fig. 1, a) and $N_c = 3$ is given by the formula [26][1]:

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{(3)} = \int d\rho\, n(\rho) \Bigg\{ \prod_{i=u,d,s} \left(m_i^{\text{cur}}\rho - \frac{4\pi}{3}\rho^3 \bar{q}_{iR} q_{iL} \right) + \\
+ \frac{3}{32}\left(\frac{4}{3}\pi^2\rho^3\right)^2 \Bigg[\left(j_u^a j_d^a - \frac{3}{4} j_{u\mu\nu}^a j_{d\mu\nu}^a \right) \left(m_s^{\text{cur}}\rho - \frac{4}{3}\pi^2\rho^3 \bar{q}_{SR} q_{sL} \right) + \\
+ \frac{9}{40}\left(\frac{4}{3}\pi^2\rho^3\right)^2 d^{abc} j_{u\mu\nu}^a j_{d\mu\nu}^b j_s^c + \text{perm.} \Bigg] + \frac{9}{320}\left(\frac{4}{3}\pi^2\rho^3\right)^3 d^{abc} j_u^a j_d^b j_s^c + \\
+ \frac{i f^{abc}}{256}\left(\frac{4}{3}\pi^2\rho^3\right)^3 j_{u\mu\nu}^a j_{d\nu\lambda}^b j_{s\lambda\mu}^c + (R \leftrightarrow L) \Bigg\}, \quad (1)
\end{aligned}$$

[1]In this section, all Lagrangians are written in the Euclidian space-time.

where $m_i^{\rm cur}$ is the quark current mass, $q_{R,L} = (1 \pm \gamma_5)q(x)/2$, $j_i^a = \bar{q}_{iR}\lambda^a q_{iL}$, $j_{i\mu\nu}^a = \bar{q}_{iR}\sigma_{\mu\nu}\lambda^a q_{iL}$, ρ is the instanton size, and $n(\rho)$ is the density of instantons. For $N_f = 2$, the 't Hooft interaction for zero current quark mass is much simpler:

$$\mathcal{L}_{\rm eff}^{(N_f=2)} = \int d\rho\, n(\rho) \left(\frac{3}{4}\pi^2\rho^3\right)^2 \bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL} \times \\ \times \left[1 + \frac{3}{32}\lambda_u^a\lambda_d^a + \frac{9}{32}\boldsymbol{\sigma}_u \cdot \boldsymbol{\sigma}_d\lambda_u^a\lambda_d^a\right] + (R \leftrightarrow L). \quad (2)$$

This Lagrangian can be obtained from Eq. (1) by connecting the strange quark legs through the quark condensate. The two Lagrangians, Eqs. (1) and (2), are considered as the bases for different versions of the Nambu–Jona-Lasinio (NJL) models [24].

The effective quark–gluon interaction induced by the instantons is (see, for example, [27, 28])

$$\mathcal{L}_{\rm eff} = \int dU\, d\rho\, n(\rho) \prod_q - 2\pi^2\rho^3\bar{q}_R \left(1 + \frac{i}{4}U_{ab}\tau^a\bar{\eta}_{b\mu\nu}\sigma_{\mu\nu}\right) q_L \times \\ \times \exp\left(-\frac{2\pi^2}{g_s}\rho^2 U_{cd}\bar{\eta}_{d\alpha\beta}G_{\alpha\beta}^c\right) + (R \leftrightarrow L), \quad (3)$$

where U is the orientation matrix of the instanton in the $SU(3)_c$ color space. From this Lagrangian one can obtain the interaction between quarks and gluons which contributes to the mixing between scalar and pseudoscalar glueballs, quarkoniums, and tetraquark states:

$$\mathcal{L}_{gq\bar{q}} = \int d\rho \frac{\pi^3\rho^4 n(\rho)}{8\alpha_s(\rho)}[(G^2 - G\widetilde{G})\mathcal{L}_{f,I} + (G^2 + G\widetilde{G})\mathcal{L}_{f,A}], \quad (4)$$

where

$$G^2 \equiv G_{\mu\nu}^a G_{\mu\nu}^a, \quad G\widetilde{G} \equiv G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a, \quad (5)$$

where $\widetilde{G}_{\mu\nu}^a(x) = 1/2\epsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}^a(x)$, $\mathcal{L}_{f,I}$ and $\mathcal{L}_{f,A}$ are the 't Hooft interaction induced by the instanton and anti-instanton, respectively.

The interaction between gluons in the scalar and pseudoscalar channels induced by the instanton is

$$\mathcal{L}_{gg} = \frac{\pi^6}{80}\int d\rho \frac{n(\rho)\rho^8}{\alpha_s^2(\rho)}[G_{\mu\nu}^a G_{\mu\nu}^a G_{\sigma\tau}^b G_{\sigma\tau}^b + G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a G_{\sigma\tau}^b \widetilde{G}_{\sigma\tau}^b]. \quad (6)$$

Due to factor $2\pi^2/g_s$ in Eq. (3), each two gluon legs produce a large enhancement factor π^6/α_s^2 in the gluon–gluon interaction induced by instantons. So one can expect that this type of interaction is much stronger in comparison with the quark–quark case.

We would like to emphasize that the instanton-induced interaction is very sensitive to the parity of the glueball and quarkonium states and, in particular, it leads to the mass splitting between scalar and pseudoscalar glueballs [29, 37]. Indeed, the single-instanton contribution to the difference of two correlators of glueball currents with opposite parities is given by

$$\Delta\Pi(Q^2)_G = i\int d^4x\, \mathrm{e}^{iqx}(\langle 0|TO_S(x)O_S(0)|0\rangle - \langle 0|T|O_P(x)O_P(0)|0\rangle) = \\ = 2^6\pi^2\int d\rho\, n(\rho)(\rho Q)^4 {K_2}^2(\rho Q), \quad (7)$$

where the glueball currents for the scalar and pseudoscalar states are the following:

$$O_S(x) = \alpha_s G^a_{\mu\nu}(x) G^a_{\mu\nu}(x), \quad (8)$$

$$O_P(x) = \alpha_s G^a_{\mu\nu}(x) \widetilde{G}^a_{\mu\nu}(x). \quad (9)$$

At $T < T_c$, the main contribution to the quark–quark and gluon–gluon interactions comes from disordered instantons, Fig. 1, in the so-called "random" phase of the instanton liquid. The phase provides spontaneous chiral symmetry breaking in QCD (see reviews [23, 24]). In this case, the chiral symmetric contribution, which is related to the correlated instanton–anti-instanton molecules, Fig. 2, is expected to be small due to the small packing fraction of instantons in the QCD vacuum [23]. Above T_c, where the chiral and $U_A(1)$ symmetries are restored, the situation is opposite. Indeed, in this case the contribution from the chirality violated interaction, Fig. 1, is proportional to the product of the current quark masses and should be small. In the chiral symmetric phase, the leading contribution comes from the strongly correlated instanton–anti-instanton molecules, Fig. 2, as supported by the calculations

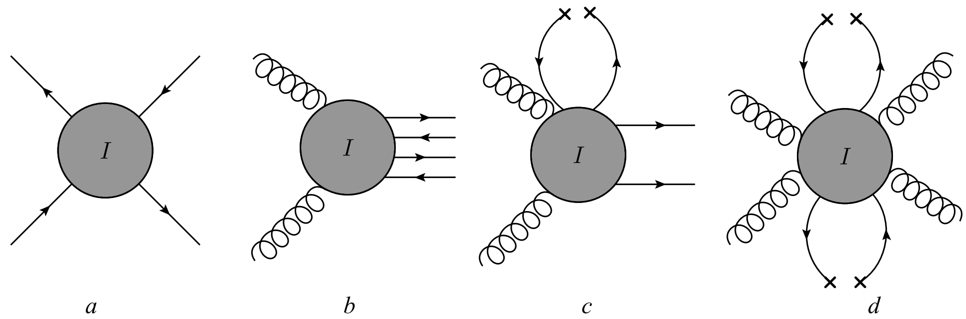

Fig. 1. Quark–quark (*a*), quark–gluon (*b*, *c*), and gluon–gluon interactions (*d*) induced by the instanton for the $N_f = 2$ case. Symbol $\times$ means connection through the quark condensate or by the current quark mass

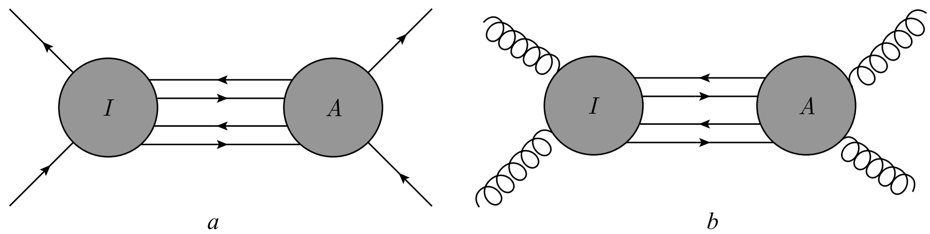

Fig. 2. Quark–quark (*a*) and gluon–gluon interactions (*b*) induced by instanton–anti-instanton molecule

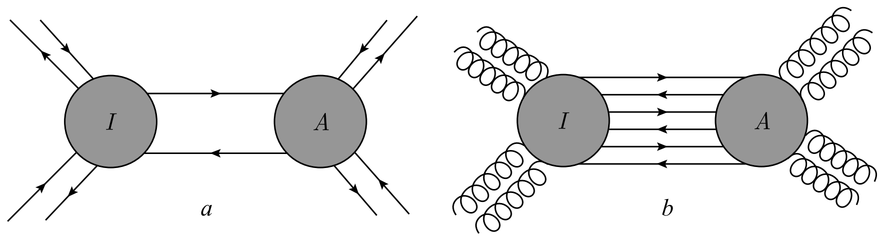

Fig. 3. Eight-quark (*a*) and eight-gluon interactions (*b*) induced by instanton–anti-instanton molecule for the $N_f = 3$ case

in [25]. The contribution from the eight-quark interaction presented in Fig. 3, *a* does not give a significant contribution to the hadron spectroscopy but it is important for the stability of the vacuum of the NJL model with the instanton-induced interaction [30, 31].

The effects of quark–quark interaction induced by instanton–anti-instanton molecules near T_c were studied in [32, 33]. It was shown that they might give a significant contribution to the binding energy of the quark–antiquark states in the deconfinement phase. The main effect, in comparison with the zero temperature case, comes from the strong polarization of instanton–anti-instanton molecules in the time direction at high temperature.

2. LOWEST MASS SCALAR GLUEBALL ABOVE T_c

At $T = 0$, there are several scalar meson states: $f_0(600)$, $f_0(980)$, $f_0(1500)$, and $f_0(1710)$ with a possible admixture of gluons [2][1]. Unfortunately, even the lattice calculation, which is based on the first principles of QCD, cannot give an accurate prediction for the scalar glueball masses with unquenched quarks. The main problem here is in the large contribution coming from the so-called disconnected diagrams [10]. In particular, they might also be responsible for the strong quarkonium–glueball mixing. Within the instanton model such mixing was shown to be large in different approaches in [11, 12]. Therefore, it is quite difficult, even impossible to find a pure glueball state at $T = 0$. Several arguments based on the analysis of sigma meson contribution to different reactions were given to consider the lowest mass scalar sigma meson $f_0(600)$ state as the state with a large mixture of the glueball [7, 8, 13]. In the line of these studies, we will also assume that at $T = 0$ the lightest glueball state is the sigma meson state with the mass $m_\sigma \approx 450$ MeV. It is also well-known that the masses of the hadron states are changing only a little in the interval $0 < T < T_c$ due to a small change of the values of quark and gluon condensates in this region. Therefore, the mass of sigma should be around 450 MeV at $T \approx T_c$. It has been discussed above that in the chiral symmetric phase above T_c the main contribution to the gluon–gluon interaction is related to the formation of the instanton–anti-instanton molecules. The example of these interactions is presented in Figs. 2, *b* and 3, *b*. Similarly to the case of the NJL model with the eight-quark interaction based on the instanton-induced multiquark interaction [30, 31], the effective Lagrangian in the gluon sector has the form

$$\mathcal{L}(\Phi) = \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi), \tag{10}$$

[1]We do not include in this list the $f_0(1370)$ state due to its controversial situation in the experiment (see the recent discussion in [9]).

where

$$V_0(\Phi) = -\frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4, \tag{11}$$

and Φ is now the scalar glueball field.

Due to the scale anomaly in QCD, the trace energy momentum tensor is not zero but it is proportional to the value of the gluon condensate

$$T^\mu{}_\mu = -\frac{9}{8}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle. \tag{12}$$

By using the parton–hadron duality principle, we obtain the relation between the gluon condensate in QCD and the minimum of the potential, Eq. (11), in the glueball sector

$$T^\mu{}_\mu = 2 \cdot 4V(\Phi_0),$$

where additional factor 2 is coming from the pseudoscalar glueball contribution to the $T_\mu{}^\mu$ (see below).

To estimate the coupling λ, we use the relation

$$\lambda = \frac{4\pi m_0^4}{9\langle \alpha_s G^2 \rangle}$$

and the value of the gluon condensate [14]

$$\langle \alpha_s G^2 \rangle \approx 0.07 \text{ GeV}^4.$$

For the mass of the glueball at $T \approx T_c$ $m_0 \approx 450$ MeV, we have $\lambda = 0.82$. Now we are in a position to calculate the temperature dependence of the scalar glueball mass above T_c. At finite temperature, the effective potential in the one-loop approximation for the $\lambda\Phi^4$ theory is given (see, for example, [15])

$$V(\Phi(T)) = V_0(\Phi(T)) + \frac{3\lambda\Phi(T)^2}{4\pi^2} \int \frac{k^2 dk}{\sqrt{k^2 + m_0^2}(\exp(\sqrt{k^2 + m_0^2}/T) - 1)}. \tag{13}$$

By using the condition of the minimum of $V(\Phi(T))$

$$\frac{\delta V(\Phi(T))}{\delta \Phi(T)} = 0 \tag{14}$$

and the definition for the mass of the glueball

$$\frac{\delta^2 V(\Phi(T))}{\delta \Phi(T)^2} = m_\Phi^2(T), \tag{15}$$

we obtain

$$m_\Phi^2(T) = m_0^2 - \frac{3\lambda}{\pi^2} \int \frac{k^2 dk}{\sqrt{k^2 + m_0^2}(\exp(\sqrt{k^2 + m_0^2}/T) - 1)} \tag{16}$$

and

$$m_\Phi^2(T) = 2\lambda \Phi_{\min}^2(T). \tag{17}$$

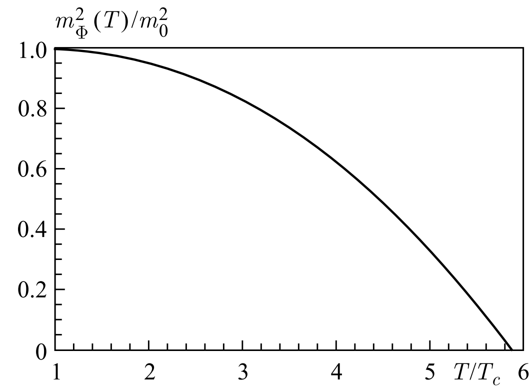

Fig. 4. Ratio of $m_\Phi^2(T)/m_0^2$ as a function of T/T_c

For the trace anomaly, we obtain

$$T^\mu{}_\mu = 4V(\Phi_{\min}(T)) = -\lambda\Phi^4_{\min}(T) = -\frac{m^4_\Phi(T)}{4\lambda}.$$

The temperature dependence of the gluon condensate in this model is given by the formula

$$\left\langle \frac{\alpha_s}{\pi} G^2(T) \right\rangle = \frac{4m^4_\Phi(T)}{9\lambda}. \tag{18}$$

In Fig. 4, the temperature dependence of the mass of the scalar glueball is presented. So one can see that at $T_{\rm scale} \approx 6\,T_c \approx 0.9$ GeV the mass of the scalar glueball vanishes and the scale invariance is restored. This can be treated as the appearance of the massless dilaton field in the limit $T^\mu{}_\mu \to 0$ (see the recent discussion of the dilaton in [43]).

We should emphasize that the justification of the use of the point-like effective interaction in Eq. (11) can be related to a very small size of the scalar glueball in QGP. Even at $T = 0$, due to the strong attraction between gluons induced by the instantons, the size of the scalar glueball is very small, $R_g \approx 2/3\rho_c \approx 0.2$ fm, where $\rho_c \approx 0.3$ fm is the average instanton size in the QCD vacuum [34]. This small size was also confirmed in the lattice calculation [35,36]. Above T_c a similar phenomenon happens as well. However, in this case the instanton–anti-instanton molecules produce a very strong attraction in the scalar glueball channel. Therefore, we expect that the size of the scalar glueball should be smaller than that of instantons in QGP, which is cutting at

$$R_G \ll \bar\rho^2(T) \approx \frac{1}{3\pi^2 T^2}, \tag{19}$$

at finite T and $N_c = N_f = 3$ [39]. We would like to emphasize that this size is smaller than the perturbative Debye screening length in QGP

$$\lambda^2_D(T) = \frac{1}{M^2_D(T)} > \frac{1}{3\pi T^2}, \tag{20}$$

where $\alpha_s(\rho_c) \approx 0.5$ [24] for $T = 0$ and $M^2_D(T) = g^2_s(N_c/3 + N_f/6)T^2$ [38] were used. Therefore, we come to the important conclusion that the Debye screening cannot destroy the binding of the scalar glueball in QGP.

3. PSEUDOSCALAR GLUEBALL ABOVE T_c

There are also several candidates for the pseudoscalar glueball below 2 GeV at $T = 0$: $\eta(1405)$, $\eta(1475)$, $\eta(1760)$, and $X(1835)$. In the lattice quenched calculation, the lowest mass pseudoscalar glueball was predicted to have the mass $M_{0^{-+}} \approx 2.6$ GeV [40, 41][1]. The large difference between scalar and pseudoscalar glueball masses $m_{0^{-+}}(T = 0) - m_{0^{++}}(T = 0) \approx 1$ GeV at low temperature can be explained by the large single-instanton contribution to these channels (see discussion in [23]). Indeed, it gives a strong attraction in the scalar channel and a strong repulsion in the pseudoscalar state. After restoration of the chiral symmetry, the masses of scalar and pseudoscalar glueballs should be equal in the limit of the zero light quark masses, $m_u = m_d = m_s = 0$. In this case, instanton–anti-instanton molecules give the same strength of attraction in both states. Therefore, the leading contribution to the splitting between the masses of two states at $T > T_c$ is determined by the density of the single instantons, which is proportional to the product of the current masses of the light quarks. We should mention that even at large temperature the single-instanton contribution is repulsive in the pseudoscalar glueball case, and its collapse to the massless state is not allowed. This is a very important difference between the properties of pseudoscalar and scalar glueballs at the temperature of the scale invariance restoration.

We can estimate the mass of the pseudoscalar glueball at $T = T_{\rm scale}$ by using the instanton model. The difference between the scalar and pseudoscalar glueball masses above T_c is determined by the instanton density $n(\rho(T))$:

$$n(\rho(T)) \sim m_u m_d m_s (\rho(T))^{b_0 - 2}, \tag{21}$$

where $b_0 = 11N_c/3 - 2N_f/3$ and m_i are the current masses of the quarks. Finally, we have

$$m_{0^{-+}}(T_{\rm scale}) \approx \frac{m_u m_d m_s}{m_u^* m_d^* m_s^*} \left(\frac{1}{\sqrt{3}\pi T_{\rm scale}\rho_c} \right)^7 (m_{0^{-+}}(T=0) - m_{0^{++}}(T=0)), \tag{22}$$

where $m_i^* = m_i + m^*$ is the so-called effective mass of the quark in the instanton vacuum related to the quark condensate. The use of $m_u = m_d \approx 4.5$ MeV, $m_s \approx 100$ MeV [42], $m^* = 170$ MeV [23], $\rho_c = 1/600$ MeV^{-1}, and $m_{0^{-+}}(T = 0) - m_{0^{++}}(T = 0) \approx 1$ GeV gives the estimation

$$m_{0^{-+}}(T_{\rm scale}) \approx 0.1 \text{ eV}. \tag{23}$$

Therefore, the mass of the pseudoscalar glueball at $T = T_{\rm scale}$ is finite but very small.

4. BOSE–EINSTEIN CONDENSATION OF THE LIGHT GLUEBALLS AND SUPERFLUIDITY OF GLUEBALL MATTER IN QGP

The Bose–Einstein condensation (BEC) of the identical bosons is the well-known phenomenon in both theoretical and experimental physics. Recently, it was suggested that the overoccupied initial state of gluons created during relativistic heavy-ion collisions might lead to BEC of gluons [44–47]. However, gluons have the color charge and spin and, additionally,

[1] The difference between the observed and lattice masses might be related to the strong mixing between the glueball and quarkonium states [12].

the number of gluons is not conserved. All of these features lead to a very complicated study of the possible formation of the gluon BEC in QGP. On the other hand, the light glueballs with zero spin and color can easily form BEC at rather high temperature because for the given boson number density n the BEC temperature depends on the mass as [48]

$$T_{\rm BEC} \approx 3.31 \frac{n^{2/3}}{m} \tag{24}$$

for the nonrelativistic case. For the arbitrary boson mass, the critical BEC density is related to the temperature by the equation (see, for example, [49] and references therein)

$$n_{\rm BEC} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left((\sqrt{k^2+m^2}-m)/T_{\rm BEC}\right)-1}. \tag{25}$$

In Fig. 5, we show the critical glueball density of the scalar and pseudoscalar glueballs in QGP by using the T-dependent glueball mass from Eq. (16)[1]. This density can be compared with the density of QGP at the time τ produced at the RHIC, which was estimated by using the multiplicity of the meson production in the Au–Au central collision [50]

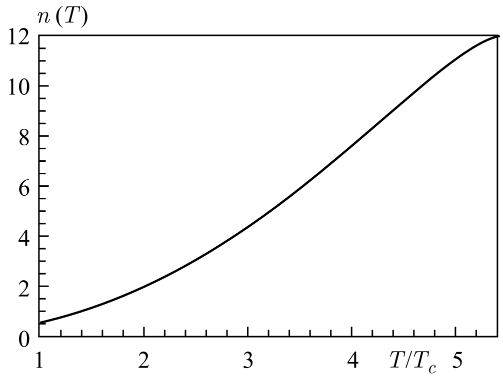

Fig. 5. T-dependency of the BEC density of glueballs in the fm^{-3} units

$$n_{\rm RHIC} \approx \frac{13}{\tau}\ (\mathrm{fm}^{-3}), \tag{26}$$

where τ is in fm. For LHC Pb–Pb central collisions at $\sqrt{s_{NN}} = 2.76$ GeV the multiplicity is approximately twice larger. Therefore, the density is

$$n_{\rm LHC} \approx \frac{26}{\tau}\ (\mathrm{fm}^{-3}). \tag{27}$$

For QGP in the thermal equilibrium we can estimate the contribution of the glueballs to the total density. In this case, we have approximately 16 gluon and 24 quark degrees of freedom ($N_f = 2$) and 2 for glueballs. For the gluons and quarks with the mass $M_{qg} \approx 3T$ [51], we have for scalar (pseudoscalar) glueball contribution to the total density at the RHIC

$$n_{\rm RHIC}^{\rm glueball} \approx \frac{3}{\tau}\ (\mathrm{fm}^{-3}) \tag{28}$$

for the initial temperature $T_0^{\rm RHIC} \approx 300$ MeV.

For the LHC energy ($T_0^{\rm LHC} \approx 400$ MeV) the estimation is

$$n_{\rm LHC}^{\rm glueball} \approx \frac{5}{\tau}\ (\mathrm{fm}^{-3}). \tag{29}$$

[1]We neglect a tiny difference between the values of the scalar and pseudoscalar glueballs in QGP.

By using the Bjorken model for the QGP expansion [52]

$$\tau T^3 = \tau_0 T_0^3, \tag{30}$$

with the thermalization time $\tau_0 \approx 1$ fm, we can estimate the BEC temperature for the RHIC and LHC

$$T_{\rm BEC}^{\rm RHIC} \approx 200~\text{MeV}, \quad T_{\rm BEC}^{\rm LHC} \approx 270~\text{MeV}. \tag{31}$$

Therefore, the formation of the glueball BEC is possible at both RHIC and LHC heavy-ion collisions. We should also mention that there is a large gap between the mass of the light glueball and the mass of the excited state in the system of two gluons in QGP, which is $m_{gg} \approx 6T > 1$ GeV. It is well-known that such a gap should lead to the phenomenon of the superfluidity of the BEC matter. So we come to the conclusion that QGP at the RHIC and LHC might be considered as a mixture of three matters. One of them is the usual "normal" QGP matter consisting of quarks and gluons and others are two superfluid matters of very light scalar and pseudoscalar glueballs.

There is also an additional mechanism which can lead to the abundance of the light glueballs production in ultrarelativistic heavy-ion collisions. Indeed, at very high temperatures at the initial stage of the production of the quark–gluon matter, the pair production of glueballs is possible by the fusion of two gluons, Fig. 6. The effective interaction responsible for such production is

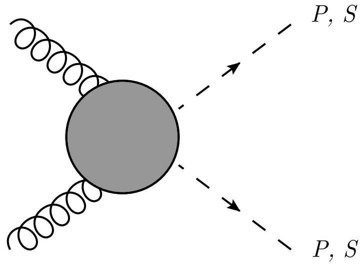

Fig. 6. Production of the scalar S and pseudoscalar P glueballs in nonequilibrium QGP

$$\mathcal{L} = \lambda_2(T)\alpha_s G^a_{\mu\nu} G^a_{\mu\nu}(S^2 + P^2), \tag{32}$$

where $S(P)$ are the scalar (pseudoscalar) fields. In the mean field approximation we have

$$\lambda_2(T) \sim \frac{m_{S,P}(T)^2}{\alpha_s G^a_{\mu\nu} G^a_{\mu\nu}(T)} \sim \frac{1}{m^2_{S,P}(T)}. \tag{33}$$

Therefore, one might expect a strong enhancement of the ultralight glueball production in the phase where the quark–gluon plasma is far away from the equilibrium.

CONCLUSIONS

In summary, we considered the properties of scalar and pseudoscalar glueballs in QGP created in relativistic heavy-ion collisions. Based on the instanton model for the QCD vacuum, we gave the arguments in favor of the existence of very light scalar and pseudoscalar glueball states above the temperature of the deconfinement transition. The estimation of the temperature of the scale invariance restoration, at which the scalar glueball becomes massless, was given. We also discussed the mechanism of the Bose–Einstein condensation and the superfluidity of the scalar and pseudoscalar glueball matter in QGP. We also showed the possibility of the abundance glueball production at the initial stage of the QGP formation. The influence of this phenomenon on the fast thermalization of QGP is the subject of our future investigation.

Acknowledgements. The author is grateful to A. E. Dorokhov, A. Di Giacomo, S. B. Gerasimov, D. G. Pak, M.-E. Ilgenfritz, M. Oka, Hee-Jung Lee, Yongseok Oh, and Pengming Zhang for useful discussions. It is a pleasure to thank Vicente Vento for numerous discussions of the properties of the glueballs in QGP. The author acknowledges cordial hospitality by the Institute of Modern Physics, CAS in Lanzhou, where this work was completed. This work was partially supported by the National Natural Science Foundation of China (Grant Nos. 11035006 and 11175215) and by the Chinese Academy of Sciences visiting professorship for senior international scientists (Grant No. 2013T2J0011). The work was initiated through the series of APCTP–BLTP JINR Joint Workshops.

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Received on October 10, 2015.