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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

LIGHT-LIKE WILSON LINE IN QCD WITHOUT PATH ORDERING

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Unlike the Wilson line in QED, the Wilson line in QCD contains path ordering. In this paper, we get rid of the path ordering in the light-like Wilson line in QCD by simplifying all the infinite number of noncommuting terms in the $SU(3)$ pure gauge. We prove that the light-like Wilson line in QCD naturally emerges when path integral formulation of QCD is used to prove factorization of the soft and collinear divergences at all orders in coupling constant in QCD processes at high-energy colliders.

В отличие от линии Вильсона в КЭД линия Вильсона в КХД содержит упорядочение по путям. В представленной статье удается избавиться от этого упорядочения для светоподобной линии Вильсона в КХД путем упрощения всего бесконечного числа некоммутирующих слагаемых в чистой $SU(3)$-калибровке. Показано, что светоподобная линия Вильсона в КХД появляется естественным образом, когда используется формулировка интегралов по путям для КХД и происходит факторизация мягкой и коллинеарной расходимостей во всех порядках разложения по константе связи в КХД-процессах при высоких энергиях коллайдеров.

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INTRODUCTION

In the Feynman diagrams the infrared divergences appear whenever the energy-momentum k^μ involved with the massless particle becomes very small. Similarly, the collinear divergences occur when the momenta $\mathbf{k}, \mathbf{p}$ of two massless particles become parallel in the region $0 < k \ll p$. Typically, the soft and collinear divergences occur in the Feynman diagrams due to momentum integration in the quantum loop diagrams involving massless propagators and due to momentum integration in the Feynman diagrams involving emission/absorption of massless particles. In quantum electrodynamics (QED) the massless particle is photon, and in quantum chromodynamics (QCD) the massless particle is gluon. The soft and collinear divergences are more severe in QCD than those in QED, because massless gluons interact with each other, whereas massless photons do not interact with each other. Since massless particle is always light-like, one finds that the soft and collinear divergences can be described by the light-like Wilson line.

However, the physical quantities measured are all soft and collinear divergences free. Hence, it is important to prove that all the noncanceling soft and collinear divergences in

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the perturbative Feynman diagrams are factorized in the definition of the (physical) gauge-invariant nonperturbative quantities in QCD, such as in the definition of the parton distribution function and fragmentation function at high-energy colliders, because the soft and collinear limit corresponds to long-distance regime. This is done by supplying the Wilson line in the definition of the parton distribution function and fragmentation function [1]. The factorization refers to separation of the short-distance effects from the long-distance effects in quantum field theory.

The proof of factorization theorem in QCD is very nontrivial by using the diagrammatic method of QCD [2,3], but it is enormously simplified by using the path integral method of QCD [4,5]. The main idea behind the path integral method of QCD to prove factorization is to study the soft and collinear behavior of nonperturbative correlation function, such as $\langle 0|\bar{\psi}(x)\psi(x')\bar{\psi}(x'')\psi(x''')\ldots|0\rangle$ in QCD, due to the presence of the light-like Wilson line in QCD. Note that the light-like quark with the light-like four-velocity l^μ produces the $SU(3)$ pure gauge potential at all the time-space points x^μ, except at the spatial position $\mathbf{x}$ transverse to the motion of the quark at the time of the closest approach [2,6,7]. The soft and collinear divergences in the Feynman diagrams in QCD can be studied by using the eikonal approximation for the propagators and vertices [1,2,8–15]. Hence, due to the eikonal approximation for the soft and collinear divergences arising from the soft and collinear gluon interactions with the light-like quark, the light-like quark finds the gluon field $A^{\mu a}(x)$ as the $SU(3)$ pure gauge [4,5]. The $U(1)$ pure gauge

$$A^\mu(x) = \partial^\mu \omega(x) \tag{1}$$

gives the light-like Wilson line in QED

$$\exp\left[ie\int_{x_i}^{x_f} dx^\mu A_\mu(x)\right], \tag{2}$$

which is used to study factorization of the soft and collinear divergences in QED [8,13]. In QCD, the $SU(3)$ pure gauge

$$T^a A^{\mu a}(x) = \frac{1}{ig}[\partial^\mu U(x)]U^{-1}(x), \quad U(x) = \mathrm{e}^{igT^a\omega^a(x)}, \tag{3}$$

gives the light-like Wilson line in QCD

$$\mathcal{P}\exp\left[igT^a\int_{x_i}^{x_f} dx^\mu A^a_\mu(x)\right], \tag{4}$$

which is used to study factorization of the soft and collinear divergences in QCD [4,5]. Note that, unlike the Wilson line in QED in Eq. (2), which does not contain path ordering $\mathcal{P}$, the Wilson line in QCD in Eq. (4) contains path ordering $\mathcal{P}$.

In this paper, we get rid of path ordering $\mathcal{P}$ in the light-like Wilson line in QCD by simplifying all the infinite number of noncommuting terms in the $SU(3)$ pure gauge in Eq. (3).

We find that the light-like Wilson line in QCD without path ordering is given by

$$\mathcal{P}\exp\left[ig\int_{x_i}^{x_f} dx^\mu A_\mu^a(x)T^a\right] = \exp\left\{igT^a\left[\frac{1}{2l\cdot D[A(x_f)]}l\cdot\frac{d[gA(x_f)]}{dg}\right]^a\right\}\times$$

$$\times\exp\left\{-igT^b\left[\frac{1}{2l\cdot D[A(x_i)]}l\cdot\frac{d[gA(x_i)]}{dg}\right]^b\right\}, \quad (5)$$

where the right-hand side of the above equation does not contain path ordering $\mathcal{P}$. In Eq. (5), $D_\mu^{ab}[A]$ is the covariant derivative, l^μ is the light-like four-velocity, and $A^{\mu a}(x)$ is the $SU(3)$ pure gauge in QCD, which, unlike the $U(1)$ pure gauge $A^\mu(x)$ in QED, contains infinite powers of g [6].

Since the light-like Wilson line in QCD does not depend on the path but depends only on the end points [4, 5], we find from Eq. (5) that the non-Abelian phase or the gauge link in QCD without path ordering is given by

$$\mathcal{P}\,\exp\left[-ig\int_0^\infty d\lambda l\cdot A^a(x+l\lambda)T^a\right] = \exp\left\{igT^a\left[\frac{1}{2l\cdot D[A(x)]}l\cdot\frac{d[gA(x)]}{dg}\right]^a\right\}, \quad (6)$$

which is used to study factorization of the soft and collinear divergences in QCD, where the right-hand side of the above equation does not contain path ordering $\mathcal{P}$.

In this paper, we will provide a derivation of Eq. (5).

In [4], we have shown that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove nonrelativistic QCD (NRQCD) factorization at all orders in coupling constant in heavy quarkonium production. Similarly, in [5], we have shown that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove factorization of the soft and collinear divergences of the gluon distribution function at high-energy colliders at all orders in coupling constant. In this paper, we will prove that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove factorization of the soft and collinear divergences of the quark distribution function at high-energy colliders at all orders in coupling constant. Hence, we find that the light-like Wilson line in QCD naturally emerges when path integral formulation of QCD is used to prove factorization of the soft and collinear divergences at all orders in coupling constant in QCD processes at high-energy colliders.

The paper is organized as follows. In Sec. 1, we derive the light-like Wilson line in QCD without path ordering as given by Eq. (5). In Sec. 2, we study the gauge transformation of the light-like Wilson line in QCD without path ordering. In Sec. 3, we prove that the light-like Wilson line in QCD naturally emerges when path integral formulation of QCD is used to prove factorization of the soft and collinear divergences at all orders in coupling constant in QCD processes at high-energy colliders. Finally, we draw conclusions.

1. THE LIGHT-LIKE WILSON LINE IN QCD WITHOUT PATH ORDERING

The $SU(3)$ pure gauge in QCD is given by Eq. (3), which contains infinite number of noncommuting terms. Simplifying all the infinite number of noncommuting terms in Eq. (3), we find that the $SU(3)$ pure gauge $A^{\mu a}(x)$ is given by [6]:

$$A^{\mu a}(x) = \partial^\mu \omega^b(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{ab}, \tag{7}$$

where

$$M_{ab}(x) = f^{abc}\omega^c(x). \tag{8}$$

Expanding the exponential in Eq. (7), we find

$$A^{\mu a}(x) = [\partial^\mu \omega^b(x)] \left[1 + \frac{g}{2!}M(x) + \frac{g^2}{3!}M^2(x) + \frac{g^3}{4!}M^3(x) + \ldots \right]_{ab}. \tag{9}$$

In QED, the $U(1)$ pure gauge potential produced by a point charge e is linearly proportional to the electric charge e [2, 6, 7], i.e.,

$$\partial^\mu \omega(x) \propto e. \tag{10}$$

Since $\omega(x)$ is linearly proportional to e, we find that $\omega^a(x)$ is linearly proportional to g [6,7]. Since $\omega^a(x)$ is linearly proportional to g, we write

$$\omega^a(x) = g\beta^a(x), \tag{11}$$

where $\beta^a(x)$ is independent of g. Using Eq. (11) in (9), we find

$$\frac{1}{g}A^{\mu a}(x) = [\partial^\mu \beta^b(x)] \left[1 + \frac{g^2}{2!}N(x) + \frac{(g^2)^2}{3!}N^2(x) + \frac{(g^2)^3}{4!}N^3(x) + \ldots \right]_{ab}, \tag{12}$$

where

$$N_{ab}(x) = f^{abc}\beta^c(x). \tag{13}$$

Multiplying $g^2 N_{ab}(x)$ in Eq. (12), we obtain

$$[gN(x)A^\mu(x)]^a = \\ = [\partial^\mu \beta^b(x)] \left[g^2 N(x) + \frac{(g^2)^2}{2!}N^2(x) + \frac{(g^2)^3}{3!}N^3(x) + \frac{(g^2)^4}{4!}N^4(x) + \ldots \right]_{ab}. \tag{14}$$

Adding $\partial^\mu \beta^b(x)$ in Eq. (14), we find

$$D^\mu[A(x)]\beta^a(x) = \\ = [\partial^\mu \beta^b(x)] \left[1 + g^2 N(x) + \frac{(g^2)^2}{2!}N^2(x) + \frac{(g^2)^3}{3!}N^3(x) + \frac{(g^2)^4}{4!}N^4(x) + \ldots \right]_{ab}, \tag{15}$$

where

$$D^{ab}_\mu[A(x)] = \delta^{ab}\partial_\mu + gf^{acb}A^c_\mu(x). \tag{16}$$

Multiplying g^2 in Eq. (12) and then taking derivative with respect to g^2, we obtain

$$\frac{1}{2g}\frac{d[gA^{\mu a}(x)]}{dg} = \left[\partial^\mu \beta^b(x)\right]\left[1 + g^2 N(x) + \frac{(g^2)^2}{2!}N^2(x) + \frac{(g^2)^3}{3!}N^3(x) + \ldots\right]_{ab}. \tag{17}$$

Since right-hand sides of Eqs. (15) and (17) are equal, we find

$$D^\mu[A(x)]\beta^a(x) = \frac{1}{2g}\frac{d[gA^{\mu a}(x)]}{dg}. \tag{18}$$

Converting $\beta^a(x)$ to $\omega^a(x)$ by using Eq. (11), we find from Eq. (18)

$$D^\mu[A(x)]\omega^a(x) = \frac{1}{2}\frac{d[gA^{\mu a}(x)]}{dg}. \tag{19}$$

Multiplying the same x^μ independent four-vector l^μ in Eq. (19), we find

$$l \cdot \frac{d[gA^a(x)]}{dg} = 2l \cdot D[A(x)]\omega^a(x). \tag{20}$$

Dividing $l \cdot D[A(x)]$ from left in Eq. (20), we obtain

$$\omega^a(x) = \left[\frac{1}{2l \cdot D[A(x)]}\right]_{ab}\frac{d[l \cdot gA^b(x)]}{dg} = \left[\frac{1}{2l \cdot D[A(x)]} l \cdot \frac{d[gA(x)]}{dg}\right]^a, \tag{21}$$

which gives the non-Abelian phase

$$\Phi(x) = \mathrm{e}^{igT^a\omega^a(x)} = \exp\left\{igT^a\left[\frac{1}{2l \cdot D[A(x)]} l \cdot \frac{d[gA(x)]}{dg}\right]^a\right\}. \tag{22}$$

From [4,5] we find that the light-like Wilson line in QCD for the soft and collinear divergences is given by

$$\begin{aligned}\mathcal{P}\exp\left[ig\int_{x_i}^{x_f} dx^\mu A^a_\mu(x)T^a\right] &= \mathrm{e}^{igT^a\omega^a(x_f)}\,\mathrm{e}^{-igT^b\omega^b(x_i)} = \\ = \left\{\mathcal{P}\exp\left[-ig\int_0^\infty d\lambda\, l \cdot A^a(x_f + l\lambda)T^a\right]\right\}&\mathcal{P}\exp\left[ig\int_0^\infty d\lambda\, l \cdot A^b(x_i + l\lambda)T^b\right]. \end{aligned} \tag{23}$$

Using Eq. (22) in Eq. (23), we find that the light-like Wilson line in QCD without path ordering is given by

$$\begin{aligned}\mathcal{P}\exp\left[ig\int_{x_i}^{x_f} dx^\mu A^a_\mu(x)T^a\right] &= \exp\left\{igT^a\left[\frac{1}{2l \cdot D[A(x_f)]} l \cdot \frac{d[gA(x_f)]}{dg}\right]^a\right\}\times \\ &\times \exp\left\{-igT^b\left[\frac{1}{2l \cdot D[A(x_i)]} l \cdot \frac{d[gA(x_i)]}{dg}\right]^b\right\}, \end{aligned} \tag{24}$$

which reproduces Eq. (5), where the right-hand side does not contain path ordering $\mathcal{P}$.

Since the light-like Wilson line in QCD does not depend on the path but depends only on the end points [4,5], we find from Eqs. (22) and (23) that the non-Abelian phase or the gauge link in QCD without path ordering is given by

$$\mathcal{P}\exp\left[-ig\int_0^\infty d\lambda l\cdot A^a(x+l\lambda)T^a\right] = \exp\left\{igT^a\left[\frac{1}{2l\cdot D[A(x)]}l\cdot\frac{d[gA(x)]}{dg}\right]^a\right\}, \tag{25}$$

which reproduces Eq. (6) used to study factorization of the soft and collinear divergences in QCD, where the right-hand side of the above equation does not contain path ordering $\mathcal{P}$.

2. NON-ABELIAN GAUGE TRANSFORMATION OF THE LIGHT-LIKE WILSON LINE IN QCD WITHOUT PATH ORDERING

In order to study the gauge transformation of the light-like Wilson line in QCD without path ordering, we proceed as follows. The non-Abelian gauge transformation is given by

$$T^aA'^a_\mu(x) = U(x)T^aA^a_\mu(x)U^{-1}(x) + \frac{1}{ig}[\partial_\mu U(x)]U^{-1}(x), \tag{26}$$

where

$$U(x) = \mathrm{e}^{igT^a\omega^a(x)}. \tag{27}$$

Since the matrices T^a are noncommuting, we find from Eq. (27)

$$T^aU^{-1}(x) = T^a\mathrm{e}^{-igT^b\omega^b(x)} = T^a\Bigg[1 + (-ig)T^b\omega^b(x) + \\ + \frac{(-ig)^2}{2!}T^bT^c\omega^b(x)\omega^c(x) + \frac{(-ig)^3}{3!}T^bT^cT^d\omega^b(x)\omega^c(x)\omega^d(x) + \\ + \frac{(-ig)^4}{4!}T^bT^cT^dT^e\omega^b(x)\omega^c(x)\omega^d(x)\omega^e(x) + \ldots\Bigg]. \tag{28}$$

By repeated use of the commutation relation

$$[T^a,\ T^b] = if^{abc}T^c \tag{29}$$

we find from Eq. (28)

$$T^aU^{-1}(x) = \Bigg[T^a + (-ig)T^b\omega^b(x)T^a + \frac{(-ig)^2}{2!}T^bT^c\omega^b(x)\omega^c(x)T^a + \\ + \frac{(-ig)^3}{3!}T^bT^cT^d\omega^b(x)\omega^c(x)\omega^d(x)T^a + \\ + \frac{(-ig)^4}{4!}T^b\omega^b(x)T^c\omega^c(x)T^d\omega^d(x)T^e\omega^e(x)T^a + \ldots + \\ + (-ig)if^{abc}\omega^b(x)T^c + (-ig)^2T^b\omega^b(x)if^{acd}\omega^c(x)T^d + \\ + \frac{(-ig)^2}{2!}if^{abd}\omega^b(x)if^{dce}\omega^c(x)T^e +$$

$$+\frac{(-ig)^3}{2!}T^b\omega^b(x)T^c\omega^c(x)if^{ade}\omega^d(x)T^e+\frac{(-ig)^3}{2!}T^b\omega^b(x)if^{ace}\omega^c(x)if^{edg}\omega^d(x)T^g+$$
$$+\frac{(-ig)^3}{3!}if^{abe}\omega^b(x)if^{ecg}\omega^c(x)if^{gdh}\omega^d(x)T^h+$$
$$+\frac{(-ig)^4}{3!}T^b\omega^b(x)T^c\omega^c(x)T^d\omega^d(x)if^{aeg}\omega^e(x)T^g+$$
$$+\frac{(-ig)^4}{2!2!}T^b\omega^b(x)T^c\omega^c(x)if^{adg}\omega^d(x)if^{geh}\omega^e(x)T^h+$$
$$+\frac{(-ig)^4}{3!}T^b\omega^b(x)if^{acg}\omega^c(x)if^{gdh}\omega^d(x)if^{hei}\omega^e(x)T^i+$$
$$\left.+\frac{(-ig)^4}{4!}if^{abg}\omega^b(x)if^{gch}\omega^c(x)if^{hdi}\omega^d(x)if^{iej}\omega^e(x)T^j+\ldots\right],\quad (30)$$

which gives after simplification

$$T^aU^{-1}(x)=\left[T^a+(-ig)T^b\omega^b(x)T^a+\frac{(-ig)^2}{2!}T^bT^c\omega^b(x)\omega^c(x)T^a+\right.$$
$$+\frac{(-ig)^3}{3!}T^bT^cT^d\omega^b(x)\omega^c(x)\omega^d(x)T^a+$$
$$+\frac{(-ig)^4}{4!}T^b\omega^b(x)T^c\omega^c(x)T^d\omega^d(x)T^e\omega^e(x)T^a+\ldots+$$
$$+\left[1+(-ig)T^b\omega^b(x)+\frac{(-ig)^2}{2!}T^b\omega^b(x)T^c\omega^c(x)+\ldots\right](-ig)if^{ade}\omega^d(x)T^e+$$
$$+\left[1+(-ig)T^b\omega^b(x)+\frac{(-ig)^2}{2!}T^b\omega^b(x)T^c\omega^c(x)+\ldots\right]\frac{(-ig)^2}{2!}if^{apd}\omega^p(x)if^{dhe}\omega^h(x)T^e+$$
$$+[1+(-ig)T^q\omega^q(x)+\ldots]\frac{(-ig)^3}{3!}if^{abe}\omega^b(x)if^{ecg}\omega^c(x)if^{gdh}\omega^d(x)T^h+$$
$$\left.+[1+\ldots]\frac{(-ig)^4}{4!}if^{abp}\omega^b(x)if^{pch}\omega^c(x)if^{hdq}\omega^d(x)if^{qes}\omega^e(x)T^s+\ldots\right].\quad (31)$$

From Eq. (31) we find

$$T^aU^{-1}(x)=U^{-1}(x)\left[T^a+(-ig)if^{ade}\omega^d(x)T^e+\frac{(-ig)^2}{2!}if^{agd}\omega^g(x)if^{dhe}\omega^h(x)T^e+\right.$$
$$+\frac{(-ig)^3}{3!}if^{abe}\omega^b(x)if^{ecg}\omega^c(x)if^{gdh}\omega^d(x)T^h+$$
$$\left.+\frac{(-ig)^4}{4!}if^{abg}\omega^b(x)if^{gch}\omega^c(x)if^{hdi}\omega^d(x)if^{iej}\omega^e(x)T^j+\ldots\right],\quad (32)$$

which gives

$$U(x)T^aU^{-1}(x)=[\mathrm{e}^{-gM(x)}]_{ab}T^b,\quad (33)$$

where $M_{ab}(x)$ is given by Eq. (8). From Eq. (33) we find

$$U(x)T^aA^a_\mu(x)U^{-1}(x)=[\mathrm{e}^{gM(x)}]_{ab}T^aA^b_\mu(x).\quad (34)$$

Similarly, by simplifying infinite number of noncommuting terms in $[\partial_\mu U(x)]U^{-1}(x)$ we find [6]:

$$\frac{1}{ig}[\partial_\mu U(x)]U^{-1}(x) = \left[\frac{\mathrm{e}^{gM(x)}-1}{gM(x)}\right]_{ab} [\partial_\mu \omega^b(x)]T^a, \tag{35}$$

where $M_{ab}(x)$ is given by Eq. (8).

Hence, by using Eqs. (34) and (35) in Eq. (26) we find

$$A'^a_\mu(x) = [\mathrm{e}^{gM(x)}]_{ab} A^b_\mu(x) + \left[\frac{\mathrm{e}^{gM(x)}-1}{gM(x)}\right]_{ab} [\partial_\mu \omega^b(x)], \tag{36}$$

which is the finite gauge transformation in QCD, where $M_{ab}(x)$ is given by Eq. (8). Under infinitesimal gauge transformation we find from Eq. (36)

$$A'^{\mu a}(x) = A^{\mu a}(x) + gf^{abc}\omega^c(x)A^{\mu b}(x) + \partial^\mu \omega^a(x), \tag{37}$$

which is the infinitesimal gauge transformation in QCD familiar in the literature [16].

When $A^{\mu a}(x)$ is the $SU(3)$ pure gauge, we find by using Eq. (7) in (36) that

$$A'^{\mu a}(x) = \left[\frac{\mathrm{e}^{2gM(x)}-1}{gM(x)}\right]_{ab} [\partial^\mu \omega^b(x)]. \tag{38}$$

By using Eq. (11) in (38) we find

$$A'^{\mu a}(x) = \left[\frac{\mathrm{e}^{2g^2N(x)}-1}{gN(x)}\right]_{ab} [\partial^\mu \beta^b(x)], \tag{39}$$

where $N_{ab}(x)$ is given by Eq. (13), which is independent of g, because $\beta^a(x)$ is independent of g, see Eq. (11). Multiplying the matrix $gN(x)$ in Eq. (39), we obtain

$$D^\mu[A'(x)]\beta^a(x) = [\mathrm{e}^{2g^2N(x)}]_{ab}\ [\partial^\mu \beta^b(x)], \tag{40}$$

where

$$D^{ab}_\mu[A'(x)] = \delta^{ab}\partial_\mu + gf^{acb}A'^c_\mu(x). \tag{41}$$

By multiplying g in Eq. (39) and then taking the derivative with respect to g, we find

$$\frac{d[gA'^{\mu a}(x)]}{dg} = 4g\ [\mathrm{e}^{2g^2N(x)}]_{ab}\ [\partial^\mu \beta^b(x)]. \tag{42}$$

Using Eq. (40) in (42), we obtain

$$\frac{d[gA'^{\mu a}(x)]}{dg} = 4gD^\mu[A'(x)]\beta^a(x). \tag{43}$$

By using Eq. (11) in (43) we find

$$\frac{d[gA'^{\mu a}(x)]}{dg} = 4D^\mu[A'(x)]\omega^a(x). \tag{44}$$

By multiplying the same x^μ independent four-vector l^μ in Eq. (44) we obtain

$$l \cdot \frac{d[gA'^a(x)]}{dg} = 4l \cdot D[A'(x)]\omega^a(x). \tag{45}$$

By dividing $l \cdot D[A'(x)]$ from left in Eq. (45) we find

$$\left[\frac{1}{2l \cdot D[A'(x)]} l \cdot \frac{d[gA'(x)]}{dg}\right]^a = 2\omega^a(x). \tag{46}$$

Under the non-Abelian gauge transformation as given by Eq. (26) we find from Eq. (22)

$$\Phi'(x) = \exp\left\{igT^a \left[\frac{1}{2l \cdot D[A'(x)]} l \cdot \frac{d[gA'(x)]}{dg}\right]^a\right\}. \tag{47}$$

Hence, from Eqs. (46), (47), (22), and (27) we find

$$\Phi'(x) = U(x)\ \Phi(x), \qquad \Phi'^\dagger(x) = \Phi^\dagger(x)\ U^{-1}(x), \tag{48}$$

which is the gauge transformation of the non-Abelian phase in QCD under the non-Abelian gauge transformation as given by Eq. (26).

From Eqs. (22), (23), and (48) we find

$$\mathcal{P}\exp\left[-ig\int_0^\infty d\lambda l \cdot A'^a(x + l\lambda)T^a\right] = U(x)\mathcal{P}\exp\left[-ig\int_0^\infty d\lambda l \cdot A^a(x + l\lambda)T^a\right], \quad U(x) = \mathrm{e}^{igT^a\omega^a(x)}, \tag{49}$$

which is the gauge transformation of the non-Abelian gauge link in QCD under the non-Abelian gauge transformation as given by Eq. (26).

From Eqs. (23) and (49) we find that, under the non-Abelian gauge transformation as given by Eq. (26), the light-like Wilson line in QCD transforms as

$$\mathcal{P}\exp\left[ig\int_{x_i}^{x_f} dx^\mu A'^a_\mu(x)T^a\right] = U(x_f)\left\{\mathcal{P}\exp\left[ig\int_{x_i}^{x_f} dx^\mu A^a_\mu(x)T^a\right]\right\}U^{-1}(x_i), \quad U(x) = \mathrm{e}^{igT^a\omega^a(x)}. \tag{50}$$

3. EMERGENCE OF THE LIGHT-LIKE WILSON LINE IN QCD IN THE PROOF OF FACTORIZATION THEOREM AT HIGH-ENERGY COLLIDERS

Note that in [4] we have shown that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove NRQCD factorization at all orders in coupling constant in heavy quarkonium production. Similarly, in [5], we have shown that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove factorization of the soft and collinear divergences of the gluon distribution function at high-energy colliders at all orders in coupling constant. In this section, we will prove that the light-like Wilson line in QCD naturally emerges when path integral formulation is used to prove factorization of the soft and collinear divergences of the quark distribution function at high-energy colliders at all orders in coupling constant.

The generating functional in the path integral method of QCD is given by [16, 17]:

$$Z[J,\eta,\bar{\eta}] = \int [dQ][d\bar{\psi}][d\psi] \det\left(\frac{\delta \partial_\mu Q^{\mu a}}{\delta \omega^b}\right) \times \\ \times \exp\left\{ i \int d^4x \Big[-\frac{1}{4} F^{a\,2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(\partial_\mu Q^{\mu a})^2 + \\ + \bar{\psi}[i\gamma^\mu \partial_\mu - m + gT^a\gamma^\mu Q^a_\mu]\psi + J\cdot Q + \bar{\eta}\psi + \bar{\psi}\eta \Big] \right\}, \quad (51)$$

where $J^{\mu a}(x)$ is the external source for the quantum gluon field $Q^{\mu a}(x)$ and $\bar{\eta}_i(x)$ is the external source for the Dirac field $\psi_i(x)$ of the quark, and

$$F^a_{\mu\nu}[Q] = \partial_\mu Q^a_\nu(x) - \partial_\nu Q^a_\mu(x) + gf^{abc}Q^b_\mu(x)Q^c_\nu(x), \quad F^{a\,2}_{\mu\nu}[Q] = F^{\mu\nu a}[Q]F^a_{\mu\nu}[Q]. \quad (52)$$

The light-like quark traveling with light-like four-velocity l^μ produces the $SU(3)$ pure gauge potential $A^{\mu a}(x)$ at all the time-space positions x^μ, except at the position $\mathbf{x}$ perpendicular to the direction of motion of the quark ($\mathbf{l}\cdot\mathbf{x} = 0$) at the time of the closest approach [2, 6, 7]. Hence, the soft and collinear behavior of the nonperturbative correlation function in QCD due to the presence of the light-like Wilson line in QCD can be studied by using path integral formulation of the background field method of QCD in the presence of the $SU(3)$ pure gauge background field [4, 5].

The background field method of QCD was originally formulated by 't Hooft [18] and later extended by Klueberg-Stern and Zuber [19, 20] and by Abbott [17]. This is an elegant formalism which can be useful to construct gauge-invariant nonperturbative Green's functions in QCD. This formalism is also useful to study quark and gluon production from classical chromofield [21] via the Schwinger mechanism [22], to compute β function in QCD [23], to perform calculations in lattice gauge theories [24], and to study evolution of QCD coupling constant in the presence of chromofield [25].

It can be mentioned here that in soft collinear effective theory (SCET) [26] it is also necessary to use the idea of background fields [17] to give well-defined meaning to several distinct gluon fields [9].

Note that a massive color source traveling at speed much less than speed of light cannot produce the $SU(3)$ pure gauge field [2, 6, 7]. Hence, when one replaces the light-like Wilson line with the massive Wilson line, one expects the factorization of soft/infrared divergences to break down. This is in conformation with the finding in [27], which used the diagrammatic method of QCD. In case of massive Wilson line in QCD, the color transfer occurs and the factorization breaks down. Note that, in case of the massive Wilson line, there are no collinear divergences.

The generating functional in the path integral formulation of the background field method of QCD is given by [17–19]:

$$Z[A,J,\eta,\bar{\eta}] = \int [dQ][d\bar{\psi}][d\psi] \det\left(\frac{\delta G^a(Q)}{\delta\omega^b}\right) \times \\ \times \exp\Bigg\{ i\int d^4x \Big[-\frac{1}{4}{F^a}^2_{\mu\nu}[A+Q] - \frac{1}{2\alpha}(G^a(Q))^2 + \\ + \bar{\psi}[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu(A+Q)^a_\mu]\psi + J\cdot Q + \bar{\eta}\psi + \bar{\psi}\eta \Big]\Bigg\}, \quad (53)$$

where the gauge fixing term is given by

$$G^a(Q) = \partial_\mu Q^{\mu a} + gf^{abc}A^b_\mu Q^{\mu c} = D_\mu[A]Q^{\mu a}, \quad (54)$$

which depends on the background field $A^{\mu a}(x)$ and

$$F^a_{\mu\nu}[A+Q] = \partial_\mu[A^a_\nu + Q^a_\nu] - \partial_\nu[A^a_\mu + Q^a_\mu] + gf^{abc}[A^b_\mu + Q^b_\mu][A^c_\nu + Q^c_\nu]. \quad (55)$$

We have followed the notations of [17–19], and accordingly denoted the quantum gluon field by $Q^{\mu a}$ and the background field by $A^{\mu a}$.

Note that the gauge fixing term $\frac{1}{2\alpha}(G^a(Q))^2$ in Eq. (53) (where $G^a(Q)$ is given by Eq. (54)) is invariant for gauge transformation of A^a_μ:

$$\delta A^a_\mu = gf^{abc}A^b_\mu\omega^c + \partial_\mu\omega^a \qquad \text{(type I transformation)}, \quad (56)$$

provided one also performs a homogeneous transformation of Q^a_μ [17, 19]:

$$\delta Q^a_\mu = gf^{abc}Q^b_\mu\omega^c. \quad (57)$$

The gauge transformation of background field A^a_μ as given by Eq. (56) along with the homogeneous transformation of Q^a_μ in Eq. (57) gives

$$\delta(A^a_\mu + Q^a_\mu) = gf^{abc}(A^b_\mu + Q^b_\mu)\omega^c + \partial_\mu\omega^a, \quad (58)$$

which leaves $(-1/4){F^a}^2_{\mu\nu}[A+Q]$ invariant in Eq. (53).

For fixed A^a_μ, i.e., for

$$\delta A^a_\mu = 0 \qquad \text{(type II transformation)}, \quad (59)$$

the gauge transformation of Q^a_μ [17, 19]:

$$\delta Q^a_\mu = gf^{abc}(A^b_\mu + Q^b_\mu)\omega^c + \partial_\mu\omega^a, \quad (60)$$

gives Eq. (58), which leaves $(-1/4){F^a}^2_{\mu\nu}[A+Q]$ invariant in Eq. (53).

It is useful to remember that, unlike QED [8], finding an exact relation between the generating functional $Z[J,\eta,\bar{\eta}]$ in QCD in Eq. (51) and the generating functional $Z[A,J,\eta,\bar{\eta}]$

in the background field method of QCD in Eq. (53) in the presence of the $SU(3)$ pure gauge background field is not easy. The main difficulty is due to the gauge fixing terms which are different in both cases. While the Lorentz (covariant) gauge fixing term $-\frac{1}{2\alpha}(\partial_\mu Q^{\mu a})^2$ in Eq. (51) in QCD is independent of the background field $A^{\mu a}(x)$, the background field gauge fixing term $-\frac{1}{2\alpha}(G^a(Q))^2$ in Eq. (53) in the background field method of QCD depends on the background field $A^{\mu a}(x)$, where $G^a(Q)$ is given by Eq. (54) [17–19]. Hence, in order to study nonperturbative correlation function in the background field method of QCD in the presence of the $SU(3)$ pure gauge background field, we proceed as follows.

By changing $Q \to Q - A$ in Eq. (53) we find

$$Z[A,J,\eta,\bar{\eta}] = \exp\left(-i\int d^4x J\cdot A\right)\int [dQ][d\bar{\psi}][d\psi]\det\left(\frac{\delta G_f^a(Q)}{\delta\omega^b}\right)\times$$
$$\times \exp\left\{i\int d^4x\left[-\frac{1}{4}F^{a2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(G_f^a(Q))^2 + J\cdot Q + \right.\right.$$
$$\left.\left. + \bar{\psi}[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q_\mu^a]\psi + \bar{\eta}\psi + \bar{\psi}\eta\right]\right\}, \quad (61)$$

where the gauge fixing term from Eq. (54) becomes

$$G_f^a(Q) = \partial_\mu Q^{\mu a} + gf^{abc}A_\mu^b Q^{\mu c} - \partial_\mu A^{\mu a} = D_\mu[A]Q^{\mu a} - \partial_\mu A^{\mu a}, \quad (62)$$

and Eq. (57) (by using Eq. (56), type I transformation [17, 19]) becomes

$$\delta Q_\mu^a = gf^{abc}Q_\mu^b\omega^c + \partial_\mu\omega^a. \quad (63)$$

Equations (62) and (63) can also be derived by using type II transformation, which can be seen as follows. By changing $Q \to Q - A$ in Eq. (53) we find Eq. (61), where the gauge fixing term from Eq. (54) becomes Eq. (62), and Eq. (60) (by using Eq. (59)) becomes Eq. (63). Hence, we obtain Eqs. (61), (62), and (63), whether we use the type I or type II transformation. Hence, we find that we will obtain the same Eq. (84), whether we use the type I or type II transformation.

The equation

$$Q'^a_\mu(x) = Q_\mu^a(x) + gf^{abc}\omega^c(x)Q_\mu^b(x) + \partial_\mu\omega^a(x) \quad (64)$$

in Eq. (63) is valid for infinitesimal transformation ($\omega \ll 1$), which is obtained from the finite equation

$$T^aQ'^a_\mu(x) = U(x)T^aQ_\mu^a(x)U^{-1}(x) + \frac{1}{ig}[\partial_\mu U(x)]U^{-1}(x), \qquad U(x) = \mathrm{e}^{igT^a\omega^a(x)}. \quad (65)$$

Simplifying infinite numbers of noncommuting terms in Eq. (65) (by using Eq. (33) and [6]), we find that

$$Q'^a_\mu(x) = [\mathrm{e}^{gM(x)}]_{ab}Q_\mu^b(x) + \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)}\right]_{ab}[\partial_\mu\omega^b(x)], \qquad M_{ab}(x) = f^{abc}\omega^c(x). \quad (66)$$

Changing the variables of integration from unprimed to the primed ones in Eq. (61), we find

$$Z[A,J,\eta,\bar{\eta}] = \exp\left(-i\int d^4x J\cdot A\right)\int [dQ'][d\bar{\psi}'][d\psi'] \det\left(\frac{\delta G_f^a(Q')}{\delta\omega^b}\right)\times$$
$$\times \exp\left\{i\int d^4x\Big[-\frac{1}{4}F^{a2}_{\mu\nu}[Q'] - \frac{1}{2\alpha}(G_f^a(Q'))^2 + J\cdot Q' +$$
$$+\bar{\psi}'[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q'^a_\mu]\psi' + \bar{\eta}\psi' + \bar{\psi}'\eta\Big]\right\}. \quad (67)$$

This is because of a change of variables from unprimed to the primed ones, the value of integration does not change.

Under the finite transformation, using Eq. (66), we find

$$[dQ'] = [dQ]\det\left[\frac{\partial Q'^a}{\partial Q^b}\right] = [dQ]\det[[\mathrm{e}^{gM(x)}]] = [dQ]\exp\Big[\mathrm{Tr}\,(\ln[\mathrm{e}^{gM(x)}])\Big] = [dQ], \quad (68)$$

where we have used (for any matrix H)

$$\det H = \exp[\mathrm{Tr}\,(\ln H)]. \quad (69)$$

The fermion field transforms as

$$\psi'(x) = \mathrm{e}^{igT^a\omega^a(x)}\psi(x). \quad (70)$$

Using Eqs. (66) and (70), we find

$$[d\bar{\psi}'][d\psi'] = [d\bar{\psi}][d\psi],$$
$$\bar{\psi}'[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q'^a_\mu]\psi' = \bar{\psi}[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q^a_\mu]\psi, \quad (71)$$
$$F^{a2}_{\mu\nu}[Q'] = F^{a2}_{\mu\nu}[Q].$$

Using Eqs. (68) and (71) in Eq. (67), we find

$$Z[A,J,\eta,\bar{\eta}] = \exp\left(-i\int d^4x J\cdot A\right)\int [dQ][d\bar{\psi}][d\psi] \det\left(\frac{\delta G_f^a(Q')}{\delta\omega^b}\right)\times$$
$$\times \exp\left\{i\int d^4x\Big[-\frac{1}{4}F^{a2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(G_f^a(Q'))^2 + J\cdot Q' +$$
$$+\bar{\psi}[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q^a_\mu]\psi + \bar{\eta}\psi' + \bar{\psi}'\eta\Big]\right\}. \quad (72)$$

From Eq. (62) we find

$$G_f^a(Q') = \partial_\mu Q^{'\mu a} + gf^{abc}A^b_\mu Q^{'\mu c} - \partial_\mu A^{\mu a}. \quad (73)$$

By using Eqs. (7) and (66) in Eq. (73) we find

$$G_f^a(Q') = \partial^\mu \left[[\mathrm{e}^{gM(x)}]_{ab} Q_\mu^b(x) + \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{ab} [\partial_\mu \omega^b(x)] \right] + \\ + gf^{abc} \left[\partial^\mu \omega^e(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{be} \right] \left[[\mathrm{e}^{gM(x)}]_{cd} Q_\mu^d(x) + \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{cd} [\partial_\mu \omega^d(x)] \right] - \\ - \partial_\mu \left[\partial^\mu \omega^b(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{ab} \right], \quad (74)$$

which gives

$$G_f^a(Q') = \partial^\mu [[\mathrm{e}^{gM(x)}]_{ab} Q_\mu^b(x)] + \\ + gf^{abc} \left[\partial^\mu \omega^e(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{be} \right] \left[[\mathrm{e}^{gM(x)}]_{cd} Q_\mu^d(x) + \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{cd} [\partial_\mu \omega^d(x)] \right]. \quad (75)$$

From Eq. (75) we find

$$G_f^a(Q') = \partial^\mu [[\mathrm{e}^{gM(x)}]_{ab} Q_\mu^b(x)] + gf^{abc} \left[\partial^\mu \omega^e(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{be} \right] [[\mathrm{e}^{gM(x)}]_{cd} Q_\mu^d(x)], \quad (76)$$

which gives

$$G_f^a(Q') = [\mathrm{e}^{gM(x)}]_{ab} \partial^\mu Q_\mu^b(x) + \\ + Q_\mu^b(x) \partial^\mu [[\mathrm{e}^{gM(x)}]_{ab}] + \left[\partial^\mu \omega^e(x) \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{be} \right] gf^{abc} [[\mathrm{e}^{gM(x)}]_{cd} Q_\mu^d(x)]. \quad (77)$$

From [6] we find

$$\partial^\mu [\mathrm{e}^{igT^a \omega^a(x)}]_{ij} = ig[\partial^\mu \omega^b(x)] \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{ab} T_{ik}^a [\mathrm{e}^{igT^c \omega^c(x)}]_{kj}, \\ M_{ab}(x) = f^{abc} \omega^c(x), \quad (78)$$

which in the adjoint representation of $SU(3)$ gives (by using $T_{bc}^a = -if^{abc}$)

$$[\partial^\mu \mathrm{e}^{gM(x)}]_{ad} = [\partial^\mu \omega^e(x)] \left[\frac{\mathrm{e}^{gM(x)} - 1}{gM(x)} \right]_{be} gf^{bac} [\mathrm{e}^{M(x)}]_{cd}, \qquad M_{ab}(x) = f^{abc} \omega^c(x). \quad (79)$$

Using Eq. (79) in (77), we find

$$G_f^a(Q') = [\mathrm{e}^{gM(x)}]_{ab} \partial^\mu Q_\mu^b(x), \quad (80)$$

which gives

$$(G_f^a(Q'))^2 = (\partial_\mu Q^{\mu a}(x))^2. \quad (81)$$

Since for $n \times n$ matrices A and B we have

$$\det(AB) = (\det A)(\det B), \tag{82}$$

we find by using Eq. (80) that

$$\det\left[\frac{\delta G_f^a(Q')}{\delta\omega^b}\right] = \det\left[\frac{\delta[[e^{gM(x)}]_{ac}\partial^\mu Q_\mu^c(x)]}{\delta\omega^b}\right] = \det\left[[e^{gM(x)}]_{ac}\frac{\delta(\partial^\mu Q_\mu^c(x))}{\delta\omega^b}\right] =$$
$$= \left[\det[[e^{gM(x)}]_{ac}]\right]\left[\det\left[\frac{\delta(\partial^\mu Q_\mu^c(x))}{\delta\omega^b}\right]\right] = \exp\left[\mathrm{Tr}\,(\ln[e^{gM(x)}])\right]\det\left[\frac{\delta(\partial_\mu Q^{\mu a}(x))}{\delta\omega^b}\right] =$$
$$= \det\left[\frac{\delta(\partial_\mu Q^{\mu a}(x))}{\delta\omega^b}\right]. \tag{83}$$

Using Eqs. (81) and (83) in Eq. (72), we find

$$Z[A,J,\eta,\bar\eta] = \exp\left(-i\int d^4x J\cdot A\right)\int[dQ][d\bar\psi][d\psi]\det\left[\frac{\delta(\partial_\mu Q^{\mu a}(x))}{\delta\omega^b}\right]\times$$
$$\times\exp\left\{i\int d^4x\left[-\frac{1}{4}F^{a2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(\partial_\mu Q^{\mu a})^2 + J\cdot Q' +\right.\right.$$
$$\left.\left. + \bar\psi[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q_\mu^a]\psi + \bar\eta\psi' + \bar\psi'\eta\right]\right\}. \tag{84}$$

From Eqs. (7) and (66) we find

$$Q'^a_\mu(x) - A^a_\mu(x) = [e^{gM(x)}]_{ab}Q^b_\mu(x), \qquad M_{ab}(x) = f^{abc}\omega^c(x). \tag{85}$$

Note that Eqs. (84) and (85) are valid, whether we use type I transformation (Eqs. (56) and (57)) or type II transformation (Eqs. (59) and (60)).

Since we have used Eq. (26) to study the gauge transformation of the Wilson line in QCD, we will use type I transformation, see Eqs. (56) and (57), in the rest of the paper, which gives for finite transformation [17, 19]:

$$J'^a_\mu(x) = [e^{gM(x)}]_{ab}J^b_\mu(x), \qquad M_{ab}(x) = f^{abc}\omega^c(x). \tag{86}$$

From Eqs. (84), (85), and (86) we find

$$Z[A,J',\eta,\bar\eta] = \int[dQ][d\bar\psi][d\psi]\det\left[\frac{\delta(\partial_\mu Q^{\mu a}(x))}{\delta\omega^b}\right]\times$$
$$\times\exp\left\{i\int d^4x\left[-\frac{1}{4}F^{a2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(\partial_\mu Q^{\mu a})^2 + J\cdot Q +\right.\right.$$
$$\left.\left. + \bar\psi[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q_\mu^a]\psi + \bar\eta\psi' + \bar\psi'\eta\right]\right\}. \tag{87}$$

Under the non-Abelian gauge transformation the fermion source transforms as [17, 19]:

$$\eta'(x) = \mathrm{e}^{igT^a\omega^a(x)}\,\eta(x). \tag{88}$$

From Eqs. (70) and (88) we find

$$\bar{\eta}'\psi' = \bar{\eta}\psi, \qquad \bar{\psi}'\eta' = \bar{\psi}\eta, \tag{89}$$

which gives from Eq. (87)

$$Z[A, J', \eta', \bar{\eta}'] = \int [dQ][d\bar{\psi}][d\psi] \det\left[\frac{\delta(\partial_\mu Q^{\mu a}(x))}{\delta\omega^b}\right] \times \\ \times \exp\left\{ i\int d^4x \Big[-\frac{1}{4}F^{a\,2}_{\mu\nu}[Q] - \frac{1}{2\alpha}(\partial_\mu Q^{\mu a})^2 + J\cdot Q + \\ + \bar{\psi}[i\gamma^\mu\partial_\mu - m + gT^a\gamma^\mu Q^a_\mu]\psi + \bar{\eta}\psi + \bar{\psi}\eta \Big]\right\}. \tag{90}$$

Hence, from Eqs. (90) and (51) we find that in QCD

$$Z[J, \eta, \bar{\eta}] = Z[A, J', \eta', \bar{\eta}'], \tag{91}$$

when the background field $A^{\mu a}(x)$ is the $SU(3)$ pure gauge as given by Eq. (3). The corresponding relation in QED is given by

$$Z[J, \eta, \bar{\eta}] = Z[A, J, \eta', \bar{\eta}'], \tag{92}$$

when the background field $A^\mu(x)$ is the $U(1)$ pure gauge as given by Eq. (1). Note that, unlike Eq. (91) in QCD, there is no J' in Eq. (92) in QED, because while the (quantum) gluon directly interacts with classical chromo-electromagnetic field, the (quantum) photon does not directly interact with classical electromagnetic field.

The nonperturbative correlation function of the type $\langle 0|\bar{\psi}(x)\psi(x')|0\rangle$ in QCD is given by [8]:

$$\langle 0|\bar{\psi}(x)\psi(x')|0\rangle = \frac{\delta}{\delta\eta(x)}\frac{\delta}{\delta\bar{\eta}(x')}Z[J, \eta, \bar{\eta}]|_{J=\eta=\bar{\eta}=0}. \tag{93}$$

Similarly, the nonperturbative correlation function of the type $\langle 0|\bar{\psi}(x)\psi(x')|0\rangle_A$ in the background field method of QCD is given by [8]:

$$\langle 0|\bar{\psi}(x)\psi(x')|0\rangle_A = \frac{\delta}{\delta\eta(x)}\frac{\delta}{\delta\bar{\eta}(x')}Z[A, J, \eta, \bar{\eta}|_{J=\eta=\bar{\eta}=0}. \tag{94}$$

When background field $A^{\mu a}(x)$ is the $SU(3)$ pure gauge as given by Eq. (3), we find from Eqs. (91), (93), (94), (86), and (88) that

$$\langle 0|\bar{\psi}(x)\psi(x')|0\rangle = \langle 0|\bar{\psi}(x)\Phi(x)\Phi^\dagger(x')\psi(x')|0\rangle_A, \tag{95}$$

which proves factorization of the soft and collinear divergences at all orders in coupling constant in QCD, where (see Eq. (23) and [4, 5])

$$\Phi(x) = \mathcal{P} \exp \left[-igT^a \int_0^\infty d\lambda\, l \cdot A^a(x + l\lambda) \right] = \mathrm{e}^{igT^a \omega^a(x)} \tag{96}$$

is the non-Abelian phase or the gauge link in QCD.

From Eq. (95) we find that the correct definition of the quark distribution function at high-energy colliders, which is consistent with the number operator interpretation of the quark and is gauge-invariant and is consistent with the factorization theorem in QCD, is given by

$$f_{q/P}(x) = \frac{1}{4\pi} \int dy^- \mathrm{e}^{-ixP^+ y^-} \times$$
$$\times \langle P | \bar{\psi}(0, y^-, 0_T) \gamma^+ \left[\mathcal{P} \exp \left[igT^a \int_0^{y^-} dz^- A^{+a}(0, z^-, 0_T) \right] \right] \psi(0) | P \rangle, \tag{97}$$

which is valid in covariant gauge, in light-cone gauge, in general axial gauges, in general noncovariant gauges, and in the general Coulomb gauge, etc., respectively [5]. In Eq. (97), $\psi(x)$ is the Dirac field of the quark and $A^{\mu a}(x)$ is the $SU(3)$ pure gauge background field as given by Eq. (3).

Hence, we find from Eq. (97) and from [4, 5] that the light-like Wilson line in QCD naturally emerges when path integral formulation of QCD is used to prove factorization of the soft and collinear divergences at all orders in coupling constant in QCD processes at high-energy colliders.

CONCLUSIONS

Unlike the Wilson line in QED, the Wilson line in QCD contains path ordering. In this paper, we have got rid of the path ordering in the light-like Wilson line in QCD by simplifying all the infinite number of noncommuting terms in the $SU(3)$ pure gauge. We have proved that the light-like Wilson line in QCD naturally emerges when path integral formulation of QCD is used to prove factorization of the soft and collinear divergences at all orders in coupling constant in QCD processes at high-energy colliders.

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