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# NEWTONIAN MOTION AS ORIGIN OF ANISOTROPY OF THE LOCAL VELOCITY FIELD OF GALAXIES

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The origins of recently reported anisotropy of the local velocity field of nearby galaxies (velocities less than 500 km/s corresponding to the distance less than 8 Mpc) are studied. The exact solution of the Newtonian equation for the expanding Universe is obtained. This solution allows us to separate the Newtonian motion of nearby galaxies from the Hubble flow by the transition to the conformal coordinates. The relation between the Hubble flow and the Newtonian motion is established. We show that the anisotropic local velocity field of nearby galaxies can be formed by such a Newtonian motion in the expanding Universe, if at the moment of the capture of galaxies by the central gravitational field their conformal energy is equal to zero.

Изучается причина недавно открытой анизотропии локального поля скоростей местных галактик (скорости менее 500 км/с, что соответствует расстоянию менее 8 Мпк). Найдено точное решение ньютоновских уравнений с учетом расширения Вселенной. Такое решение позволяет отделить ньютоновское движение местных галактик от хаббловского потока с помощью перехода к конформным координатам. Тем самым, устанавливается соотношение между ньютоновским движением и локальным полем скоростей галактик. Показано, что анизотропия локального поля скоростей местных галактик может быть получена благодаря такому ньютоновскому движению в расширяющейся Вселенной, если в момент захвата галактик центральным гравитационным полем их конформная энергия равна нулю.

## INTRODUCTION

Recent observation of the local velocity field of galaxies gives a three-dimensional ellipsoid with different values of the Hubble parameter, clearly showing its anisotropic character [3,4].

In this paper, we present a possible point of view that this local velocity field of galaxies can be explained by their Newtonian motions.

The analysis of the observational data will be based on the radial velocities of nearby galaxies, belonging to the Local Group. Our paper is organized in the following manner. In Sec. 1, the cosmic evolution is described. In Sec. 2, we introduce the Newtonian motion and separate this motion from the cosmic one. In Sec. 3, the initial data of the galaxies capture by a central gravitational field is considered. In Sec. 4, the simplest example is given to elucidate our results. The paper ends with the conclusion.

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## 1. COSMIC EVOLUTION OF A FREE PARTICLE

Effects of cosmic evolutions are considered in the Friedmann–Lemâitre–Robertson–Walker (FLRW) metrics

$$(ds^2) = (dt)^2 - a(t)^2(dx^i)^2. \tag{1}$$

The formulation of the Newtonian problem in this metrics proposes a choice of physical variables and coordinates. The modern cosmology uses two choices of such variables: the conformal time $(\eta)$ and coordinate distance $(x^i)$ with the interval in terms of which the interval (1) takes the form

$$(ds^2) = a(\eta)^2[(d\eta)^2 - (dx^i)^2], \tag{2}$$

and the Friedmann time $(t)$ and distance $X^i = ax^i$ in terms of which the interval (1) takes the form

$$(ds^2) = (dt)^2 - [dX^i - H(t)X^i dt]^2, \tag{3}$$

where $H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter, we have used here the formula of differential calculus: $adx = d(ax) - xda$. Both the sets are mathematically equivalent[1].

In terms of the Friedmann variables $X^i = ax^i$ the Newton action in the space with the interval (3) takes the form

$$S_A = \int\limits_{t_I}^{t_0} dt \left[P_i(\dot{X}^i - HX^i) - \frac{P_i^2}{2m_0}\right]. \tag{4}$$

The equations of motion

$$\dot{X}^i - HX^i = \frac{P_i}{m_0}, \quad \dot{P}^i + HP^i = 0 \tag{5}$$

have the simplest solution $X^i = ax_0^i$, where $x_0^i$ is a constant.

The Hubble flow (or the Hubble velocity field) can be defined by

$$H_{\text{tot}} = \frac{\dot{R}}{R} \quad \left(R = \sqrt{X_1^2 + X_2^2 + X_3^2}\right). \tag{6}$$

The transition to the conformal variables (2) $x^i = X^i/a$, $p_i = aP_i$ gives the action[2]

$$S_A = \int\limits_{\eta_I}^{\eta_0} d\eta \left[p_i \frac{dx^i}{d\eta} - \frac{p_i^2}{2m_0 a(\eta)}\right] \tag{7}$$

for a particle with the running mass $m_0 a(\eta)$ (with the present-day value $a(\eta_0) = a_0 = 1$).

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[1] Note that $a(t) = a(\eta(t))$ is connected with $a(\eta)$ by equation $dt = a(\eta)d\eta$.

[2] Actions (4) and (6) differ from the action (7.4) in monograph [1], that is not compatible with quantum field theory in the conformed flat metric (2) [2].

## 2. NEWTONIAN MOTION IN AN EXPANDING UNIVERSE

Let us consider the action

$$S_A = \int\limits_{t_I}^{t_0} dt \left[ P_i(\dot{X}^i - HX^i) - \frac{P_i^2}{2m_0} + \frac{\alpha}{R} \right], \tag{8}$$

where $\alpha = M_O m_0 G$ is a constant of a Newtonian interaction of a galaxy with a mass $m_0$ in a gravitational field of a central mass $M_O$. Action (8) for radial momentum $P_R$ and orbital moment $P_\theta$ in the cylindrical coordinates

$$X^1 = R \cos\theta, \quad X^2 = R \sin\theta, \quad X^3 = 0 \tag{9}$$

takes the form

$$S_A = \int\limits_{t_I}^{t_0} dt \left[ P_R(\dot{R} - HR) + P_\theta \dot{\theta} - \frac{P_R^2}{2m_0} - \frac{P_\theta^2}{2m_0 R^2} + \frac{\alpha}{R} \right]. \tag{10}$$

To separate the Newtonian motion from the Hubble velocity field (6), we use the conformal variables $p_r = P_R a(t)$, $r = R/a(t)$, $d\eta = dt/a(t)$. In terms of these variables the action takes the form

$$S_A = \int\limits_{\eta_I}^{\eta_0} d\eta \left[ p_r r' + P_\theta \theta' - \frac{p_r^2}{2m_0 a(\eta)} - \frac{P_\theta^2}{2m_0 a(\eta) r^2} + \frac{\alpha}{r} \right]. \tag{11}$$

Substituting $R(\eta) = a(\eta) r(\eta)$ in the definition of the total Hubble flow $H_{\rm tot}$ (6) we get this Hubble flow in the following form

$$H_{\rm tot} = \frac{1}{R}\frac{dR}{dt} = \frac{1}{a}\frac{da}{dt} + \frac{1}{r}\frac{dr}{dt} = H + \Delta H. \tag{12}$$

One can see that the total Hubble flow differs from the classical one $H = \dot{a}/a$ by the value

$$\Delta H = \frac{1}{r}\frac{dr}{dt}. \tag{13}$$

This transition (12) is just the main idea of our paper to carry out the separation of the Hubble velocity field (13) from possible Newtonian motion.

## 3. THE CAPTURE OF GALAXIES BY CENTRAL FIELD

The energy of a particle with the running mass $m(\eta) = a(t)m_0$ described by the action (11)

$$E(\eta) = \frac{p_r^2}{2m_0 a(\eta)} + \frac{P_\theta^2}{2m_0 a(\eta) r^2} - \frac{\alpha}{r} \tag{14}$$

is not conserved in the contrast to the energy of particle with a constant mass in the Newtonian mechanics [5]. In our case (14), if the scale factor $a(\eta)$ increases, the energy (14) runs from its positive values to negatives ones. There is a moment of a time $\eta = \eta_I$ when the energy (14) is equal to zero:

$$\frac{E(\eta_I)}{m_0 a(\eta_I)} \equiv \frac{(r_I')^2 + v_I^2}{2} - w_I^2 = 0, \tag{15}$$

where $r_I' = p_r(\eta_I)/m_I r_I$ is radial initial velocity; $m_I = m_0 a(\eta_I),\ r_I = r(\eta_I)$ are the initial conformal mass and coordinate distance, and

$$v_I = \frac{P_\theta}{m_I r_I}, \quad w_I = \sqrt{\frac{\alpha}{m_I r_I}} \tag{16}$$

are the orbital velocity and Newtonian one, respectively. It is known that the change of a sign of the energy means the change of an unrestricted motion of a particle by a finite motion in the central field. Therefore, the time $\eta_I$ can be treated as the time of the capture of a particle (cosmic object) by the central gravitational field.

If the initial radial velocity is also equal to zero $r_I' = 0$, the zero energy constraint (15)

$$v_I^2 = 2w_I^2 \tag{17}$$

becomes the equation for the initial data $m_0 a(\eta_I) r(\eta_I) \equiv m_I r_I$. The solution of this equation $m_I r_I = P_\theta^2/(2\alpha)$ can give an orbital velocity

$$v_I = \frac{2\alpha}{P_\theta} = \mathrm{const} \tag{18}$$

of the captured cosmic objects. The fact of the universality of the orbital velocity (18) for all ellipsoidal trajectories (due to the zero energy initial data of the formation of a local Universe) gives us a possibility of explaining both the numerous observational data on the law of the constant orbital velocity [4–7] and the anisotropy of the local velocity field [3,4].

The anisotropy of the local velocity field $\Delta H = 0$ [1,2] is not compatible with the class of the isotropic circular trajectories $r(\eta_0) \equiv r_0$, $r' = r'' = 0$ with the equation of motion

$$v_0^2 = w_0 \ \left(v_0 = \frac{P_\theta}{m_0 r_0}, w_0 = \sqrt{\frac{\alpha}{m_0 r_0}}\right) \tag{19}$$

known as the «virial theorem» [7–9] where the initial radius $r_I$ can be identified with the observational radius[1]. If trajectories belong to a class of ellipsoidal ones, the initial radius $r_I$ does not coincide with the observational radius.

[1]Remind that the hypothesis of the Cold Dark Matter was proposed in [7,8] to explain the law of the constant orbital velocity in the class of the circular trajectories in a nonexpanding Universe, where (19) is valid.

## 4. THE EXACT SOLUTION

Let us consider a solution of the Kepler problem, i. e., a motion of an object in the central field in the expanding Universe for the rigid state when the densities of energy and pressure are equal. The cosmic scale factor $a$ can be written as [10]

$$a(\eta)^2 = [1 + 2H_0(\eta - \eta_0)], \tag{20}$$

which describes the recent Supernova data [11–13] in the relative units [10]. Equation (20) can be considered as a change of the evolution parameter $\eta \to a(\eta)$. In terms of the new variables

$$y = r/r_0, \quad p_y = p_r/m_0, \quad v_0 = P_\theta/r_0 m_0,$$

where $r(\eta_0) = r_0$ is the present-day data, action (11) takes the form

$$S_A = r_0 m_0 \int\limits_{a_I}^{1} da \left\{ p_y \frac{dy}{da} + v_0 \frac{d\theta}{da} - \frac{1}{c_0} \left[ \frac{p_y^2 + v_0^2/y^2}{2} - \frac{a w_0^2}{y} \right] \right\}, \tag{21}$$

where $w_0 = \sqrt{\alpha/m_0 r_0}$, $c_0 = H_0 r_0$ are the Newtonian velocity and the Hubble one, respectively, and $a_I = 1/(1 + z_I)$ is determined with the redshift $z_I$ at the moment of formation $\eta = \eta_I$. The total energy of the system (14) in terms of the new variable takes the form

$$\frac{E(a)}{r_0 m_0} = \frac{1}{c_0} \left[ \frac{p_y^2 + v_0^2/y^2}{2} - \frac{a w_0^2}{y} \right]. \tag{22}$$

It is easy to see that $v_0$ is a constant of the motion $dv_0/da = 0$. The equations of the radial motion

$$p_y = c_0 \frac{dy}{da}, \quad c_0 \frac{dp_y}{da} = \frac{v_0^2}{y^3} - \frac{a w_0^2}{y^2} \tag{23}$$

can be written in the Lagrangian form

$$\frac{d^2 y}{da^2} = \left( \frac{w_0}{c_0} \right)^2 \left[ \left( \frac{v_0}{w_0} \right)^2 \frac{1}{y^3} - \frac{a}{y^2} \right]. \tag{24}$$

At the present-day time $a = a_0 = 1$, $y_0 = 1$ this equation determines the second derivative

$$\left[ \frac{d^2 y}{da^2} \right] \Bigg|_{a=1} = \left( \frac{w_0}{c_0} \right)^2 \left[ \left( \frac{v_0}{w_0} \right)^2 - 1 \right] \tag{25}$$

in terms of three velocities $w_0$, $v_0$, $c_0$, and it allows us to choose any relations between $w_0$ and $v_0$, in particular $v_0^2 = 2w_0^2$, in contrast to the circular trajectory, where $v_0^2 = w_0^2$.

The general solution of this equation (24) can be obtained in the parametrical form:

$$a(\tau) = c_1 \frac{N_2(\tau)}{\tau^{2/3} N(\tau)}, \quad y(\tau) = c_2 \tau^{2/3} N(\tau), \tag{26}$$

where $\tau$ is the parameter of solution with the initial date $r_0 = r(\eta_0)$, and

$$N(\tau) = \alpha_1 U^2(\tau) + \beta_1 U(\tau)V(\tau) + \gamma_1 V^2(\tau), \tag{27}$$

$$N_2(\tau) = \left(\tau \frac{dN(\tau)}{d\tau} + \frac{2}{3}N(\tau)\right)^2 \pm 4\tau^2 N^2(\tau) + \omega^2\Delta, \tag{28}$$

$$\Delta = 4\alpha_1\gamma_1 - \beta_1^2, \tag{29}$$

$$c_1 = \left(\frac{v_0}{w_0}\right)\left(\frac{3c_0}{4w_0}\right)^{1/3} \frac{1}{2\omega\Delta^{1/2}}, \tag{30}$$

$$c_2 = \left(\frac{v_0}{w_0}\right)\left(\frac{4w_0}{3c_0}\right)^{1/3} \frac{1}{\omega\Delta^{1/2}}. \tag{31}$$

Three constants $\alpha_1, \beta_1, \gamma_1 = \mathrm{const}$ can be found from the following system of three equations:

$$a|_{\tau=0} = a|_{\eta=\eta_I} = a_I = \frac{1}{1+z_I}, \quad y|_{\tau=0} = \frac{r_I}{r_0} = 1 + z_I, \quad \left.\frac{dy}{da}\right|_{\tau=0} = 0; \tag{32}$$

here for the upper sign (restricted solution at infinity, $\tau = +\infty$) in (28)

$$U(\tau) = J_{1/3}(\tau), \quad V(\tau) = Y_{1/3}(\tau), \quad \text{and} \quad \omega = \frac{2}{\pi}, \tag{33}$$

where $J_{1/3}(\tau)$ and $Y_{1/3}(\tau)$ are the Bessel functions of the first and second (or Niemann function) kind, while for the lower sign (unrestricted solution at infinity, $\tau = +\infty$) in (28)

$$U(\tau) = I_{1/3}(\tau), \quad V(\tau) = K_{1/3}(\tau), \quad \text{and} \quad \omega = -1, \tag{34}$$

where $I_{1/3}(\tau)$ and $K_{1/3}(\tau)$ are the modified Bessel functions of the first and second (or MacDonald function) kind (see, e. g., [14]).

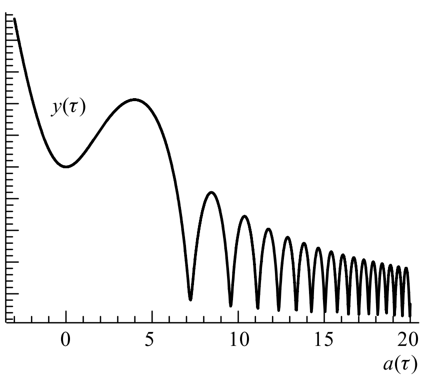


Fig. 1. Graph of function $y(\tau) = a(\tau)r(\tau)/r_0$ (26) at $v_0^2/c_0^2 = 0.2$ and $w_0^2/c_0^2 = 0.1$

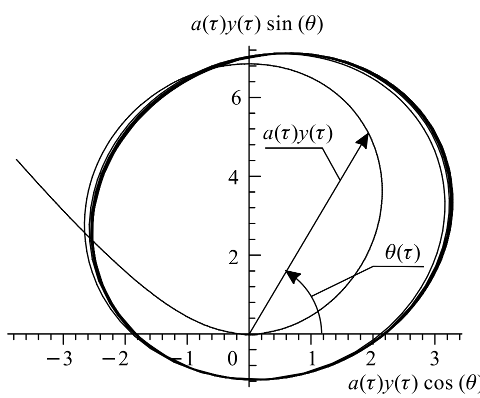


Fig. 2. Graph of functions product $a(\tau)y(\tau)$ (26) at $v_0^2/c_0^2 = 0.2$ and $w_0^2/c_0^2 = 0.1$ for $-3 < a(\tau) < 14$

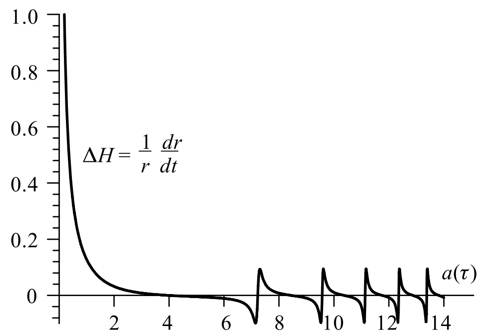


Fig. 3. Graph of function $\Delta H$ (13) in units $H_0$ at $v_0^2/c_0^2 = 0.2$ and $w_0^2/c_0^2 = 0.1$

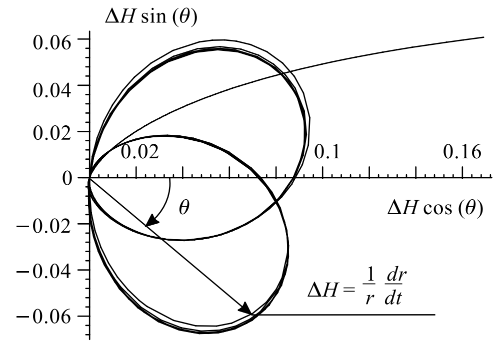


Fig. 4. Graph of function $\Delta H$ (13) in units $H_0$ at $v_0^2/c_0^2 = 0.2$ and $w_0^2/c_0^2 = 0.1$ for $0.8 < a(\tau) < 14$

A solution of the equation of motion following from the action (11) is given in Figs. 1 and 2. We use this solution for construction of two plots.

Figure 3 gives the values of the correction (13) to the Friedmann Hubble flow resulting from taking into account Eq. (13). It is clearly seen that the corrections are dumped with time and have a quasi-periodical character.

In Fig. 4 the angular distribution of the Hubble flow correction, as given by (13), is presented. The correction is anisotropic. We consider the 2-dimensional case while Karachentsev's anisotropy is observed in 3-dimensions. Nevertheless, our 2-dimensional analysis allows one to see the anisotropy of the Hubble flow and to estimate the order of the magnitude of the anisotropy.

We have considered the case when the formation of a galaxy began from the zero energy state (the initial data $E_0 = 0$, and velocity $y_0' = 0$). These data correspond to the relation $v_0^2 = 2w_0^2$.

## CONCLUSION

Our paper was motivated by the finding that in the local Universe the velocity field is anisotropic [3, 4]. This effect is difficult for explanation. The only possible suggestion, but rejected by Karachentsev, was rotation [4]. We are trying to find the origin of this anisotropy. In ordered to do this, we consider the general uniform expansion of the Universe. Since this is the nearby (less than 8 Mpc) part of the Universe around us, we use the Newtonian approach. We studied the motion of the test massive particle in the central gravitational field on the background of cosmic evolution of the type of FLRW space-time with uniform expansion. We assume the rigid state of the matter when densities of energy and pressure are equal and which corresponds to conformal cosmology [10] compatibles with Supernova data [11]. We obtained the exact solution of the above-mentioned Kepler problem, to find the difference between the uniform Hubble flow and our case. We have shown that this difference was anisotropic. In such a way we explained the anisotropy of the local velocity field by the Newtonian motion of galaxies in the central field. Of course, our 2-D consideration shows a possible mechanism of the observed 3-D anisotropy.

Having the solution of the Kepler problem we admit the rotation of galaxies around the centre of the local Universe. This local Universe must be regarded as the Local Group of galaxies. In such a way, we support the picture in which galaxies rotate around the centre of the Local Group in the class of ellipsoidal trajectories.

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