

УДК 539.172.12: 539.171.017

## JET ENERGY DENSITY IN HADRON–HADRON COLLISIONS AT HIGH ENERGIES

*J. V. Ilić<sup>a</sup>, G. P. Škoro<sup>b,1</sup>, M. V. Tokarev<sup>c,2</sup>*

<sup>a</sup> Institute of Nuclear Sciences «Vinča», Belgrade, Serbia and Montenegro

<sup>b</sup> Faculty of Physics, University of Belgrade, Belgrade, Serbia and Montenegro

<sup>c</sup> Joint Institute for Nuclear Research, Dubna

The average particle multiplicity density  $dN_{\text{ch}}/d\eta$  is the dynamical quantity that reflects some regularities of particle production in low- $p_T$  range. The quantity is an important ingredient of  $z$  scaling. Experimental results on charged particle density are available for  $pp$ ,  $pA$  and  $AA$  collisions, while experimental properties of the jet density are still an open question. The goal of this work is to find the variable that will reflect the main features of the jet production in low transverse energy range and play the role of the scale factor for the scaling function  $\psi(z)$  and variable  $z$  in data  $z$ -presentation. The appropriate candidate is the variable we called «scaled jet energy density». The scaled jet energy density is the probability to have a jet with defined  $E_T$  in defined  $x_T$  and pseudorapidity regions. The PYTHIA6.2 Monte Carlo generator is used for calculation of the scaled jet energy density in proton–proton collisions over a high energy range ( $\sqrt{s} = 200–14000$  GeV) and at  $\eta = 0$ . The properties of the new variable are discussed and sensitivity to «physical scenarios» applied in the standard Monte Carlo generator is noted. The calculations results of scaled jet energy density at LHC energies are presented and compared with predictions based on  $z$  scaling.

Динамическая характеристика, средняя плотность числа частиц,  $dN_{\text{ch}}/d\eta$  отражает особенности рождения частиц в области малых  $p_T$ . Эта величина является важной компонентой  $z$ -скейлинга. В литературе имеются экспериментальные данные для плотности числа заряженных частиц, образующихся в  $pp$ -,  $pA$ - и  $AA$ -взаимодействиях, в отличие от аналогичных данных для рождения струй. Цель работы — найти такую переменную, которая отражала бы особенности рождения струй при малых поперечных энергиях и выполняла бы роль масштабного фактора для скейлинговой функции и скейлинговой переменной в  $z$ -представлении экспериментальных данных. В качестве такой переменной в работе предлагается использовать «масштабированную плотность энергии струй». Эта величина характеризует вероятность рождения струи с поперечной энергией  $E_T$  в заданном диапазоне по  $x_T$  и псевдобыстроте  $\eta$ . Для определения масштабированной плотности энергии струй, рожденных в протон-протонных взаимодействиях при энергиях  $\sqrt{s} = 200–14000$  ГэВ и  $\eta = 0$  использовался монте-карловский генератор событий PYTHIA6.2. Обсуждаются свойства этой переменной и отмечается «чувствительность» к различным физическим сценариям, заложенным в генераторе событий PYTHIA6.2. Представлены результаты расчетов масштабированной плотности энергии струй при энергиях LHC и сравнение с предсказаниями  $z$ -скейлинга.

### INTRODUCTION

For the description of particle production in high- $p_T$   $pp$ ,  $\bar{p}p$  and  $pA$  collisions at high energies, the  $z$ -scaling concept is proposed in [1]. In the framework of  $z$  scaling, such

---

<sup>1</sup>E-mail: goran@ff.bg.ac.yu

<sup>2</sup>E-mail: tokarev@sunhe.jinr.ru

experimental observables as inclusive cross section  $Ed^3\sigma/dp^3$  and the average charged particle multiplicity density  $\rho \equiv dN_{\text{ch}}/d\eta$  are used to construct the scaling function  $\psi(z)$  and variable  $z$ . The scaling, known as  $z$  scaling, reveals interesting properties. These are the independence of the scaling function  $\psi(z)$  from collision energy and an angle of produced objects (hadron, photon). A general concept of the scaling is based on such fundamental principles as self-similarity, locality, fractality, and scale relativity [2, 3]. Because the scaling function  $\psi(z)$  is well defined in hadron–hadron collisions and expressed via two experimental observables, it is clear that the quantity can be used to study the properties of jet production, too.

In  $z$ -scaling concept, the average charged particle multiplicity density plays the role of the scale factor,  $z \sim 1/\rho(s)$ , and  $\psi(z) \sim 1/\rho(s, \eta)$ . Experimental results on charged particle density are available for  $pp$ ,  $pA$  and  $AA$  collisions, while experimental properties of the jet density are still an open question. In the case of jets, there are a lot of uncertainties (knowledge of parton distribution and fragmentation functions, knowledge of factorization, renormalization and fragmentation scales, uncertainties in the parton shower modelling, etc.) causing the problems in understanding of jet behavior at very high energies. The goal of this work is to find the variable that will reflect the main features of the jet production in low transverse energy range at a given energy and play the role of the scale factor.

The paper is organized as follows. A basic description of a scale factor in  $z$ -scaling concept, as well as results of Monte Carlo simulations on a scale factor in the case of charged particle production, is given in Sec. 1. New results on a scale factor for jet production based on the analysis of the experimental data and Monte Carlo simulations are described in Sec. 2. Discussion of the obtained results at the LHC energies is presented in Sec. 3. Conclusions are summarized in the final section.

## 1. SCALE FACTOR IN $z$ -SCALING CONCEPT

One of the most interesting problems in the modern particle physics is a search for general properties of quark and gluon interactions in collisions of leptons, hadrons, and nuclei. Universal approach to description of the processes allows us detailed understanding of the physical phenomena underlying the secondary particle production. Up to date, the investigation of hadron properties in the high energy collisions has revealed widely known scaling regularities. Some of the most popular and famous are the Feynman scaling [4] for inclusive hadron production, the Bjorken scaling observed in deep inelastic scattering (DIS) [5],  $y$  scaling valid in DIS on nuclei [6], limiting fragmentation established for nuclei fragmentation [7], scaling behavior of the cumulative particle production [8–10], KNO scaling [11], and others. However, detailed experimental study of the established scaling laws has shown certain violations of these. The domains in which the observed regularities are violated are of great interest. These can be relevant in searching for new physical phenomena — quark compositeness, new interactions, quark–gluon plasma, and others.

The concept of the  $z$  scaling is introduced in [1] for the description of inclusive production cross sections in  $pp/\bar{p}p$  interactions at high energies and high  $p_T$  values of secondary particles. The scaling function  $\psi(z)$  is expressed via the invariant inclusive cross section  $Ed^3\sigma/dp^3$  and the average charged particle multiplicity density  $\rho(s, \eta)$ . The function  $\psi(z)$  is found to be independent of collision energy  $\sqrt{s}$  and an angle  $\theta$  of the inclusive particle. The scaling was also applied for the analysis of the inclusive particle productions in  $pA$  collisions [2],

jet productions [12], etc. The scaling function of direct photon production was found to reveal the power behavior of  $\psi(z) \sim z^{-\beta}$  [13]. The properties of the scaling are assumed to reflect the fundamental properties of particle structure, interaction, and production. The scaling function describes the probability to form the produced particle with formation length  $z$ . The existence of the scaling itself means that the hadronization mechanism of particle production reveals such fundamental properties as self-similarity, locality, fractality, and scale relativity.

But it was also found that there is a strong sensitivity of the scaling behavior to the energy dependence of the scale factor  $\rho(s)$  at  $\eta = 0$ . The experimental results show that the scale factor  $\rho(s)$  (the average charged particle multiplicity density) is a well defined quantity (at least up to Tevatron energies) and that simulation results of standard Monte Carlo generators (such as PYTHIA) are in nice agreement with available experimental data. But it is clear that this scale factor cannot be used for description of processes in the case of jet production at high energies and that corresponding variable for jets must be found. This variable should represent the main properties of jet production at low  $E_T$  and must be, as much as possible, independent of jet energy  $E_T$ . It should be noted that the scale factor  $\rho(s, \eta)$  in the case of particle production has such properties. Because of that and for the sake of completeness, we start the story about the jet scale factor with short description of the properties of charged particle multiplicity density based on the results of Monte Carlo simulations.

The PYTHIA Monte Carlo generator [14] is used for calculations of charged particle multiplicity density in hadron–hadron ( $pp$ ,  $\pi p$ ) collisions in high energy range and at pseudo-rapidity  $\eta = 0$ . In both the cases, the dependence of density  $\rho$  on energy  $\sqrt{s}$  at  $\eta = 0$  was fitted by the function:  $\rho(s) = a \cdot s^b$ , where  $a$  and  $b$  are free parameters. Choice of the fitting function reflects the experimentally observed power law dependence of charged particle density on energy. On the other hand, the properties of this power law should be

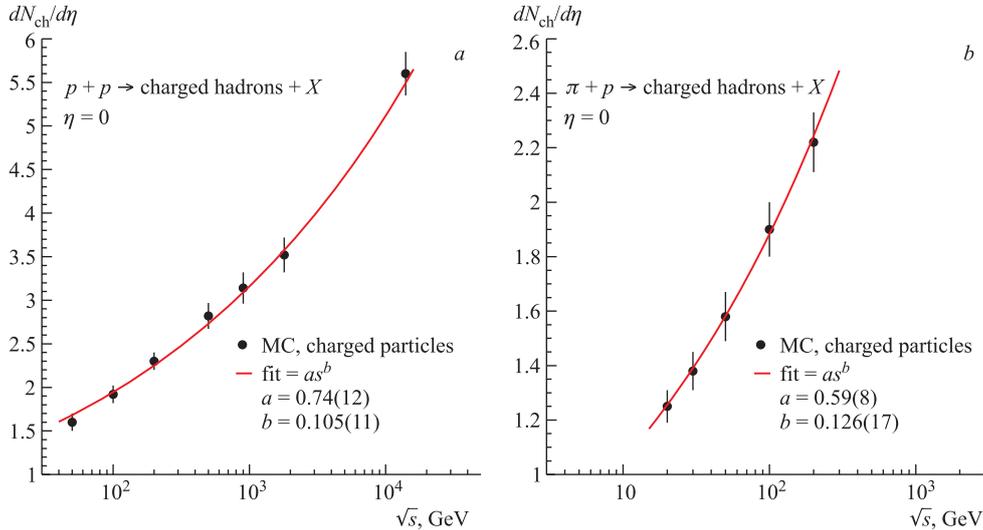


Fig. 1. Charged particle multiplicity density  $dN_{\text{ch}}/d\eta$  at  $\eta = 0$  as a function of energy  $\sqrt{s}$  in  $pp$  collisions (a) and in  $\pi p$  collisions (b)

a consequence of the Pomeron trajectory with intercept  $\Delta = \alpha_P - 1$ . Based on analysis of available experimental data, the value of the quantity was found to be 0.105. Charged particle density  $\rho(s)$  in  $pp$  interactions was simulated in the energy range  $\sqrt{s} = 50 \div 14000$  GeV. The value of  $d\sigma_{\text{ch}}/d\eta$  for every energy was normalized to the corresponding value of the inelastic cross section  $\sigma_{\text{inel}}$ . The results of simulations are shown in Fig. 1, *a*. As can be seen, the fit is satisfactory, with parameters equal to  $a = 0.74(12)$  and  $b = 0.105(11)$ . This result fully agrees with theoretical predictions and available experimental results. It is expected that the multiplicity density of charged particles at  $\sqrt{s} = 14$  TeV will follow the same energy dependence, but it is, in principle, still an open question. The Monte Carlo simulations of charged particle density at LHC, which are in progress (see, for example, ATLAS TDR [15, p. 480]), give results for multiplicity density at  $\eta = 0$  in the range from 4.5 up to 10. Charged particle density  $dN_{\text{ch}}/d\eta$  in  $\pi p$  collisions was simulated in the energy range  $\sqrt{s} = 10 \div 200$  GeV. The chosen energy range is relatively narrow, but it is, at the moment, experimentally available. The results of simulations are presented in Fig. 1, *b*. The parameter values were found to be  $a = 0.59(8)$  and  $b = 0.126(17)$ . For  $pp$  and  $\pi p$  collisions, we have obtained practically the same value for parameter  $b$  (within the errors). Also, in the energy region from 50 to 200 GeV there is no difference between densities in  $pp$  and  $\pi p$  collisions.

The power law dependence of charged particle density on energy  $\sqrt{s}$  is valid for  $pA$  too. In the case of  $pA$  collisions, the densities of charged particles can be parameterized [2] by the formula  $dN_{\text{ch}}/d\eta \simeq 0.67A^{0.18}s^{0.105}$ , where  $A$  is the atomic weight of the corresponding nucleus.

## 2. JET ENERGY DENSITY

In the case of jets, the situation is much more complicated. For example, in [12] the average jet multiplicity density dependence on energy  $\rho(s)$ , resulting from requirements of  $z$  scaling, is used for analysis of jet production at high energies. The authors used different experimental results on jet cross sections to produce semi-empirical energy dependence of jet scale factor. The result of that analysis is reproduced in Table 1. Also, the authors give the prediction of jet multiplicity density at LHC energies but emphasized that high-accuracy measurements of absolute cross-section normalization and the jet density are very important to verify the energy independence of the scaling function  $\psi(z)$ . On the other hand, the search for the «universal» jet scale factor is complicated because the cross sections for jet production have nontrivial behavior. The cross sections for production of jets with a fixed transverse energy  $E_T$  rise with  $\sqrt{s}$ . This is because the important  $x$  values decrease and there are more partons at smaller  $x$ . But cross sections for jets with transverse momentum that is a fixed fraction of  $\sqrt{s}$  fall with  $\sqrt{s}$ . This is mostly because the partonic cross sections fall with  $E_T$  like  $E_T^{-2}$ .

Table 1. The average jet multiplicity density  $\rho(s)$  normalized to the value at  $\sqrt{s} = 1800$  GeV in  $\bar{p}p$  and  $pp$  collisions over the central pseudorapidity range as a function of collision energy [12]

$\sqrt{s}$ , GeV	63	200	630	1000	1800	7000	14000
Average jet density (normalized)	0.35	0.5	0.67	0.84	1	1.57	1.95

Keeping all that in mind, we performed Monte Carlo analysis of jet production and found the variable that satisfied all the criteria. The detailed description is given below and started with definitions of the variables used in jet production analysis.

**2.1. Main Definitions.** Jets are experimentally defined as the amount of energy deposited in the cone of radius  $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$  in the space  $(\eta, \phi)$ , where  $\Delta\eta$  and  $\Delta\phi$  specify the extent of the cone in the pseudorapidity and azimuth. The pseudorapidity  $\eta$  is determined via the center mass angle  $\theta$  by the formula  $\eta = -\ln(\tan(\theta/2))$ . In this work, the value of cone radius was taken to be  $R = 0.7$ .

The inclusive jet cross section measures the probability of observing a hadronic jet with a given  $E_T$  and  $\eta$  in a hadron–hadron collision. The inclusive jet cross section is usually expressed in terms of the invariant cross section

$$E \frac{d^3\sigma}{dp^3}. \quad (1)$$

In the experiments [16, 17], the measured variables are the transverse energy ( $E_T$ ) and pseudorapidity ( $\eta$ ). In terms of these variables, the cross section is expressed as follows:

$$\frac{d^2\sigma}{dE_T d\eta}. \quad (2)$$

The quantities (1) and (2) are related by

$$E \frac{d^3\sigma}{dp^3} = \frac{1}{2\pi E_T} \frac{d^2\sigma}{dE_T d\eta}. \quad (3)$$

Expression (3) follows if the jets are assumed to be massless. For most measurements, the cross section is averaged over some range of pseudorapidity. In this paper, as in [16, 17], we analyzed jets in the central pseudorapidity region  $|\eta| < 0.5$ .

**2.2. Results.** The PYTHIA6.2 Monte Carlo generator [14] is used for calculation of inclusive jet cross sections in hadron–hadron ( $pp$ ,  $\bar{p}p$ ) collisions in high energy range and for pseudorapidity  $\eta = 0$ . As a first step, we simulated the inclusive jet cross sections at Tevatron energies  $\sqrt{s} = 1800$  and 630 GeV. The comparison between the Monte Carlo simulations and experimental data [17] is shown in Fig. 2. Points denote experimental data, while crosses denote Monte Carlo results. The agreement is very good. But this agreement can be obtained only if the higher-order effects are included in PYTHIA code. This can be done in PYTHIA6.2 by including the so-called  $K$  factor. The  $K$  factor is the ratio of NLO cross section and LO cross section. In this case, we used the model with separate factors for ordinary and colour annihilation graphs. As expected, we can see very strong dependence on the jet transverse energy  $E_T$ .

In order to compare jets cross sections at two different colliding energies, the so-called scaled dimensionless cross section (SDCS) is used. This variable reads

$$\text{SDCS} = E_T^4 E \frac{d^3\sigma}{dp^3}. \quad (4)$$

The scaling hypothesis, which is motivated by the Quark-Parton Model, predicts that this variable plotted against  $x_T = (2E_T/\sqrt{s})$  will be independent of the collision energy  $\sqrt{s}$ .

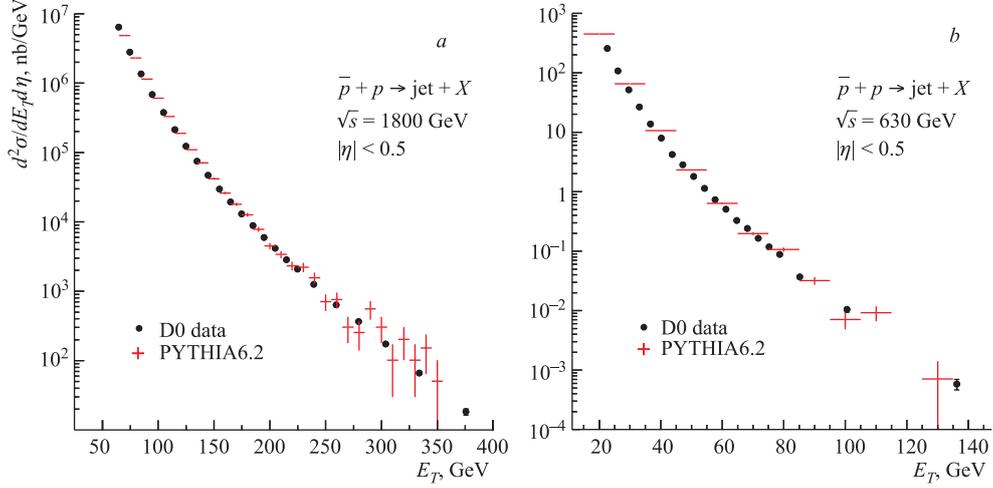


Fig. 2. The comparison between the MC simulations and experimental data [17] on inclusive jet cross sections for  $|\eta| < 0.5$  in  $\bar{p}p$  collisions at Tevatron energies  $\sqrt{s} = 1800$  GeV (a) and 630 GeV (b). Points — experiment; crosses — Monte Carlo results

However, QCD leads to scaling violation through the running coupling constant  $\alpha_s$  and the evolution of the PDFs [17]. Theoretically, the scaled dimensionless cross sections at different collision energies should be nearly exponential and close to one another [17], or in other words, the ratio of CDCSs for different energies should be a constant when plotted as a function of  $x_T$ . Figure 3, a shows the ratio of dimensionless inclusive jet cross sections at  $\sqrt{s} = 630$  and 1800 GeV and for  $|\eta| < 0.5$ , as well as corresponding results of Monte Carlo simulations<sup>1</sup>.

This variable was our starting point, because it practically does not depend on  $x_T$ . It can be seen in Fig. 3, a that  $\text{SDCS}(630)/\text{SDCS}(1800) > 1$ . It means that the SDCS decreases with increasing colliding energy. On the other hand, the scale factor for jets  $\rho_{\text{jet}}$  should take into account the rise of the jet  $E_T$  with increasing  $\sqrt{s}$  (for the same  $x_T$  bin) and the behavior of minijets production cross sections which increase with energy approximately as  $s^\delta \ln(s)$ , where  $\delta$  value is between 0 and 0.4 (from QCD expectations and HERA results).

So, the next step was to find the variable similar to the SDCS with taking into account the above requirements. The natural choice was to multiply the SDCS value by corresponding jet  $E_T$  and to divide by number of jets in  $\eta$  region. The variable we called «scaled jet energy density» then has the form

$$\text{Scaled jet energy density} = \text{SDCS} \frac{E_T}{N_{\text{jet}}}. \quad (5)$$

<sup>1</sup>J. Womersley wrote about these results: «Both CDF and D0 have measured the ratio of jet cross sections, exploiting a short period of data taking at the latter center-of-mass energy at the end of Run I. This ratio is expected to be a rather straightforward quantity to measure and to calculate. Unfortunately, the two experiments are not obviously consistent with each other (especially at low  $x_T$ ) or with the NLO QCD expectation for the ratio. At least two explanations have been suggested for the discrepancy. It seems that more work, both theoretical and experimental, is needed before this question can be resolved».

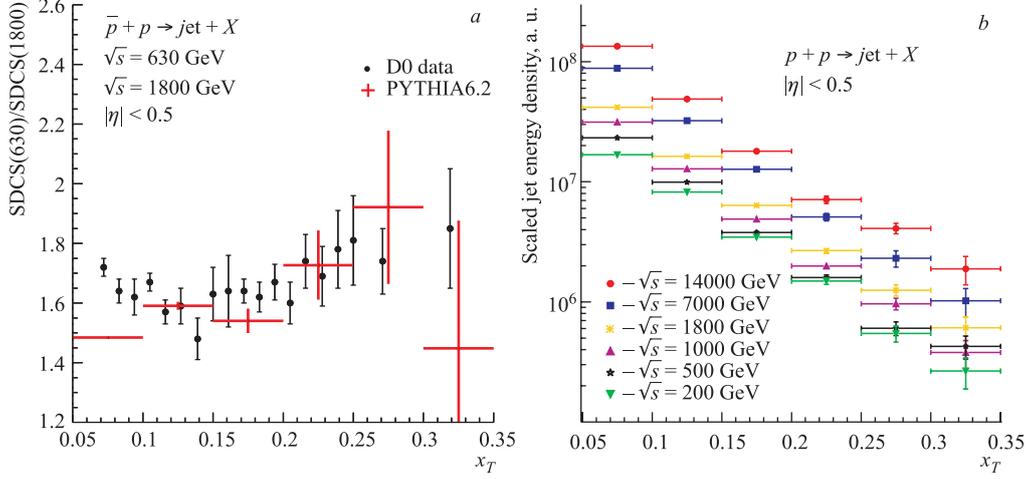


Fig. 3. *a*) The ratio of dimensionless inclusive jet cross sections at  $\sqrt{s} = 630$  and  $1800$  GeV and for  $|\eta| < 0.5$  in comparison with corresponding results of Monte Carlo simulations. *b*) The scaled jet energy density in the central pseudorapidity region for different collision energies (from 200 to 14000 GeV) as a function of  $x_T$

The shape of this variable for different colliding energies is  $x_T$ -independent. It increases with energy  $\sqrt{s}$  (from 200 to 14000 GeV). The results are shown in Fig. 3, *b*. This variable has the straightforward interpretation — it reflects the probability to have a jet with defined  $E_T$  in defined  $x_T$  and pseudorapidity region (in this case we talk about the central region). The  $x_T$  independence of scaled jet energy density is clearly seen when the values are normalized. Following the procedure described in [12] we applied the normalization by dividing the scaled jet energy density values for different  $\sqrt{s}$  by the corresponding value for  $\sqrt{s} = 1800$  GeV. The results are given in Fig. 4, *a*. These ratios show one interesting property: they do not depend on the value of cone radius  $R$  in jet finder algorithm. This is an important feature

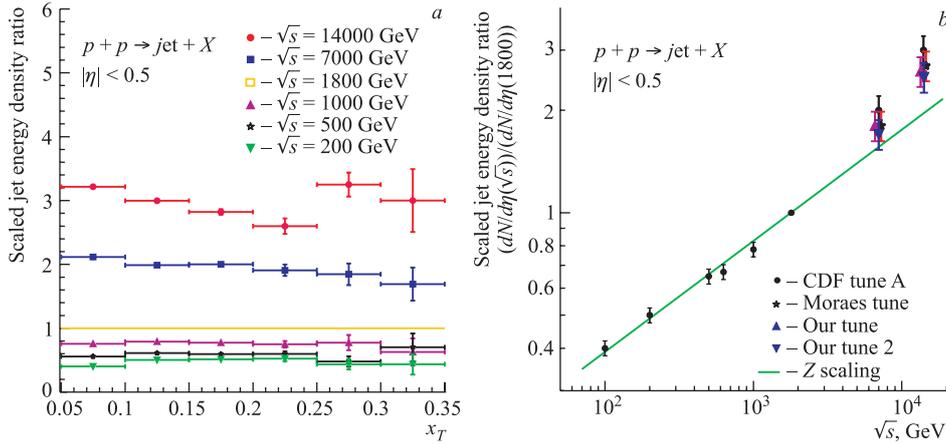


Fig. 4. The scaled jet energy density ratio (see text) in the central pseudorapidity region for different collision energies (from 200 to 14000 GeV) as a function of  $x_T$  (*a*) and  $\sqrt{s}$  (*b*)

because there is an indication that jets at LHC will be broader than expected [15]. So, on the basis of these variable properties, we conclude that this variable can be a good candidate for the role of scale factor ( $\rho_{\text{jet}}$ ) for jet analysis in the framework of  $z$ -presentation.

Thus, Fig. 4, *b* shows the ratio  $\rho_{\text{jet}}(\sqrt{s})/\rho_{\text{jet}}(1800)$  as a function of  $\sqrt{s}$  compared with predictions of  $z$  scaling. The first impression is that corresponding values agree very well up to Tevatron energies. In other words, introduction of the new variable fully confirms  $z$ -scaling predictions for jets for available energies.

### 3. DISCUSSION

However, we should be very careful with the results of Monte Carlo simulations at LHC energy. Extrapolations to LHC energies, based on measurements at the Tevatron, show the importance of taking into account the processes wherein (relatively) small transverse momenta are involved. The description of this problem is given in [18]: «Most of the time the protons will pass through each other with low amount of momentum (low- $p_T$ ) being transferred between the interacting partons. Occasionally there will be a hard parton-parton collision, resulting in large transverse momentum outgoing particles. Perturbative QCD is highly successful when applied to hard processes (large- $p_T$ ) but cannot be applied to soft interactions (low- $p_T$ ). Alternative approaches to describe soft processes are therefore required. PYTHIA's model for hadron-hadron collisions attempts to extend perturbative (high- $p_T$ ) picture down to low- $p_T$  region considering the possibility that multiple parton scattering takes place in hadron-hadron collisions». These «soft» processes can violate expected distribution even in hard processes. For example, the violation of KNO scaling is also attributed to secondary processes taking place in the hadron scattering.

The problem of accounting low- $p_T$  processes is present at the Tevatron energies, too. It was found that the default PYTHIA settings do not describe the minimum bias and underlying event data at CDF and D0 experiments. But, with appropriate tunings for PYTHIA [18, 19], those minimum bias and underlying event data can be described. So-called CDF tune A is the best model describing experimental data from the Tevatron. However, it fails to reproduce several minimum bias distributions at lower energies. On the other hand, tune from [18] gives reasonable description of underlying event data and nice description of minimum bias distributions. The relevant PYTHIA6.2 parameters values in different tunings [18, 19] are shown in Table 2.

This problem is interesting also in the case of jet production. It should be noted that the results for jet energy density shown in Figs. 2–4 are obtained with CDF tune A parameters with fixed  $p_T$  cut for multiple interactions at different collision energies. The changes in the results compared with default PYTHIA values are small at energies of up to 1800 GeV, but the situation could be quite different at the LHC.

At the LHC the important topic will be multiple parton scattering, i.e., the simultaneous occurrence of two independent hard (semihard, soft) scatterings in the same interaction. On the other hand, in a hard scattering process, the underlying event has a hard component (initial + final-state radiation and particles from the outgoing hard scattered partons) and a soft component (beam-beam remnants). In case of such extreme colliding energies the small differences in «physical scenarios» can produce sizeable differences in scaled jet energy density. Analyzing the values of the parameters in Table 2, all the tunes assume smooth transition

Table 2. The relevant PYTHIA6.2 parameters values in different tunings [18, 19]

Parameter	Default	CDF tune A	Moraes tune	Our tune
PARP(67)	1.0	4.0	1.0	1.0
MSTP(82)	1	4	4	4
PARP(82)	1.9	2.0	1.8	1.8
PARP(84)	0.2	0.4	0.5	0.6
PARP(85)	0.33	0.9	0.33	0.66
PARP(86)	0.66	0.95	0.66	0.66
PARP(89)	1000	1800	1000	1000
PARP(90)	0.16	0.25	0.16	0.16

between high- and low- $p_T$  regions ( $MSTP(82) = 4$ ) instead of cut on  $p_T$  ( $MSTP(82) = 1$ ). It can be seen that main changes are for values of parameters PARP(84), PARP(85) and PARP(86). PARP(84) regulates the size of the hadron core if the double Gaussian matter distribution in hadrons is assumed. PARP(85) and PARP(86) describe the probability that multiple parton scattering produces two gluons with color connections to the nearest neighbors or as a closed gluon loop. We also applied our tune by increasing the probability of producing two gluons with color connections to the nearest neighbors in multiple interactions and by increasing the size of the hadrons core (right column in Table 2). This results in decrease of the scaled jet energy density ratio at the LHC energies and the corresponding values are very close to prediction of  $z$  scaling. Comparing the values of jet energy density at the LHC energy (Fig. 4, *b*) simulated with different tunings [18, 19] and our tune, it can be concluded that this variable is sensitive (at the level of  $10 \div 20\%$ ) to the changes of these parameters.

## CONCLUSIONS

In this work we tried to find the variable that will reflect the main features of the jet production in low transverse energy range at a given energy and play the role of the scale factor for description of jets in the framework of  $z$  scaling. The PYTHIA6.2 Monte Carlo generator was used for calculation of jet production in proton–proton collisions over a high energy range ( $\sqrt{s} = 200–14000$  GeV) and for pseudorapidity  $\eta = 0$ . We introduced the variable we called the «scaled jet energy density». The scaled jet energy density is the probability to have a jet with defined  $E_T$  in defined  $x_T$  and pseudorapidity regions. Its definition is related to the «scaled dimensionless cross section» and its features (for example,  $x_T$  independence) show that this variable can be used in the studies of jet production at high energies. The important result is that the properties of the new variable fully confirms  $z$ -scaling predictions for jet production at available energies. Detailed analysis of the variable behavior at the LHC energies show that it is sensitive to relatively small differences in applied «physical scenarios» in standard Monte Carlo generators. The fact is that there are sizeable uncertainties in LHC predictions generated by different models so the alternative approach like  $z$  scaling is very important for understanding of inclusive processes of jet production at high energies.

**Acknowledgements.** One of the authors (M.T.) would like to thank I.Zborovský for numerous fruitful and stimulating discussions of the obtained results.

REFERENCES

1. Zborovsky I. *et al.* // Phys. Rev. D. 1996. V. 54. P. 5548.
2. Zborovsky I. *et al.* // Phys. Rev. C. 1999. V. 59. P. 2227.
3. Nottale L. Fractal Space Time and Microphysics. Singapore: World Scientific, 1993.
4. Feynman R. P. // Phys. Rev. Lett. 1969. V. 23. P. 1415.
5. Bjorken J. D. // Phys. Rev. 1969. V. 179. P. 1547;  
Bjorken J. D., Paschos E. A. // Ibid. V. 185. P. 1975.
6. Bosted P. *et al.* // Phys. Rev. Lett. 1982. V. 49. P. 1380.
7. Benecke J. *et al.* // Phys. Rev. 1969. V. 188. P. 2159.
8. Baldin A. M. // Part. Nucl. 1977. V. 8. P. 429.
9. Stavinsky V. S. // Part. Nucl. 1979. V. 10. P. 949.
10. Leksin G. A. Preprint ITEP-147. M., 1976. 12 p.; Proc. of the XVIII Intern. Conf. on High Energy Physics, Tbilisi, 1976. Dubna, 1977. P. A6-3.
11. Koba Z., Nielsen H. B., Olesen P. // Nucl. Phys. B. 1972. V. 40. P. 317.
12. Tokarev M., Dedovich T. // Intern. J. Mod. Phys. A. 2000. V. 15. P. 3495.
13. Tokarev M. V., Potrebenikova E. V. // Comp. Phys. Commun. 1999. V. 117. P. 229.
14. Sjostrand T. *et al.* // Comp. Phys. Commun. 2001. V. 135. P. 238.
15. ATLAS TDR 15. CERN/LHCC/99-15. V. II.
16. Abe F. *et al.* (CDF Collab.) // Phys. Rev. Lett. 1996. V. 77. P. 438.
17. Abbott B. *et al.* (D0 Collab.) // Phys. Rev. Lett. 1999. V. 82. P. 2451; Phys. Rev. D. 2001. V. 64. P. 032003; FERMILAB-PUB-00/216-E (DEC 2000).
18. Moraes A., Buttar C., Dawson I. Talk given at the Meeting «HERA and the LHC», CERN, March 26, 2004.
19. Korytov A. Soft QCD Phenomena in High- $E_T$  Jet Events at CDF. Talk given at the EPS 2003 Meeting, Aachen, July 17–23, 2003.

Received on December 16, 2004.