

BOSE–EINSTEIN CORRELATIONS OF LIGHT HADRONS AND THE STOCHASTIC SCALE OF PARTICLE EMITTER SOURCE

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Based on quantum field theory at finite temperature, we obtained new results for two-particle Bose–Einstein correlation (BEC) function $C_2(Q)$ in case of light hadrons. The important parameters of BEC function related to the size of emitting source, mean multiplicity, stochastic forces range and correlation radius with the particle energy and the mass dependence, and the temperature of the source are obtained in analytical form for the first time. Not only is the correlation between identical bosons explored but also the off-correlation between non-identical particles is proposed.

На основе квантовой теории поля при конечной температуре получены новые результаты для двухчастичной корреляционной функции $C_2(Q)$ Бозе–Эйнштейна. Впервые получены важные параметры корреляционной функции $C_2(Q)$, связанные с размерами области источника частиц с учетом влияния возмущающих стохастических сил при конечной температуре, средней множественности частиц, зависимости размеров области источника от массы и энергии частиц. Проведено исследование идентичных частиц, а также получены новые корреляционные эффекты для частиц с противоположными зарядами.

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INTRODUCTION

To explore the correlations of Bose–Einstein type (BEC), one needs to use the properties of a particle detector, e.g., its tracking system to study the hadron processes in some energy region. Such a study will be done soon in the next papers.

This paper describes an attempt to address the problems of BEC within the theoretical aspects prior to analysis of the real data.

Over the past few decades, a considerable number of studies have been done on the phenomena of multiparticle correlations observed in high-energy particle collisions (see the review in [1]). It is well understood that the studies of correlations between produced particles, the effects of coherence and chaoticity, an estimation of particle emitting source size and the temperature play an important role in this branch of high-energy physics.

By studying the Bose–Einstein correlations of identical particles (e.g., like-sign charged particles of the same sort) or even off-correlations with respect to different charged bosons, it is possible to predict and even experimentally determine the time and spatial region over which particles do not have the interactions. Such a surface is called decoupling one. In fact, for an evolving system such as, e.g., $p\bar{p}$ collisions, it is not really a surface, since at each

time there is a spread out surface due to fluctuations in the final interactions, and the shape of this surface evolves even in time. The particle source is not approximately constant because of energy-momentum conservation constraint.

More than half a century ago Hanbury-Brown and Twiss [2] used BEC between photons to measure the size of distant stars. In the papers [3–10], the master equations for evolution of thermodynamic system created at the final state of the (very) high multiplicity process were established. The equations have the form of the field operator evolution equation (Langevin-like [11]) that allows one to gain the basic features of the emitting source space-time structure. In particular, it has been conjectured and further confirmed that the BEC is strongly affected by nonclassical off-shell effect.

The shapes of BEC function were experimentally established in the LEP experiments ALEPH [12], DELPHI [13] and OPAL [14], and by ZEUS collaboration at HERA [15], which also indicated a dependence of the measured so-called correlation radius on the hadron (π , K) mass. The results for $\pi^\pm\pi^\pm$ and $\pi^\pm\pi^\mp$ correlations with $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV were published by E735 collaboration in [16].

One of the aims of this paper is to carry out the extended model of BEC in the framework of quantum field theory at finite temperature (QFT $_\beta$) approach to be applied later to real experimental data on two-particle BEC. It is known that the effective temperature of the vacuum or the ground state or even the thermalized state of particles distorted by external forces is occurring in models quantized in external fields. One of the main parameters of the model considered here is the temperature of the particle source under the random source operator influence.

Among the results obtained in this paper we mention a theoretical estimate accessible to experimental measurements of two-particle BEC and proof that quantum-statistical evolution of particle–antiparticle correlations is not an artifact of the standard formalism but quite a general property of particle physics. The effect (called surprising one) for non-identical particles correlations was predicted already in [17].

1. TWO-PARTICLE BEC

A pair of bosons with the mass m produced incoherently (in ideal nondisturbed, non-interacting cases) from an extended source will have an enhanced probability $C_2(p_1, p_2) = N_{12}(p_1, p_2)/[N_1(p_1)N_2(p_2)]$ to be measured (in terms of differential cross section σ), where

$$N_{12}(p_1, p_2) = \frac{1}{\sigma} \frac{d^2\sigma}{d\Omega_1 d\Omega_2} \quad (1)$$

to be found close in 4-momentum space \mathfrak{R}_4 when detected simultaneously, as compared to the case where they are detected separately with

$$N_i(p_i) = \frac{1}{\sigma} \frac{d\sigma}{d\Omega_i}, \quad d\Omega_i = \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_{p_i}}, \quad E_{p_i} = \sqrt{\mathbf{p}_i^2 + m^2}, \quad i = 1, 2. \quad (2)$$

The following relation can be used to retrieve the BEC function $C_2(Q)$:

$$C_2^{ij}(Q) = \frac{N^{ij}(Q)}{N^{\text{ref}}(Q)}, \quad i, j = +, -, 0, \quad (3)$$

where $N^{ij}(Q)$ in general case refer to the numbers $N^{\pm\pm}(Q)$ for like-sign charged particles (e.g., $\pi^\pm\pi^\pm, K^\pm K^\pm, \dots$); $N^{\pm\mp}(Q)$ — for different charged bosons (e.g., $\pi^\pm\pi^\mp, K^\pm K^\mp, \dots$) or even for neutral charged particles $N^{00}(Q)$ (e.g., $\pi^0\pi^0, K^0K^0, \dots$) with

$$Q = \sqrt{-(p_1 - p_2)_\mu(p_1 - p_2)^\mu} = \sqrt{M^2 - 4m^2}. \quad (4)$$

In formulas (3) and (4) N^{ref} is the number of pairs without BEC and p_{μ_i} ($i = 1, 2$) are four-momenta of produced particles, $M = \sqrt{(p_1 + p_2)_\mu^2}$ is the invariant mass of the pair of bosons. For reference sample, $N^{\text{ref}}(Q)$, the like-sign pairs from different events can be used. It is commonly assumed that the maximum of two-particle BEC function $C_2^{ii}(Q)$ is 2 for $\mathbf{p}_1 = \mathbf{p}_2$ if no any distortion and final-state interactions are taken into account.

In general, the shape of BEC $C_2(Q)$ function is model-dependent. The most simple form of Goldhaber-like parameterization for $C_2(Q)$ [18, 19] has been used for data fitting:

$$C_2(Q) = C_0(1 + \lambda e^{-Q^2 R^2})(1 + \varepsilon Q), \quad (5)$$

where C_0 is the normalization factor; λ is the so-called chaoticity strength factor, meaning $\lambda = 1$ for fully chaotic and $\lambda = 0$ for fully coherent sources; the parameter R is interpreted as a radius of the particle source, often called the «correlation radius», and assumed to be spherical in this parameterization. The linear term in (5) is often supposed to be accounted for long-range correlations outside the region of BEC. However, the origin of these long-range correlations, as well as the value of ε , is unknown yet. Note that distribution of, e.g., pions and kaons can be far from isotropic, usually concentrated in narrow jets, and further complicated by the fact that the light particles with masses less than 1 GeV often come from decays of long-lived heavier resonances and also are under the random chaotic interactions caused by other fields in the thermal bath. In the parameterization (5) all of these problems are embedded in the random chaoticity parameter λ .

We obtained the $C_2(Q)$ function within QFT $_\beta$ approach [3–7] in the form

$$C_2(Q) = \xi(N) \left[1 + \frac{2\alpha}{(1+\alpha)^2} \sqrt{\tilde{\Omega}(Q)} + \frac{1}{(1+\alpha)^2} \tilde{\Omega}(Q) \right] F(Q, \Delta x), \quad (6)$$

where $\xi(N)$ depends on the multiplicity N as

$$\xi(N) = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2}. \quad (7)$$

The consequence of Bogolyubov's principle of weakening of correlations at large distances [20] is given by the function $F(Q, \Delta x)$ of weakening of correlations at large spread of relative position Δx :

$$F(Q, \Delta x) = \frac{f(Q, \Delta x)}{f(p_1)f(p_2)} = 1 + r_f Q + \dots \quad (8)$$

normalized as $F(Q, \Delta x = \infty) = 1$. Here, $f(Q, \Delta x)$ is the two-particle distribution function with Δx , while $f(p_i)$ are one-particle probability functions with $i = 1, 2$; r_f is a measure of weakening of correlations with Δx : $r_f \rightarrow 0$ as $\Delta x \rightarrow \infty$.

The important parameter α in (6) summarizes our knowledge of other than space-time characteristics of the particle emitting source.

The $\tilde{\Omega}(Q)$ in (6) has the following structure in momentum space:

$$\tilde{\Omega}(Q) = \Omega(Q)\gamma(n), \quad (9)$$

where

$$\Omega(Q) = \exp(-\Delta_{p\mathfrak{R}}) = \exp[-(p_1 - p_2)^\mu \mathfrak{R}_{\mu\nu} (p_1 - p_2)^\nu] \quad (10)$$

is the smearing smooth dimensionless generalized function, $\mathfrak{R}_{\mu\nu}$ is the (nonlocal) structure tensor of the space-time size (BEC formation domain), and it defines the spherically similar domain of emitted (produced) particles.

The function $\gamma(n)$ in (9) reflects the quantum features of BEC pattern and is defined as

$$\gamma(n) = \frac{n^2(\bar{\omega})}{n(\omega) n(\omega')}, \quad n(\omega) \equiv n(\omega, \beta) = \frac{1}{e^{(\omega-\mu)\beta} - 1}, \quad \bar{\omega} = \frac{\omega + \omega'}{2}, \quad (11)$$

where $n(\omega, \beta)$ is the mean value of quantum numbers for BE statistics particles with the energy ω and the chemical potential μ in the thermal bath with statistical equilibrium at the temperature $T = 1/\beta$. The following condition $\sum_f n_f(\omega, \beta) = N$ is evident, where the discrete index f reflects the one-particle state f .

Note that it has been commonly assumed for a long time that there are no correlation effects among non-identical particles (e.g., among different charged particles). This assumption is often used in normalizing the experimental data on C_2^{ii} with respect to C_2^{ij} . In the absence of interference or correlation effects between, e.g., π^+ and π^- mesons it is supposed that $C_2^{+-} = 1$.

In terms of time-like R_0 , longitudinal R_L and transverse R_T components of the space-time size R_μ , the distribution $\Delta_{p\mathfrak{R}}^{ij}$ looks like ($i, j = +, -, 0$)

$$\Delta_{p\mathfrak{R}}^{ij} \rightarrow \Delta_{pR}^{ij} = (\Delta p^0)^2 R_0^2 + (\Delta p^L)^2 R_L^2 + (\Delta p^T)^2 R_T^2. \quad (12)$$

Seeking for simplicity, one has ($R_L = R_T = R$)

$$\Delta_{pR}^{ii} = (p_1^0 - p_2^0)^2 R_0^2 + (\mathbf{p}_1 - \mathbf{p}_2)^2 \mathbf{R}^2 \quad (13)$$

for like-sign charged bosons, while

$$\Delta_{pR}^{ij} = (p_1^0 + p_2^0)^2 R_0^2 + (\mathbf{p}_1 + \mathbf{p}_2)^2 \mathbf{R}^2 \quad (14)$$

for different charged particles.

Obviously, the BEC effect with $\Omega^{ij} = \exp(-\Delta_{pR}^{ij})$ is smaller than that defined by $\Omega^{ii} = \exp(-\Delta_{pR}^{ii})$. The distribution Ω^{ij} gives rise to an off-correlation pattern between different charged particles. The evidence of C_2^{ij} correlation represents a quantum-statistical correlation between a particle and an antiparticle. Since we did not follow special assumptions on the quantum operator level for C_2 from the initial stage, it may correspond to a physically real and observable effect. This pattern may lead to a new squeezing state of correlation region. We obtain that within the QFT $_\beta$ the BEC is more generally sensitive to particle–antiparticle correlations than it would be expected from the two-particle (symmetrized) wave function which never leads to such correlations.

2. GREEN FUNCTION

In this paper, we would like to focus on the role of the particle mass, which influences the correlations between particles. To explore this problem, one must derive the memory history of evolution of particles produced in high-energy collisions using the general properties of QFT at finite temperature.

For definite calculations we consider the thermal complex scalar field $\Phi(x)$ that corresponds to π^\pm mesons with the standard definition of the Fourier transformed propagator $F[\tilde{G}(p)]$:

$$F[\tilde{G}(p)] = G(x - y) = \text{Tr} \{T[\Phi(x)\Phi(y)]\rho_\beta\}, \quad (15)$$

with $\rho_\beta = e^{-\beta H}/\text{Tr} e^{-\beta H}$ being the density matrix of a local system in equilibrium at temperature $T = \beta^{-1}$ under the Hamiltonian H .

We consider the interaction of $\Phi(x)$ with the external scalar field given by the potential U . In contrast to an electromagnetic field, this potential is a scalar one, but it is not a component of the four-vector. The Lagrangian density can be written as

$$L(x) = \partial_\mu \Phi^*(x) \partial^\mu \Phi(x) - (m^2 + U)\Phi^*(x)\Phi(x)$$

and the equation of motion is

$$(\nabla^2 + m^2)\Phi(x) = -J(x), \quad (16)$$

where $J(x) = U\Phi(x)$ is the source density operator. A simple model like this allows one to investigate the origin of the unstable state of the thermalized equilibrium in a nonhomogeneous external field under the influence of source density operator $J(x)$. For example, the source can be considered as δ -like generalized function $J(x) = \tilde{\mu} \rho(x, \epsilon)\Phi(x)$ in which $\rho(x, \epsilon)$ is a δ -like succession giving the δ function as $\epsilon \rightarrow 0$ (where $\tilde{\mu}$ is some massive parameter). This model is useful because the δ -like potential $U(x)$ provides the model conditions for restricting the particle emission domain (or the deconfinement region). We suggest the following form:

$$J(x) = -\Sigma(i\partial_\mu)\Phi(x) + J_R(x),$$

where the source $J(x)$ decomposes into a regular systematic motion part $\Sigma(i\partial_\mu)\Phi(x)$ and the random source $J_R(x)$. Thus, the equation of motion (16) becomes

$$[\nabla^2 + m^2 - \Sigma(i\partial_\mu)]\Phi(x) = -J_R(x),$$

and the propagator satisfies the following equation:

$$[-p_\mu^2 + m^2 - \tilde{\Sigma}(p_\mu)]\tilde{G}(p_\mu) = 1. \quad (17)$$

The random noise is introduced with a random operator $\eta(x) = -m^{-2}\Sigma(i\partial_\mu)$, for that the equation of motion looks like

$$\{\nabla^2 + m^2[1 + \eta(x)]\}\Phi(x) = -J_R(x). \quad (18)$$

We assume that $\eta(x)$ varies stochastically with the certain correlation function (CF), e.g., the Gaussian CF:

$$\langle \eta(x)\eta(y) \rangle = C \exp(-z^2 \mu_{\text{ch}}^2), \quad z = x - y,$$

where C is the strength of the noise described by the distribution function $\exp(-z^2/L_{\text{ch}}^2)$ with L_{ch} being the noise characteristic scale. Both C and μ_{ch} define the influence of the (Gaussian) noise on the correlations between particles that «feel» an action of an environment. The solution of Eq. (18) is

$$\Phi(x) = - \int dy G(x, y) J_R(y), \quad (19)$$

where the Green function obeys the equation

$$\{\nabla^2 + m^2[1 + \eta(x)]\}G(x, y) = \delta(x - y).$$

The final aim might have been to find the solution of Eq. (19), and then average it over random operator $\eta(x)$. Note that the operator $M(x) = \nabla^2 + m^2[1 + \eta(x)]$ in the causal Green function

$$G(x, y) = \frac{1}{M(x) + i0} \delta(x - y)$$

is not definitely positive. However, we shall formulate another approach, where the random force influence is introduced on the particle operator level.

We introduce the general non-Fock representation in the form of the operator generalized functions

$$b(x) = a(x) + r(x), \quad (20)$$

$$b^+(x) = a^+(x) + r^+(x), \quad (21)$$

where the operators $a(x)$ and $a^+(x)$ obey the canonical commutation relations (CCR):

$$[a(x), a(x')] = [a^+(x), a^+(x')] = 0,$$

$$[a(x), a^+(x')] = \delta(x - x').$$

The operator-generalized functions $r(x)$ and $r^+(x)$ in (20) and (21), respectively, include random features describing the action of the external forces.

Both b^+ and b obviously define the CCR representation. For each function f from the space $S(\mathfrak{R}_\infty)$ of smooth decreasing functions, one can establish new operators $b(f)$ and $b^+(f)$:

$$b(f) = \int f(x)b(x) dx = a(f) + \int f(x)r(x) dx,$$

$$b^+(f) = \int \bar{f}(x)b^+(x) dx = a^+(f) + \int \bar{f}(x)r^+(x) dx.$$

The transition from the operators $a(x)$ and $a^+(x)$ to $b(x)$ and $b^+(x)$, obeying those commutation relations as $a(x)$ and $a^+(x)$, leads to linear canonical representations.

3. EVOLUTION EQUATION

Referring to [3–7] for details, let us recapitulate here the main points of our approach in the quantum case: the collision process produces a number of particles, out of which we select only one (we assume for simplicity that we are dealing only with identical bosons) and describe it by stochastic operators $b(\mathbf{p}, t)$ and $b^+(\mathbf{p}, t)$, carrying the features of annihilation and creation operators, respectively. The rest of the particles are then assumed to form a kind of heat bath, which remains in an equilibrium characterized by a temperature T (one of our parameters). We also allow for some external (relative to the above heat bath) influence on our system. The time evolution of such a system is then assumed to be given by a Langevin-type equation [3–7] for stochastic operator $b(\mathbf{p}, t)$

$$i\partial_t b(\mathbf{p}, t) = A(\mathbf{p}, t) + F(\mathbf{p}, t) + P \quad (22)$$

(and a similar conjugate equation for $b^+(\mathbf{p}, t)$). We assume an asymptotic free undistorted operator $a(\mathbf{p}, t)$, and that the deviation from the asymptotic free state is provided by the random operator $r(\mathbf{p}, t)$: $a(\mathbf{p}, t) \rightarrow b(\mathbf{p}, t) = a(\mathbf{p}, t) + r(\mathbf{p}, t)$. This means, e.g., that the particle density number (a physical number) $\langle n(\mathbf{p}, t) \rangle_{\text{ph}} = \langle n(\mathbf{p}) \rangle + O(\epsilon)$, where $\langle n(\mathbf{p}, t) \rangle_{\text{ph}}$ means the expectation value of a physical state, while $\langle n(\mathbf{p}) \rangle$ denotes that of an asymptotic state. If we ignore the deviation from the asymptotic state in equilibrium, we obtain an ideal fluid. One otherwise has to consider the dissipation term; this is why we use the Langevin scheme to derive the evolution equation, but only on the quantum level. We derive the evolution equation in an integral form that reveals the effects of thermalization.

Equation (22) is supposed to model all aspects of the hadronization processes (or even deconfinement). The combination $A(\mathbf{p}, t) + F(\mathbf{p}, t)$ in the r.h.s of (22) represents the so-called *Langevin force* and is therefore responsible for the internal dynamics of particle emission, as the memory term A causes dissipation and is related to stochastic dissipative forces [3–7]

$$A(\mathbf{p}, t) = \int_{-\infty}^{+\infty} d\tau K(\mathbf{p}, t - \tau) b(\mathbf{p}, \tau),$$

with $K(\mathbf{p}, t)$ being the kernel operator describing the virtual transitions from one (particle) mode to another. At any dependence of the field operator K on the time, the function $A(\mathbf{p}, t)$ is defined by the behavior of the system at the precedent moments. The operator $F(\mathbf{p}, t)$ in (22) is responsible for the action of a heat bath of absolute temperature T on a particle in the heat bath, and under the appropriate circumstances is given by

$$F(\mathbf{p}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \psi(p_\mu) \hat{c}(p_\mu) e^{-i\omega t}.$$

The heat bath is represented by an ensemble of coupled oscillators, each described by the operator $\hat{c}(p_\mu)$ such that $[\hat{c}(p_\mu), \hat{c}^+(p'_\mu)] = \delta^4(p_\mu - p'_\mu)$, and is characterized by the noise spectral function $\psi(p_\mu)$. Here, the only statistical assumption is that the heat bath is canonically distributed. The oscillators are coupled to a particle, which is in turn acted upon by an outside force. Finally, the constant term P in (22) (representing *an external source* term in

the Langevin equation) denotes a possible influence of some external force. This force would result, e.g., in a strong ordering of phases leading therefore to a coherence effect.

The solution of equation (22) is given in $S(\mathfrak{R}_4)$ by

$$\tilde{b}(p_\mu) = \frac{1}{\omega - \tilde{K}(p_\mu)} [\tilde{F}(p_\mu) + \rho(\omega_P, \epsilon)], \quad (23)$$

where ω in $\rho(\omega, \epsilon)$ was replaced by new scale $\omega_P = \omega/P$. It should be stressed that the term containing $\rho(\omega_P, \epsilon)$ as $\epsilon \rightarrow 0$ yields the general solution to Eq.(22). Notice that the distribution $\rho(\omega_P, \epsilon)$ indicates the continuous character of the spectrum, while the arbitrary small quantity ϵ can be defined by the special physical conditions or the physical spectra. On the other hand, this $\rho(\omega_P, \epsilon)$ can be understood as temperature-dependent succession $\rho(\omega, \epsilon) = \int dx \exp(i\omega - \epsilon)x \rightarrow \delta(\omega)$, in which $\epsilon \rightarrow \beta^{-1}$. Such a succession yields the restriction on the β -dependent second term in the solution (23), where at small enough T there is a narrow peak at $\omega = 0$.

From the scattering matrix point of view, the solution (23) has the following physical meaning: at a sufficiently outgoing past and future, the fields described by the operators $\tilde{a}(p_\mu)$ are free and the initial and the final states of the dynamic system are thus characterized by constant amplitudes. Both states, $\varphi(-\infty)$ and $\varphi(+\infty)$, are related to one another by an operator $S(\tilde{r})$ that transforms state $\varphi(-\infty)$ to state $\varphi(+\infty)$ while depending on the behaviour of $\tilde{r}(p_\mu)$:

$$\varphi(+\infty) = S(\tilde{r})\varphi(-\infty).$$

In accordance with this definition, it is natural to identify $S(\tilde{r})$ as the scattering matrix in the case of arbitrary sources that give rise to the intensity of \tilde{r} .

Based on QFT point of view, relation (20) indicates the appearance of the terms containing nonquantum fields that are characterized by the operators $\tilde{r}(p_\mu)$. Hence, there are terms with \tilde{r} in the matrix elements, and these \tilde{r} cannot be realized via real particles. The operator function $\tilde{r}(p_\mu)$ could be considered as the limit on an average value of some quantum operator (or even a set of operators) with an intensity that increases to infinity. The latter statement can be visualized in the following mathematical representation:

$$\tilde{r}(p_\mu) = \sqrt{\alpha \Xi(p_\mu, p_\mu)}, \quad \Xi(p_\mu, p_\mu) = \langle \tilde{a}^+(p_\mu) \tilde{a}(p_\mu) \rangle_\beta,$$

where α is the coherence (chaotic) function that gives the strength of the average $\Xi(p_\mu, p_\mu)$.

In principle, interaction with the fields described by \tilde{r} is provided by the virtual particles, the propagation process of which is given by the potentials defined by the \tilde{r} operator function.

The condition $M_{\text{ch}} \rightarrow 0$ (or $\Omega_0(R) \sim 1/M_{\text{ch}}^4 \rightarrow \infty$) in the representation

$$\lim_{p_\mu \rightarrow p'_\mu} \Xi(p_\mu, p'_\mu) = \lim_{Q^2 \rightarrow 0} \Omega_0(R) n(\bar{\omega}, \beta) \exp(-q^2/2) \rightarrow \frac{1}{M_{\text{ch}}^4} n(\omega, \beta),$$

with

$$\Omega_0(R) = \frac{1}{\pi^2} R_0 R_L R_T^2$$

means that the role of the arbitrary source characterized by the operator function $\tilde{r}(p_\mu)$ in $\tilde{b}(p_\mu) = \tilde{a}(p_\mu) + \tilde{r}(p_\mu)$ disappears.

4. GREEN FUNCTION AND KERNEL OPERATOR

Let us go to the thermal field operator $\Phi(x)$ by means of the linear combination of the frequency parts $\phi^+(x)$ and $\phi^-(x)$:

$$\Phi(x) = \frac{1}{\sqrt{2}} [\phi^+(x) + \phi^-(x)], \quad (24)$$

composed of the operators $\tilde{b}(p_\mu)$ and $\tilde{b}^+(p_\mu)$ as the solutions of equation (22) and conjugate to it, respectively:

$$\begin{aligned} \phi^-(x) &= \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2(\mathbf{p}^2 + m^2)^{1/2}} \tilde{b}^+(p_\mu) e^{ipx}, \\ \phi^+(x) &= \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2(\mathbf{p}^2 + m^2)^{1/2}} \tilde{b}(p_\mu) e^{-ipx}. \end{aligned}$$

The function $\Phi(x)$ obeys the commutation relation

$$[\Phi(x), \Phi(y)]_- = -iD(x)$$

with [21]

$$D(x) = \frac{1}{2\pi} \epsilon(x^0) \left[\delta(x^2) - \frac{m}{2\sqrt{x_\mu^2}} \Theta(x^2) J_1 \left(m\sqrt{x_\mu^2} \right) \right],$$

where $\epsilon(x^0)$ and $\Theta(x^2)$ are the standard unit and the step functions, respectively, while $J_1(x)$ is the Bessel function. On the mass shell, $D(x)$ becomes

$$D(x) \simeq \frac{1}{2\pi} \epsilon(x^0) \left[\delta(x^2) - \frac{m^2}{4} \Theta(x^2) \right].$$

One can easily find two equations of motion for the Fourier transformed operators $\tilde{b}(p_\mu)$ and $\tilde{b}^+(p_\mu)$ in $S(\mathfrak{R}_4)$:

$$[\omega - \tilde{K}(p_\mu)] \tilde{b}(p_\mu) = \tilde{F}(p_\mu) + \rho(\omega_P, \epsilon), \quad (25)$$

$$[\omega - \tilde{K}^+(p_\mu)] \tilde{b}^+(p_\mu) = \tilde{F}^+(p_\mu) + \rho^*(\omega_P, \epsilon), \quad (26)$$

which are transformed into new equations for the frequency parts $\phi^+(x)$ and $\phi^-(x)$ of the field operator $\Phi(x)$ (24):

$$i\partial_0 \phi^+(x) + \int_{\mathfrak{R}_4} K(x-y) \phi^+(y) dy = f(x), \quad (27)$$

$$-i\partial_0 \phi^-(x) + \int_{\mathfrak{R}_4} K^+(x-y) \phi^-(y) dy = f^+(x), \quad (28)$$

where

$$f(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3 (\mathbf{p}^2 + m^2)^{1/2}} [\tilde{F}(p) + \rho(\omega_P, \epsilon)] e^{-ipx}$$

and

$$f^+(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 (\mathbf{p}^2 + m^2)^{1/2}} [\tilde{F}^+(p) + \rho^*(\omega_P, \epsilon)] e^{ipx}.$$

Here, the field components $\phi^+(x)$ and $\phi^-(x)$ are under the effect of the nonlocal form factors $K(x-y)$ and $K^+(x-y)$, respectively. In general, these form factors can admit the description of locality for nonlocal interactions.

At this stage, it must be stressed that we have new generalized evolution Eqs. (27) and (28), which retain the general features of the propagating and interacting of the quantum fields with mass m that are in the heat bath (reservoir) and are chaotically distorted by other fields. For further analysis, let us rewrite Eqs. (27) and (28) in the following form:

$$i\partial_0\phi^+(x) + K(x) \star \phi^+(x) = f(x), \quad (29)$$

$$-i\partial_0\phi^-(x) + K^+(x) \star \phi^-(x) = f^+(x), \quad (30)$$

where $A(x) \star B(x)$ is the convoluted function of the generalized functions $A(x)$ and $B(x)$. Applying the direct Fourier transformation to both sides of Eqs. (29) and (30) with the following properties of the Fourier transformation:

$$F[K(x) \star \phi^+(x)] = F[K(x)]F[\phi^+(x)],$$

we get two equations

$$[p^0 + \tilde{K}(p_\mu)]\tilde{\phi}^+(p_\mu) = F[f(x)], \quad (31)$$

$$[-p^0 + \tilde{K}^+(p_\mu)]\tilde{\phi}^-(p_\mu) = F[f^+(x)]. \quad (32)$$

Multiplying Eqs. (31) and (32) by $-p^0 + \tilde{K}^+(p_\mu)$ and $p^0 + \tilde{K}(p_\mu)$, respectively, we find

$$[-p^0 + \tilde{K}^+(p_\mu)][p^0 + \tilde{K}(p_\mu)]\tilde{\Phi}(p_\mu) = T(p_\mu), \quad (33)$$

where

$$T(p_\mu) = [-p^0 + \tilde{K}^+(p_\mu)]F[f(x)] + [p^0 + \tilde{K}(p_\mu)]F[f^+(x)].$$

We are now at the stage of the main strategy: we have to identify the field $\Phi(x)$ introduced in Eq. (15) and the field $\Phi(x)$ (24) built up of the fields ϕ^+ and ϕ^- as the solutions of generalized Eqs. (27) and (28). The next step is our requirement that the Green function $\tilde{G}(p_\mu)$ in Eq. (17) and the function $\Gamma(p_\mu)$, which satisfies Eq. (33)

$$[-p^0 + \tilde{K}^+(p_\mu)][p^0 + \tilde{K}(p_\mu)]\tilde{\Gamma}(p_\mu) = 1, \quad (34)$$

must be equal to each other, where the full Green function $\tilde{G}(p^2, g^2, m^2)$

$$\tilde{G}(p_\mu) \rightarrow \tilde{G}(p^2, g^2, m^2) \simeq \frac{1 - g^2 \xi(p^2, m^2)}{m^2 - p^2 - i\epsilon} \quad (35)$$

has the same pole structure at $p^2 = m^2$ as the free Green function [21] with g being the scalar coupling constant and ξ is the one-loop correction of the scalar field. The dimensionless function $1 - g^2 \xi(p^2, m^2)$ is finite at $p^2 = m^2$.

We define the operator kernel $\tilde{K}(p_\mu)$ in (25) from the condition of the nonlocal coincidence of the Green function $\tilde{G}(p_\mu)$ in Eq. (17), and the thermodynamic function $\tilde{\Gamma}(p_\mu)$ from (34) in $S(\mathfrak{R}_4)$

$$F[\tilde{G}(p_\mu) - \tilde{\Gamma}(p_\mu)] = 0.$$

We can easily derive the kernel operator $\tilde{K}(p_\mu)$ in the form

$$\tilde{K}^2(p) = \frac{m^2 + \mathbf{p}^2 - g^2 \xi(p^2, m^2) p^{0^2}}{1 - g^2 \xi(p^2, m^2)}, \quad (36)$$

where [21]

$$\xi(m^2) = \frac{1}{96\pi^2 m^2} \left(\frac{2\pi}{\sqrt{3}} - 1 \right), \quad p^2 \simeq m^2,$$

and

$$\xi(p^2, m^2) = \frac{1}{96\pi m^2} \left(i \sqrt{1 - \frac{4m^2}{p^2}} + \frac{\pi}{\sqrt{3}} \right), \quad p^2 \simeq 4m^2.$$

The ultraviolet behaviour at $|p^2| \gg m^2$ leads to

$$\xi(p^2, m^2) \simeq \frac{-1}{32\pi^2 p^2} \left[\ln \frac{|p^2|}{m^2} - \frac{\pi}{\sqrt{3}} - i\pi\Theta(p^2) \right].$$

5. STOCHASTIC FORCES SCALE

In paper [10] it has been emphasized that two different scale parameters are in the model that we consider here. One of them is the so-called «correlation radius» R introduced in (5) and (6) with (9) and (12), (13), (14). In fact, this R parameter gives the pure size of the particle emission source without the external distortion and interaction coming from other fields. The other (scale) parameter is the stochastic scale L_{st} which carries the dependence of the particle mass, the α -coherence degree and, what is very important, the temperature T dependence:

$$L_{st} = \left[\frac{1}{\alpha(N) |p^0 - \tilde{K}(p)|^2 \bar{n}(m, \beta)} \right]^{1/5}. \quad (37)$$

It turns out that this scale L_{st} defines the range of stochastic forces acting on the particles in the emission source. This effect is given by $\alpha(N)$ -coherence degree which can be estimated from the experiment within the two-particle BE correlation function $C_2(Q)$ as Q close to zero, $C_2(0)$, at fixed value of mean multiplicity $\langle N \rangle$:

$$\alpha(N) \simeq \frac{2 - \bar{C}_2(0) + \sqrt{2 - \bar{C}_2(0)}}{\bar{C}_2(0) - 1}, \quad \bar{C}_2(0) = \frac{C_2(0)}{\xi(N)}. \quad (38)$$

In formula (37) $\bar{n}(m, \beta)$ is the thermal relativistic particle number density:

$$\bar{n}(m, \beta) = 3 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} n(\omega, \beta) = 3 \frac{\mu^2 + m^2}{2\pi^2} T \sum_{l=1}^{\infty} \frac{1}{l} K_2 \left(\frac{l}{T} \sqrt{\mu^2 + m^2} \right), \quad (39)$$

where K_2 is the modified Bessel function. For definite calculations we consider correlations between charged pions. The result can be extended to heavy particles case, e.g., for charged and neutral gauge bosons, which is essential program for the LHC. The stochastic scale L_{st} tends to infinity in case of particles on mass shell, i.e., $|p^0 - \tilde{K}(p)| \rightarrow 0$ which enters the L_{st} 's denominator (37). However, L_{st} will be bounded due to stochastic forces acting on the particles where

$$|p^0 - \tilde{K}(p)|^2 \simeq \Delta\epsilon_p^2 = \epsilon_p^2 \left| \frac{p^0}{\epsilon_p} - 1 - \frac{g^2 \xi(p^2, m^2)}{2} \left(1 - \frac{p^{0^2}}{\epsilon_p^2} \right) \right|^2, \quad \epsilon_p = \sqrt{m^2 + \mathbf{p}^2}$$

as $g^2 \xi(p^2, m^2) < 1$.

Within our aim to explore the correlation between charged pions, L_{st} has the form

$$L_{\text{st}} \simeq \left[\frac{e^{\sqrt{\mu^2 + m^2}/T}}{3\alpha(N)\Delta\epsilon_p^2(\mu^2 + m^2)^{3/4} \left(\frac{T}{2\pi}\right)^{3/2} \left(1 + \frac{15}{8} \frac{T}{\sqrt{\mu^2 + m^2}}\right)} \right]^{1/5}, \quad (40)$$

where the condition $l\beta\sqrt{m^2 + \mu^2} > 1$ for any integer l in (39) was taken into account. The only lower temperatures will drive L_{st} within formula (40) even if $\mu = 0$ and $l = 1$ with the condition $T < m$. Note that the condition $\mu < m$ is a general restriction in the relativistic «Bose-like gas», and $\mu = m$ corresponds to the Bose–Einstein condensation.

For large enough T no dependence of the chemical potential μ is found for L_{st} :

$$L_{\text{st}} \simeq \left[\frac{\pi^2}{3\zeta(3)\alpha(N)\Delta\epsilon_p^2 T^3} \right]^{1/5}, \quad (41)$$

where the condition $T > l\sqrt{\mu^2 + m^2}$, $l = 1, 2, \dots$ is taken into account. The origin of formula (41) comes from

$$\bar{n}(m, \beta) \rightarrow \bar{n}(\beta) = \frac{3T^3}{\pi^2} \zeta(3), \quad (42)$$

where neither a pion mass m - nor μ -dependence occurred; $\zeta(3) = \sum_{l=1}^{\infty} l^{-3} = 1.202$ is the zeta function with the argument 3. For high momentum pions ($p^2 \simeq 4m^2$) the actual mass dependence occurred for L_{st} :

$$L_{\text{st}} \simeq \left[\frac{e^{\sqrt{\mu^2 + m^2}/T}}{3\alpha(N)m^2(\mu^2 + m^2)^{3/4} \left(\frac{T}{2\pi}\right)^{3/2} \left(1 + \frac{15}{8} \frac{T}{\sqrt{\mu^2 + m^2}}\right)} \right]^{1/5} \quad (43)$$

at low T , and

$$L_{\text{st}} \simeq \left[\frac{\pi^2}{3\zeta(3)\alpha(N)m^2 T^3} \right]^{1/5} \quad (44)$$

at high temperatures, if $g^2\xi(m^2) \ll 1$, $\xi(m^2) \sim O(0.01/m^2)$ and $(\mathbf{p}^2/4m^2) \ll 1$ are valid in both temperature regime cases. Formula (42) reproduces the $\sim T^3$ behavior which is the same as the thermal distribution (in terms of density) for a gas of free relativistic massless particles. Such behavior is expected anyway in high temperature limit if the particles can be considered as asymptotically free in that regime.

Actually, the increasing of T leads to squeezing of L_{st} , and $L_{\text{st}}(T = T_0) = R$ at some effective temperature T_0 . The higher temperatures, $T > T_0$, satisfy more squeezing effect and at the critical temperature T_c the scale $L_{\text{st}}(T = T_c)$ takes its minimal value. Obviously T_c defines the phase transition where the deconfinement will occur. Since all the masses tend to zero (chiral symmetry restoration) and $\alpha \rightarrow 0$ at $T > T_c$, one should expect the sharp expansion of the region with $L_{\text{st}}(T > T_c) \rightarrow \infty$. The following condition $\tilde{n}(m, \beta)v_\pi = 1$ provides the phase transition (transition from hadronizing phase to deconfinement one) with the volume $v_\pi = (4\pi r_\pi^3/3)$, where r_π is the pion charge radius. Actually, the temperature of phase transition essentially depends on the charge (vector) radius of the pion which is a fundamental quantity in hadron physics. A recent review on r_π values is presented in [22].

What do we know about the source size estimation from experiments? DELPHI and L3 collaborations at LEP established that the correlation radius R decreases with transverse pion mass m_t as $R \simeq a + b/\sqrt{m_t}$ for all directions in the Longitudinal Center-of-Mass System (LCMS). ZEUS collaboration at HERA did not observe the essential difference among the values of R parameter in $\pi^\pm\pi^\pm$, $K_s^0K_s^0$ and $K^\pm K^\pm$ pairs, namely $R_{\pi\pi} = (0.666 \pm 0.009(\text{stat.}) + 0.022 - 0.033(\text{syst.}))$ fm, $R_{K_sK_s} = (0.61 \pm 0.08(\text{stat.}) + 0.07 - 0.08(\text{syst.}))$ fm and $R_{K^\pm K^\pm} = (0.57 \pm 0.09(\text{stat.}) + 0.15 - 0.06(\text{syst.}))$ fm, respectively. The ZEUS data are in good agreement with the LEP for radius R . However, no evidence for \sqrt{s} dependence of R is found. It is evident that more experimental data are appreciated. However, the comparison of experiments is difficult mainly due to reference samples used and the Monte Carlo corrections.

Finally, our theoretical results first predict the L_{st} in (40) and (41), and both mass- and temperature dependences are obtained clearly. This can serve as a good approximation to explain the LEP, Tevatron and ZEUS (HERA) experimental data. We need that the pion energies at the colliders are sufficient to carry out these studies (since the $\Delta\epsilon_p$ dependence). Careful simulation of their (pions) signal and background is needed. The more precisely measured pion momentum may be of some help. Also, determination of the final state interactions may clarify what is happening.

CONCLUSION

To summarize, we have found the time dependence of the correlation function $C_2(Q)$ calculated in time-dependent external field provided by the operator $r(\mathbf{p}, t)$ and the chaotic coherence function $\alpha(m, \beta)$. Based on this approach, we emphasize the explanation of the dynamic origin of the coherence in BEC, the origin of the specific shape of the correlation $C_2(Q)$ functions, and finding the dependence on the particle energy (and the mass) due to coherence function α , as seen from the QFT $_\beta$. Actually, the stochastic scale L_{st} decreases with the particle energy (the mass m). This has already been confirmed by the data of LEP, Tevatron and HERA (ZEUS) with respect to the size of particle source.

In the framework of QFT $_{\beta}$ the numerical analysis of experimental data can be carried out with a result where important parameters of $C_2^{--}(Q)$ and $C_2^{++}(Q)$ functions are retrieved (e.g., $C_0, R, \lambda, \epsilon, N, \alpha, L_{st}, T$).

The correlations of non-identical particles pairs can be observed and the corresponding C_2 parameters are retrieved. The off-correlation effect is given by the space-time distribution (14) containing the sum $\mathbf{p}_1 + \mathbf{p}_2$, and this effect is sufficient if the factor containing the sum $p_1^0 + p_2^0$ in (10) is not too small. The off-correlation effect is possible if the particle energies $p_i^0 (i = 1, 2)$ are small enough.

Like, e.g., $\pi^{\pm}\pi^{\pm}$ BEC, the correlations $\pi^{\pm}\pi^{\mp}$ can also serve as tools in the determination of parameters of the particle source. The latter correlations play a particularly important role in the detection of random chaotic correction to BEC.

The stochastic scale L_{st} decreases with increasing temperatures slowly at low temperatures, and it decreases rather abruptly when the critical temperature is approached.

We claim that the experimental measuring of R (in fm) can provide the precise estimation of the effective temperature T_0 which is the main thermal character in the particle's pair emitter source (given by the effective dimension R) with the particle mass and its energy at given α fixed by $C_2(Q = 0)$ and $\langle N \rangle$. Actually, T_0 is the true temperature in the region of multiparticle production with dimension $R = L_{st}$, because at this temperature it is exactly the creation of two particles that occurred, and these particles obey the criterion of BEC.

We have found the squeezing of the particle source due to decreasing of the correlation radius R in the case of opposite charged particles. The off-correlated system of non-identical particles is less sensitive to the random force influence (α dependence).

The results obtained in this paper can be compared with the static correlation function (see, e.g., [23] and the references therein relevant to heavy-ion collisions).

Finally, we should stress new features of particle–antiparticle BEC which can emerge from the data. It is a highly rewarding task to measure experimentally non-identical particles.

There is much to be done for $C_2(Q)$ investigation at hadron colliders. The time is ripe for dedicated searches for new effects in $C_2(Q)$ function at hadron colliders to discover, or rule out, in particular, the $\alpha(N)$ dependence.

In conclusion, the correlations of two bosons in 4-momentum space presented in this paper offer useful and instructive complimentary viewpoints of theoretical and experimental works in multiparticle femtoscopy and interferometry measurements at hadron colliders.

REFERENCES

1. *Weiner R. M.* // Phys. Rep. 2000. V. 327. P. 249.
2. *Hanbury-Brown R., Twiss R. Q.* // Nature. 1956. V. 178. P. 1046.
3. *Kozlov G. A., Utyuzh O. V., Wilk G.* // Phys. Rev. C. 2003. V. 68. P. 024901.
4. *Kozlov G. A.* // Phys. Rev. C. 1998. V. 58. P. 1188.
5. *Kozlov G. A.* // J. Math. Phys. 2001. V. 42. P. 4749.
6. *Kozlov G. A.* // New J. Phys. 2002. V. 4. P. 23.
7. *Kozlov G. A.* // Part. Nucl. 2005. V. 36. P. 108.

8. *Kozlov G. A. et al.* hep-ph/0510027.
9. *Kozlov G. A. et al.* hep-ph/0710.3710.
10. *Kozlov G. A.* hep-ph/0512184.
11. *Langevin P.* // C. R. Acad. Sci. 1908. V. 146. P. 530.
12. *ALEPH Collab.* // Eur. Phys. J. C. 2004. V. 36. P. 147; Phys. Lett. B. 2005. V. 611. P. 66.
13. *DELPHI Collab.* // Phys. Lett. B. 1996. V. 379. P. 330.
14. *Abbiendi G. et al. (OPAL Collab.)* // Phys. Lett. B. 2003. V. 559. P. 131.
15. *ZEUS Collab.* // Phys. Lett. B. 2004. V. 583. P. 231.
16. *Alexopoulos T. et al. (E735 Collab.)* // Phys. Rev. D. 1993. V. 48. P. 1931.
17. *Andreev I., Plumer M., Weiner R.* // Phys. Rev. Lett. 1991. V. 67. P. 3475.
18. *Goldhaber G. et al.* // Phys. Rev. Lett. 1959. V. 3. P. 181.
19. *Goldhaber G. et al.* // Phys. Rev. 1960. V. 120. P. 300.
20. *Bogolyubov N. N.* JINR, D-781. Dubna, 1961.
21. *Bogolyubov N. N., Shirkov D. V.* An Introduction to the Theory of Quantized Fields, N.Y.: John Wiley and Sons, Inc. Intersci., 1959.
22. *Bernard V., Kaiser N., Meissner U.-G.* // Phys. Rev. C. 2000. V. 62. P. 028201.
23. *Wong C. Y., Zhang W. N.* // Phys. Rev. C. 2007. V. 76. P. 034905.

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