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DISCOVERY OF NEW PHYSICS IN RADIATIVE PION DECAYS?

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Recently a strong indication for a deviation from the Standard Model (SM) has been obtained by the PIBETA collaboration. Namely, SM fails to describe the energy distribution and the branching ratio of the radiative decays of the positive pions at rest in the high-E_γ/low-E_e kinematic region.

The previous experiment at the ISTRA facility, testing the radiative decays of negative pions in flight in a wide kinematic region, has alarmed about the same effect, although statistically less significant. The present PIBETA result indicates a deficit of the branching ratio of the radiative pion decay in the specified kinematic region at 8σ level in comparison with SM prediction, while in the other kinematic regions both the branching ratios and the energy distributions are compatible with the $V-A$ interaction.

We argue that this effect can result only from a small admixture of new tensor interactions. They may arise due to an exchange of new spin-1 *chiral bosons* which interact *anomalously* with matter.

Недавно в измерениях коллаборации ПИБЕТА обнаружилось серьезное отклонение от стандартной модели (СМ). Оказывается, СМ не способна объяснить распределение по энергии и парциальные ширины радиационных распадов положительно заряженных пионов в покое в кинематической области с большим E_γ и низким E_e.

Предыдущий эксперимент на установке ИСТРА, в котором измерялись радиационные распады отрицательно заряженных пионов на лету в широкой кинематической области, давал тот же эффект, хотя на более низком статистическом уровне. Результат эксперимента ПИБЕТА указывает на дефицит парциальной ширины радиационного распада пиона в специфической кинематической области на уровне 8σ по сравнению с СМ-предсказанием, в то время как в остальной кинематической области и парциальные ширины, и энергетическое распределение совместимы с $V-A$-взаимодействием.

Мы отмечаем, что указанный эффект может возникнуть от небольшой примеси новых тензорных взаимодействий. Они могут генерироваться из-за обмена новых киральных бозонов со спином 1, которые аномально взаимодействуют с материей.

INTRODUCTION

The Standard Model (SM) includes three different types of fundamental interactions: electromagnetic, weak and strong ones. The corresponding forces arise due to an exchange of spin-1 gauge bosons, described by four-vector fields V_α. For the completeness of SM at least one more scalar boson is required. However, due to its weak coupling to the ordinary matter and its big mass, this particle has not been detected yet by experiments.

Meanwhile, the existence of many other bosons has been predicted theoretically and tested experimentally. The supersymmetry, for example, suggests a rich variety of new particles, which, however, have not been yet observed experimentally.

In this letter we show that the recent PIBETA result [1] for the radiative pion decays (RPD) $\pi^+ \to e^+\nu\gamma$ indicates that a different kind of fundamental spin-1 *chiral bosons* may be present in Nature. They have not been discussed intensively in literature till now.

These particles were first mentioned as a different type of spin-1 bosons in Ref. [2]. They appear naturally in the analysis of possible Yukawa couplings of spin-1 bosons to fermion currents. In the relativistic physics, where two different types of spin-1/2 fermions exist, namely left-handed $\psi_L = \frac{1}{2}(1-\gamma^5)\psi$ and right-handed $\psi_R = \frac{1}{2}(1+\gamma^5)\psi$, two different types of Lorentz invariant interactions are possible: *à la gauge* vector interactions

$$\mathcal{L}_V = \left(g_{LL}\,\overline{\psi_L}\gamma^\alpha\psi_L + g_{RR}\,\overline{\psi_R}\gamma^\alpha\psi_R\right)V_\alpha \tag{1}$$

and tensor interactions

$$\mathcal{L}_T = g_{LR}\,\overline{\psi_L}\sigma^{\alpha\beta}\psi_R T^+_{\alpha\beta} + g_{RL}\,\overline{\psi_R}\sigma^{\alpha\beta}\psi_L T^-_{\alpha\beta}, \tag{2}$$

where $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^\alpha, \gamma^\beta]$, $T^\pm_{\alpha\beta}$ are antisymmetric tensor fields and g denotes dimensionless coupling constants.

Both the vector fields V_α and the tensor fields $T^\pm_{\alpha\beta}$ describe particles with spin one. However, they interact absolutely differently with matter. The gauge particles are chirally neutral and, hence, they preserve the helicities of incoming and outgoing fermions. On the contrary, the tensor bosons carry a *chiral charge* that leads to helicity flip for the interacting fermions. Moreover, self-interactions of the *chiral bosons* exist even for an Abelian case, leading to a negative contribution to the β function [3].

An example of the presence of the new kind of *chiral particles* in Nature is the existence of the axial-vector meson resonance $b_1(1235)$, which has only anomalous tensor interactions (2) with quarks. The successful description of the dynamical properties of the hadron systems [4] hints that the same phenomenon may take place at the electroweak scale as well, similar to the gauge-like hadron resonances ρ and a_1, which have served as prototypes for the photon and the weak bosons [5].

The PIBETA result confirms the anomaly already observed at the ISTRA facility [6] for the negative pion decays $\pi^- \to e^-\tilde{\nu}\gamma$ in flight. It is remarkable that these two experiments with absolutely different systematics revealed the same effect: the deficit in RPD yield. Moreover, the PIBETA experiment has measured simultaneously absolute total $\pi \to e\nu$, $\pi^+ \to \pi^0 e^+\nu$, $\mu \to e\nu\tilde{\nu}$, as well as partial $\mu \to e\nu\tilde{\nu}\gamma$ branching ratios, which are in excellent agreement with SM predictions, with better than 1% accuracy [7]. The only deviation from SM is observed in the high-E_γ/low-E_e kinematic region exactly where the effect has been predicted [8, 9]. The effect is so big that it cannot be explained within SM [10] and its supersymmetric extensions [11].

Historically various types of *local* four-fermion effective interactions have been introduced to test the eventual presence of a new physics. Here we will follow also the phenomenological approach of the effective interactions because the complete theory for the interacting tensor particles is not constructed yet. However, we argue that the effective interactions should include new *nonlocal* momentum-dependent tensor interactions, in order to describe the PIBETA result without contradiction with the present experimental data.

1. NEW TENSOR INTERACTIONS

First of all we define the Lorentz structure of the new interactions on the basis of PIBETA and ISTRA results. Since both collaborations have observed a deficit of events in comparison with SM expectation, it should stem from destructive interference between SM and the new interactions in RPD.

It is well known [12] that the dominant contribution in RPD comes from the inner bremsstrahlung (IB) process near the edge $E_\gamma + E_e \simeq m_\pi/2$ of the kinematically allowed regions, which include the high-E_γ/low-E_e kinematic region, where the deficit of the events was observed. We assume that the new interactions interfere destructively with this process leading to the deficit of events. Hence, the new interactions should have the same chiral structure as IB which is different from $V-A$ interactions. Therefore, they do not interfere with the latter due to the smallness of the electron mass. This is one of the reasons why such type of interactions are hard to detect and why they have not been observed before.

Let us consider all possible current–current interactions which lead to a helicity flip. There are only three possible Lorentz structures obeying this property: scalar, pseudoscalar and tensor currents. The matrix element of the pion–photon transition for the scalar quark current $\langle\gamma|\bar{u}d|\pi\rangle$ is zero for kinematic reasons and does not contribute to RPD. On the other hand, the contribution of the pseudoscalar quark current to the matrix element of the ordinary pion decay

$$i\langle 0|\bar{u}\gamma^5 d|\pi\rangle = \frac{m_\pi^2}{(m_d + m_u)m_e} m_e f_\pi \simeq 3.8 \cdot 10^3 \; m_e f_\pi \tag{3}$$

is enormously enhanced in comparison with the standard chirally suppressed $V-A$ contribution, and it is severely constrained by the experimental data [13]. On the contrary, the matrix element of the tensor quark current $\langle 0|\bar{u}\sigma_{\alpha\beta}d|\pi\rangle$ is zero for kinematic reasons and does not contribute directly to the pion decay, thus escaping the experimental constraints.

Hence, only less constrained tensor interactions are possible candidates for the explanation of RPD anomaly. We argue that particularly nonlocal momentum-dependent tensor interactions are responsible for the detected anomaly.

In order to explain the ISTRA anomaly, new quark–lepton tensor interactions

$$\mathcal{L}_T^{\rm loc} = -\sqrt{2} f_T G \; \bar{u}\sigma_{\alpha\beta}d \; \bar{e}\sigma_{\alpha\beta}\nu_L + \text{h.c.} \tag{4}$$

with the effective coupling constant $f_T \simeq 10^{-2}$ have been introduced [9], here $G = G_F V_{ud}$. The dimensionless coupling constant f_T determines the strength of the new tensor interactions relative to the ordinary weak interactions, governed by the Fermi coupling constant G_F. Although such tensor interactions, introduced ad hoc, can explain the ISTRA anomaly, it has been pointed out [14] that the necessary value of the coupling constant f_T contradicts the ordinary pion decay $\pi \to e\nu$. This happens because, owing to the electromagnetic radiative corrections, the pseudotensor quark current $\bar{u}\sigma_{\alpha\beta}\gamma^5 d$ leads to a generation of the pseudoscalar quark current $\bar{u}\gamma^5 d$, to which pion decay is very sensitive.

The generation of the pseudotensor term $\bar{u}\sigma_{\alpha\beta}\gamma^5 d$ cannot be avoided for derivative free local four-fermion interactions even if we have started with a parity-conserving quark current $\bar{u}\sigma_{\alpha\beta}d$ in Eq. (4). Owing to the identity $\bar{u}\sigma_{\alpha\beta}d_R \; \bar{e}\sigma_{\alpha\beta}\nu_L \equiv 0$, the Lagrangian (4) effectively reads

$$\mathcal{L}_T^{\rm loc} = -\sqrt{2} f_T G\ \bar{u}\sigma_{\alpha\beta} d_L\ \bar{e}\sigma_{\alpha\beta}\nu_L + {\rm h.c.}, \tag{5}$$

where the chiral structure shows itself in the quark current.

The solution of this problem was found in Ref. [8] via introducing nonlocal momentum-dependent tensor interactions

$$\begin{aligned}
\mathcal{L'}_T = & - \sqrt{2} f_T G\ \bar{u}\sigma_{\lambda\beta} d_L\ \bar{e}\sigma_{\lambda\beta}\nu_L - \\
& - \sqrt{2} f'_T G\ \bar{u}\sigma_{\lambda\alpha} d_R\ \frac{4Q_\alpha Q_\beta}{Q^2}\ \bar{e}\sigma_{\lambda\beta}\nu_L + {\rm h.c.} = \\
= & - \sqrt{2} f'_T G\ \bar{u}\sigma_{\lambda\alpha} d\ \frac{4Q_\alpha Q_\beta}{Q^2}\ \bar{e}\sigma_{\lambda\beta}\nu_L + {\rm h.c.},
\end{aligned} \tag{6}$$

where Q_α is the momentum transfer between quark and lepton currents, and $f'_T = f_T$ are positive dimensionless coupling constants. In this case the second term in the second row of Eq. (6) is no longer equal to zero, despite the different chiral structures in quark and lepton currents. And due to the identity

$$\bar{u}\sigma_{\alpha\beta} d_L\ \bar{e}\sigma_{\alpha\beta}\nu_L \equiv \bar{u}\sigma_{\lambda\alpha} d_L\ \frac{4Q_\alpha Q_\beta}{Q^2}\ \bar{e}\sigma_{\lambda\beta}\nu_L, \tag{7}$$

it can compensate the opposite chiral quark structure of the first term. Then the terms with the pseudotensor quark currents $\bar{u}\sigma_{\alpha\beta}\gamma^5 d$ cancel out in Eq. (6), and the tensor current $\bar{u}\sigma_{\alpha\beta} d$ does not contribute to pseudoscalar pion decay because of parity conservation in electromagnetic interactions.

The two different terms in the effective Lagrangian (6) come from exchanges of new spin-1 bosons $T^{\pm}_{\alpha\beta}$ and $U^{\pm}_{\alpha\beta}$ with opposite chiral charges, which are necessary to avoid a chiral anomaly [8]. The peculiar Lorentz structure of the new tensor interactions reflects the Lorentz structure of the propagators for the chiral bosons. This structure can be obtained, for example, from the one-loop radiative corrections to the self-energy of the tensor fields using the interactions (2). This follows from the fact that in case of dimensionless coupling constants the theory is formally renormalizable and the radiative corrections should reproduce the Lorentz structure of the kinetic terms [16] in the bare initial Lagrangian for the tensor fields.

In general, the coupling constants f_T and f'_T can be different; however, we assume their equality to avoid the experimental constraint from the ordinary pion decay. In the following we will keep different notations for the coupling constants in order to compare the effects from the two different Lagrangians (4) and (6).

2. RADIATIVE PION DECAY

The most general matrix element of RPD $\pi^- \to e^-\tilde{\nu}\gamma$ reads

$$M = M_{\rm IB} + M_{\rm SD} + M_T + M'_T, \tag{8}$$

where, besides SM matrix elements for IB process

$$M_{\rm IB} = i\frac{eG}{\sqrt{2}} f_\pi m_e \varepsilon_\alpha\ \bar{e}\left[\frac{2p_\alpha}{pq} - \frac{2k_\alpha - i\sigma_{\alpha\beta}q_\beta}{kq}\right]\nu_L \tag{9}$$

and for the structure-dependent (SD) radiation

$$M_{\rm SD} = i\frac{\sqrt{2}eG}{m_\pi}\varepsilon_\alpha\{F_A\left[(pq)g_{\alpha\beta} - p_\alpha q_\beta\right] + iF_V\epsilon_{\alpha\beta\rho\sigma}p_\rho q_\sigma\}\ \bar{e}\gamma_\beta\nu_L, \tag{10}$$

the new tensor contributions

$$M_T = -\sqrt{2}eGF_T\varepsilon_\alpha q_\beta\bar{e}\sigma_{\alpha\beta}\nu_L \tag{11}$$

and

$$M'_T = -\sqrt{2}eGF'_T\left(q_\alpha\varepsilon_\lambda - q_\lambda\varepsilon_\alpha\right)\frac{Q_\lambda Q_\beta}{Q^2}\bar{e}\sigma_{\alpha\beta}\nu_L \tag{12}$$

are present. Here ε_α is the photon polarization vector; p, k, and q are pion, electron and photon momenta, correspondingly.

The first term in Eq. (8) $M_{\rm IB}$ describes a gauge-invariant QED process of the photon radiation from the external charged particles, which is free from the effects of the strong interactions. It contains only one well known phenomenological parameter: the pion decay constant $f_\pi = (130.7 \pm 0.4)$ MeV.

The second term $M_{\rm SD}$ corresponds to the photon emission from hadronic intermediate states, governed completely by the strong-interaction physics. It is parametrized by the two form factors F_V and F_A of the π–γ matrix elements for the vector quark current

$$\langle\gamma(q)|\bar{u}\gamma_\alpha d|\pi^-(p)\rangle = -\frac{e}{m_\pi}\varepsilon_\beta F_V\epsilon_{\alpha\beta\rho\sigma}p_\rho q_\sigma \tag{13}$$

and for the axial-vector quark current

$$\langle\gamma(q)|\bar{u}\gamma_\alpha\gamma^5 d|\pi^-(p)\rangle = i\frac{e}{m_\pi}\varepsilon_\beta F_A\left[(pq)g_{\alpha\beta} - q_\alpha p_\beta\right] + ie\varepsilon_\alpha f_\pi. \tag{14}$$

Assuming CVC hypothesis, the vector form factor F_V is directly related to the $\pi^0 \to \gamma\gamma$ amplitude [15] and can be extracted from the experimental width of this decay:

$$F_V = \frac{1}{\alpha}\sqrt{\frac{2\Gamma(\pi^0 \to \gamma\gamma)}{\pi m_{\pi^0}}} = 0.0262 \pm 0.0009. \tag{15}$$

This value is in a fair agreement with the calculations in the relativistic quark model [17] and with the leading-order calculations of the chiral perturbation theory (CHPT) [18]

$$F_V = \frac{1}{4\pi^2}\frac{m_\pi}{F_\pi} \approx 0.0270. \tag{16}$$

The value of the axial form factor F_A is model-dependent and its determination is a matter of experimental measurements. The ratio of the axial to the vector form factors $\gamma = F_A/F_V$ has been measured in the previous experiments [19] in kinematic regions where the contribution of the new tensor terms is not essential. The average value $\gamma = 0.448 \pm 0.062$ at fixed $F_V = 0.0259 \pm 0.0005$ [20] is also in agreement with the calculations in CHPT [21].

The matrix elements M_T and M'_T follow from the new interactions between quark and lepton tensor currents (6). The matrix element for the quark tensor current

$$\langle\gamma(q)|\bar{u}\sigma_{\alpha\beta}\gamma^5 d|\pi^-(p)\rangle = -\frac{e}{2}F^0_T\left(q_\alpha\varepsilon_\beta - q_\beta\varepsilon_\alpha\right) \tag{17}$$

can be calculated [11] applying the QCD sum rules techniques and the PCAC hypothesis. So

$$F_T^0 = \frac{2}{3}\frac{\chi\langle 0|\bar{q}q|0\rangle}{f_\pi} \approx 0.4 \tag{18}$$

is expressed in terms of the magnetic susceptibility [22, 23] $\chi = -(5.7 \pm 0.6)$ GeV^{-2} of the quark condensate and its vacuum expectation value $\langle 0|\bar{q}q|0\rangle \approx -(0.24 \text{ GeV})^3$. Then the tensor form factors in Eqs. (11) and (12) read $F_T = -(f_T + f'_T)F_T^0$ and $F'_T = -2f'_T F_T^0$.

In general, all form factors depend on the square of momentum transfer to the lepton pair $Q^2 = (p-q)^2$. However, these dependences are weak and, hence, the form factors can be assumed as constants.

The differential decay width of RPD

$$\frac{d^2\Gamma_{\pi\to e\nu\gamma}}{dxdy} = \frac{\alpha}{2\pi}\,\frac{\Gamma_{\pi\to e\nu}}{(1-r)^2}\,\rho(x,y) \tag{19}$$

can be expressed in terms of the ordinary pion decay width $\Gamma_{\pi\to e\nu}$, where the kinematic variables $x = 2pq/m_\pi^2$, $y = 2pk/m_\pi^2$ and the ratio $r = (m_e/m_\pi)^2 \approx 1.34\cdot 10^{-5}$ are introduced. The Dalitz plot distribution is defined by the density

$$\begin{aligned} \rho(x,y) \;&=\; \rho_{\rm IB}(x,y) + \rho_{\rm SD}(x,y) + \rho_{\rm IBSD}(x,y) + \\ &+\; \rho_T(x,y) + \rho_{{\rm SD}T}(x,y) + \rho_{{\rm IB}T}(x,y), \end{aligned} \tag{20}$$

where

$$\begin{aligned} \rho_{\rm IB} &= {\rm IB}(x,y), \\ \rho_{\rm SD} &= a^2\left[(1+\gamma)^2{\rm SD}^+(x,y) + (1-\gamma)^2{\rm SD}^-(x,y)\right], \\ \rho_{\rm IBSD} &= 2a\sqrt{r}\left[(1+\gamma)G^+(x,y) + (1-\gamma)G^-(x,y)\right], \\ \rho_T &= a^2\;T(x,y), \\ \rho_{{\rm SD}T} &= 2a^2\sqrt{r}\left[(1+\gamma)J^+(x,y) + (1-\gamma)J^-(x,y)\right], \\ \rho_{{\rm IB}T} &= 2a\;I(x,y). \end{aligned} \tag{21}$$

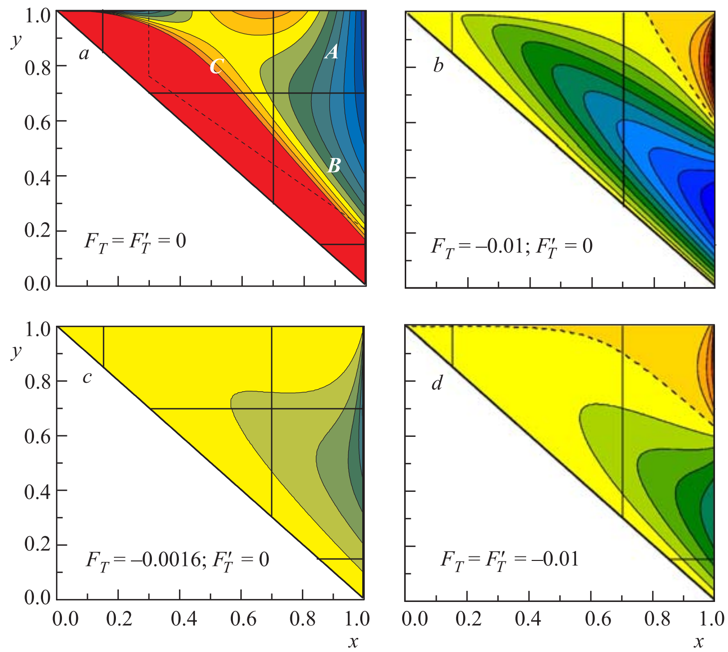
The explicit forms of the functions ${\rm IB}(x,y)$, ${\rm SD}^\pm(x,y)$, $G^\pm(x,y)$, $T(x,y)$, $J^\pm(x,y)$ and $I(x,y)$ are given in the Appendix. The constant

$$a = \frac{m_\pi^2}{2f_\pi m_e}F_V = \frac{m_\pi^3}{8\pi^2 f_\pi^2 m_e} \approx 3.945 \tag{22}$$

defines the strength of IB contribution relative to other contributions.

3. DISCUSSION AND CONCLUSIONS

Based on the previous consideration, we discuss now the experimental data and their interpretation. To investigate the most interesting part of RPD, namely the SD radiation, and to extract γ, all previous experiments [19] have been fulfilled in a restricted kinematic



a) SM distribution (ISTRA region is marked out by dashed lines); *b, c, d*) isocurves (with 10% step) corresponging to relative strength of tensor contribution for different values of F_T and F'_T (the dashed curve corresponds to zero)

region compatible with region A of PIBETA experiment (figure, *a*), which is an intersection of regions B and C.

In figure, *a* the isocurves for pure SM contributions ($F_T = F'_T = 0$) are shown. The IB contribution dominates near the edge $x + y \simeq 1$ of the kinematically allowed region and SD$^+$ reaches its maximum near the point $(2/3, 1)$, while their interference and SD$^-$ contribution are small.

Almost the whole kinematically allowed region (figure, *a*) has been investigated first at the ISTRA facility [6]. A large deficit of events $(33 \pm 10)\%$ in comparison with SM prediction has been observed. Even with a poor statistics, they were able to establish the kinematic region, where a lack of events occurs. This region corresponds to the bottom part of region B of PIBETA experiment, where only the IB and the small SD$^-$ contributions are expected.

The introduction of the tensor matrix element M_T (11) with $F_T \simeq -0.01$ [9,24] explains the lack of events and leads to a considerable improvement of the χ^2. However, such a big tensor form factor contradicts [14] the present experimental data. Moreover, in order to explain the $(19.1 \pm 2.5)\%$ deficit of events in region B of PIBETA experiment, using the same form (11) for the tensor matrix element, an order of magnitude smaller value of the tensor form factor $F_T = -0.0016(3)$ is required [25]. These different F_T values indicate an inadequate description of the new interaction.

To analyze the problem further, we compare the relative strength of the tensor contribution with respect to SM contribution for two different values of the tensor form factor F_T (figure, *b, c*). The main difference between these two plots is the following: At the biggest value of $|F_T|$ both negative and positive contributions are present depending on the region, while for

$|F_T| \lesssim 2 f_\pi m_e / m_\pi^2 \approx 0.0069$ the tensor matrix element leads to the only negative contribution to the whole kinematically allowed region.

As one can see from figure, *b* the tensor contribution with the biggest value $|F_T|$ affects significantly regions A and C, besides region B. The latter is in contradiction with PIBETA results. The lowest absolute value of F_T allows one to evade this problem, but leads to another latent difficulty of γ determination.

Indeed, PIBETA claims two different values for the ratios of axial to vector form factors $\gamma = 0.443 \pm 0.015$ and $\gamma = 0.480 \pm 0.016$. The first one corresponds to the SM fits [1] to the entire data set, while the second to region A data only. However, the 8σ deviation in the branching ratio from SM prediction in region B requires an introduction of the new tensor contributions for proper γ determination. Then taking into account the correction for the destructive interference, the experimental branching ratio in region A should be increased. This increases the γ value, as follows from the top panel of Fig. 4 in Ref. [1]. However, if we believe in CHPT calculations [21], γ should be decreased in order to approach the lowest value. In other words, the fits made with $F'_T = 0$ are inappropriate.

The case $F_T = F'_T \simeq -0.01$ allows one to describe both ISTRA and PIBETA anomalies as well, without contradiction to the experimental data. The corresponding tensor contribution has a slight slope near its zero value (figure, *d*) in regions A and C, which is in accordance with PIBETA results. Moreover, contrary to PIBETA fit, mainly the positive contribution in region A leads to the right direction for γ correction, corresponding to decreasing its value. While the tensor contribution can lead up to 40% deficit in region B.

The nonzero form factors F_T and F'_T indicate an existence of the quark–lepton tensor interactions (6) with the coupling constant $f_T = f'_T \simeq 0.013$. Although such type of interactions can be generated through radiative corrections, it is impossible to get the tensor coupling constant to be larger than 10^{-9}–10^{-8} in SM and 10^{-4}–10^{-3} in its SUSY extensions [11]. Moreover, the particular form (6) cannot arise as a result of Fierz transformations from a leptoquark exchange [26] as well. The only natural source to produce the effective interaction (6) is the exchange of the new *chiral bosons* [8], interacting *anomalously* with matter.

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APPENDIX

The analytical expressions for the functions $\mathrm{IB}(x,y)$, $\mathrm{SD}^\pm(x,y)$, $G^\pm(x,y)$, $T(x,y)$, $J^\pm(x,y)$ and $I(x,y)$ read

$$\begin{aligned}
\mathrm{IB}(x,y) &= \frac{y_1(1-r)}{x^2(x-y_1)}\left[\frac{x^2}{1-r} + 2(1-x) - \frac{2rx}{x-y_1}\right], \\
\mathrm{SD}^+(x,y) &= (x-y_1)\left[(x-y_1)(1-x) - rx\right], \\
\mathrm{SD}^-(x,y) &= y_1\left[y_1(1-x) + rx\right],
\end{aligned}$$

$$
\begin{aligned}
G^{+}(x,y) &= \frac{y_1}{x(x-y_1)}\left[(x-y_1)(1-x)-rx\right], \\
G^{-}(x,y) &= \frac{y_1}{x(x-y_1)}\left[y_1(1-x)-(1-r)x\right], \\
T(x,y) &= 2\left[(\gamma_T-\gamma'_T)^2+\gamma_T^2\right]y_1(x-y_1)+ \\
&\quad + \gamma'^2_T\frac{rx}{1-x}\left[x-2y_1-\frac{rx}{1-x}\right], \\
J^{+}(x,y) &= -\gamma'_T x\left[x-y_1-\frac{rx}{1-x}\right], \\
J^{-}(x,y) &= (\gamma'_T-2\gamma_T)xy_1, \\
I(x,y) &= \gamma'_T y_1+2(\gamma_T-\gamma'_T)y_1\left[\frac{1}{x}-\frac{r}{x-y_1}\right],
\end{aligned}
\tag{23}
$$

where $y_1 = 1 - y + r$, $\gamma_T = F_T/F_V$ and $\gamma'_T = F'_T/F_V$.

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