Письма в ЭЧАЯ. 2004. Т. 1, № 5(122). С. 24–29

УДК 539.12.01

QCD VACUUM TOPOLOGICAL SUSCEPTIBILITY IN A NONLOCAL CHIRAL QUARK MODEL

A. E. Dorokhov[1]

Joint Institute for Nuclear Research, Dubna

The topological susceptibility of QCD vacuum is studied in the framework of a covariant chiral quark model with nonlocal quark–quark interaction. The relation of the first moment of topological susceptibility $\chi'(0)$ and the «spin crisis» problem is briefly discussed. It is shown, in particular, that one always gets the inequality $\chi'(0) > \chi'_{\mathrm{OZI}}$.

Топологическая восприимчивость вакуума КХД изучена в рамках ковариантной киральной кварковой модели с нелокальным взаимодействием кварков. Кратко обсуждается роль первого момента топологической восприимчивости в решении проблемы «спинового кризиса». В частности показано, что всегда выполняется неравенство $\chi'(0) > \chi'_{\mathrm{OZI}}$.

It is well known that due to $U_A(1)$ axial Adler–Bell–Jackiw anomaly the isosinglet axial-vector current

$$J_{\mu 5}^{(0)} = \sum_f \overline{q}_f \gamma_\mu \gamma_5 q_f \tag{1}$$

is not conserved even in the chiral limit, and its divergence equals

$$\partial_\mu J_{\mu 5}^{(0)}(x) = 2N_f Q_5(x), \tag{2}$$

where

$$Q_5(x) = (\alpha_s/8\pi) G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x) \tag{3}$$

is the topological charge density. The correlator of singlet currents is defined as

$$\Pi_{A,\mu\nu}^{(0)}(q) = i \int d^4x\, \mathrm{e}^{iqx} \left\langle 0 \left| T\left\{ J_{\mu 5}^{(0)}(x) J_{\nu 5}^{(0)}(0)^\dagger \right\} \right| 0 \right\rangle = \\ = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi_{A,T}^{(0)}(Q^2) + q_\mu q_\nu \Pi_{A,L}^{(0)}(Q^2). \tag{4}$$

In the chiral limit the longitudinal part of the correlator defines the topological susceptibility, i.e., the correlator of the topological charge densities, $Q_5(x)$,

$$\chi(Q^2) = i \int d^4x\, \mathrm{e}^{iqx} \langle 0 \left| T\left\{ Q_5(x) Q_5(0) \right\} \right| 0 \rangle, \tag{5}$$

[1]E-mail: dorokhov@thsun1.jinr.ru

with the relation (see, e.g., [1])

$$\Pi_L^{A,0}(Q^2) = \frac{(2N_f)^2}{Q^2}\chi(Q^2). \tag{6}$$

At high Q^2 the operator product expansion (OPE) predicts [2]

$$\chi\left(Q^2 \to \infty\right) = -\frac{\alpha_s}{16\pi}\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle + \mathcal{O}(Q^{-2}), \tag{7}$$

where the perturbative contribution has been subtracted.

At low Q^2, $\chi(Q^2)$ is represented as a sum of contributions coming purely from QCD and from (π,η)-mesonic resonances [1]:

$$\begin{aligned}\left[\chi(Q^2)\right]_{\text{full QCD}} = \frac{m_u m_d}{m_u+m_d}\left\langle \overline{u}u\right\rangle - \chi'(0)Q^2 - \\ - \frac{f_\pi^2}{4}Q^2\left[\left(\frac{m_u-m_d}{m_u+m_d}\right)^2 \frac{m_\pi^2}{Q^2+m_\pi^2} + \frac{1}{3}\frac{m_\eta^2}{Q^2+m_\eta^2}\right] + \mathcal{O}(Q^4),\end{aligned} \tag{8}$$

where the first term has been found in [3] and its chiral limit follows the Crewther theorem [4] maintaining that $\chi\,(0) = 0$ in any theory where at least one massless quark exists.

The estimates of $\chi'(0)$ existing in the literature are rather controversial:

$$\chi'(0) = (48 \pm 6 \text{ MeV})^2 \ [5], \qquad \chi'(0) = (26 \pm 4 \text{ MeV})^2 \ [6]. \tag{9}$$

Both estimates were found within the QCD sum rules method. These values of the first moment of topological susceptibility have to be compared with the value obtained in the Okubo–Zweig–Iizuka (OZI) case, the case free of axial anomaly, which is

$$\chi'_{\text{OZI}}(0) = \frac{f_\pi^2}{2N_f} \approx (39 \text{ MeV})^2 .$$

The principal point is that smallness of $\chi'(0)$ is the base for the one of the mechanisms explaining «proton spin crisis» problem [7]. Indeed, within this approach it is assumed that the flavor singlet axial charge $a_0\left(Q^2\right)$ is proportional to the product of the first moment of the QCD topological susceptibility taken at scale Q^2 and an RG-invariant coupling of «OZI Goldstone boson» with nuclon:

$$a_0(Q^2) = \frac{1}{2m_N}6\sqrt{\chi'(0)}\,\widehat{\Gamma}_{\eta_0 NN}. \tag{10}$$

This mechanism has been, however, criticized in [8]. All this makes important further model estimates of $\chi'(0)$.

Within the chiral quark model[1] [9] based on the nonlocal structure of instanton QCD

[1]The explicit calculations below are performed in $SU(2)$ sector of the model.

vacuum [11] the full isosinglet axial-vector vertex becomes [10]

$$\Gamma^0_{\mu 5}(k,q,k'=k+q) = \left[\gamma_\mu - (k+k')_\mu \frac{\left(\sqrt{M(k')}-\sqrt{M(k)}\right)^2}{k'^2-k^2} - \right.$$
$$\left. - \frac{q_\mu}{q^2} 2\sqrt{M(k')M(k)}\,\frac{G'}{G}\frac{1-GJ_{PP}(q^2)}{1-G'J_{PP}(q^2)}\right]\gamma_5, \quad (11)$$

where $M(k)$ is dynamical, momentum-dependent quark mass; G and G' are 4-quark couplings in isotriplet and isosinglet channels, correspondingly, and

$$J_{PP}(q^2)\delta_{ab} = -\frac{i}{M_q^2}\int \frac{d^4k}{(2\pi)^4} M(k)M(k+q)\,\mathrm{Tr}\,\left[S(k)\gamma_5\tau^a S\,(k+q)\,\gamma_5\tau^b\right]. \quad (12)$$

In (12) the (inverse) quark propagator is $S^{-1}(p)=\hat{p}-M(p)$. Because of axial anomaly the singlet current does not contain massless pole, since as $q^2\to 0$ one has

$$\frac{1-GJ_{PP}(q^2)}{-q^2} = G\frac{f_\pi^2}{M_q^2}, \quad (13)$$

where f_π is pion weak decay constant and $M_q = M(0)$. The cancellation of the massless pole occurs with the help of the gap equation. Instead, the current develops a pole at the η'-meson mass[1], $1-G'J_{PP}(q^2=-m^2_{\eta'})=0$, thus solving the $U_A(1)$ problem. The vertex (11) satisfies the anomalous Ward–Takahashi identity:

$$q_\mu\Gamma^{(0)}_{\mu 5}(k,q,k'=k+q) = \gamma_5 S_F^{-1}(k') + S_F^{-1}(k)\gamma_5 + \gamma_5\frac{2\sqrt{M(k')M(k)}}{1-G'J_{PP}(q^2)}\left(1-\frac{G'}{G}\right), \quad (14)$$

where the last term is due to the anomaly. Thus, the QCD pseudoscalar gluonium operator is interpolated by the pseudoscalar effective quark field operator with coefficient expressed in terms of dynamical quark mass. This is a consequence of the fact that in the effective quark model the connection between quark and integrated gluon degrees of freedom is fixed by the gap equation.

For completeness we display the vertex corresponding to the conserved isotriplet axial-vector current

$$\Gamma^a_{\mu 5}(k,q,k'=k+q) = T^a\left[\gamma_\mu - q_\mu\frac{M(k')+M(k)}{q^2} - \right.$$
$$\left. - \left(k+k'-q\frac{k'^2-k^2}{q^2}\right)_\mu \frac{\left(\sqrt{M(k')}-\sqrt{M(k)}\right)^2}{k'^2-k^2}\right]\gamma_5 \quad (15)$$

satisfying the axial Ward–Takahashi identity

$$q_\mu\Gamma^a_{\mu 5}(k,q,k') = \gamma_5 S_F^{-1}(k')T^a + T^a S_F^{-1}(k)\gamma_5. \quad (16)$$

[1] See the previous footnote.

The axial-vector vertex (15) has a kinematical pole at $q^2 = 0$, a property that follows from the spontaneous breaking of the chiral symmetry in the limit of massless u and d quarks. Evidently, this pole corresponds to the massless Goldstone pion.

The quark matrix elements of currents corresponding to vertices (11) and (15) can be expressed in terms of real form factors

$$\left\langle p's' \left| A^{(0,3)}_{\mu 5}(0) \right| ps \right\rangle = \overline{u}_{s'}(p') T^{(0,3)} \left[\gamma_\mu \gamma_5 G^{(0,3)}_1(q^2) - q_\mu \gamma_5 G^{(0,3)}_2(q^2) \right] u_s(p), \tag{17}$$

where $T^{(0,3)} = (1, \tau^3/2)$; $u_s(p)$ are spinor solutions of the Dirac equation for free quarks, and the currents are defined as

$$A^{(0,3)}_{\mu 5}(q) = \int \frac{d^4k}{(2\pi)^4} \psi^\dagger(k) \Gamma^{(0,3)}_{\mu 5}(k, q, k' = k+q) \psi(k+q), \tag{18}$$

with $\psi(k)$ being the solutions of the Dirac equation

$$\left[\widehat{k} - M(k) \right] \psi(k) = 0. \tag{19}$$

By using the Dirac equation, one gets

$$q^\mu A^{(3)}_{\mu 5}\left(q^2\right) = 0, \quad q^\mu A^{(0)}_{\mu 5}(q^2) = \frac{(1 - G'/G)}{1 - G' J_{PP}(q^2)} \int \frac{d^4k}{(2\pi)^4} 2\sqrt{M(k')M(k)} \psi^\dagger(k) \gamma_5 \psi(k+q).$$

Comparison with (17) leads to the relations for form factors (taken in the local limit $M(k) \approx M_q$)

$$G^{(3)}_1\left(q^2\right) = 1, \qquad G^{(3)}_2\left(q^2\right) = 2M_q/q^2, \qquad G^{(0)}_1(0) = 1, \qquad G^{(0)}_2(0) = 0,$$

resembling the results for a model of free massive quarks.

Full model calculations lead to the following expression for the topological susceptibility [10]

$$\begin{aligned}
-\left(2N_f\right)^2 \chi(Q^2) = & \\
= 2N_f \left(1 - \frac{G'}{G}\right) \Bigg\{ & Q^2 J_{\pi A}\left(Q^2\right) \left[1 - \frac{G' J_{AP}(Q^2)}{M_q^2} + \frac{1}{1 - G' J_{PP}(Q^2)} \right] + \\
& + M_q^2 J_{PP}\left(Q^2\right) \left(1 - \frac{G'}{M_q^2} J_{AP}(Q^2)\right) \left[\frac{G J_{AP}(Q^2)}{M_q^2} - \frac{G - G'}{G\left[1 - G' J_{PP}(Q^2)\right]} \right] + \\
& + \frac{G}{M_q^2} \left[4N_c N_f \int \frac{d^4k}{(2\pi)^4} \frac{M(k)}{D(k)} \left[M(k) - \sqrt{M(k+Q)M(k)} \right] \right]^2 \Bigg\},
\end{aligned} \tag{20}$$

where $D(k) = k^2 + M^2(k)$ and the integrals $J_{AP}(q^2)$ and $J_{\pi A}(q^2)$ are defined by

$$\begin{aligned}
J_{AP}(q^2) &= 4N_c N_f \int \frac{d^4l}{(2\pi)^4} \frac{M(l)}{D(l)} \sqrt{M(l+q)M(l)}, \\
J_{\pi A}(q^2)\delta_{ab} &= \frac{q_\mu}{q^2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[S(k) \widetilde{\Gamma}^{5a}_\mu(k, q, k+q) S(k+q) \Gamma^a_\pi(k+q, k) \right].
\end{aligned}$$

At large Q^2 one obtains the power-like behavior consistent with the OPE prediction (7), namely

$$-\left(2N_f\right)^2 \chi\left(Q^2 \to \infty\right) = \frac{2N_f M_q^2}{G}\left(1 - \frac{G'}{G}\right). \tag{21}$$

At zero momentum the topological susceptibility is zero:

$$\chi\left(0\right) = 0, \tag{22}$$

in accordance with the Crewther theorem. For the first moment of the topological susceptibility we obtain [10]

$$\chi'\left(0\right) = \frac{1}{2N_f}\left\{f_\pi^2\left(2 - \frac{G'}{G}\right) + \left(1 - \frac{G'}{G}\right)^2 J'_{AP}(0)\right\}. \tag{23}$$

If the OZI rule were exact in the flavor singlet channel and there were no anomaly, one would have $G' = G$ and $\chi'(0) = \chi'_{\mathrm{OZI}}(0)$. But actually one has strong attraction in the isotriplet channel and strong repulsion due to the anomaly in the isosinglet channel that means that one has always the inequality $G' < G$. The second, negative term in (23) is numerically suppressed with respect to the first, positive term, $J'_{AP}(0)/f_\pi^2 \approx -0.24$. Thus, from the existence of the anomaly we always have the inequality

$$\chi'(0) > \chi'_{\mathrm{OZI}}(0), \tag{24}$$

and it is impossible to get anomalously small $\chi'(0)$. At this point we also mention other alternative approaches to the spin crisis problem based on screening of topological charge in the QCD vacuum [12, 13] (see for review, e.g., [14]).

The constants G and G' are fixed with the help of the meson spectrum. Approximately one has $G' \approx 0.1$ G. As a profile for the dynamical quark mass we take a Gaussian form

$$M(u) = M_q \exp\left(-2u/\Lambda^2\right) \tag{25}$$

with the model parameters $M_q = 0.3$ GeV, $\Lambda = 1.085$ GeV. Then the estimate for the first moment of the topological susceptibility is [10]

$$\chi'(0) = \left(50\ \mathrm{MeV}\right)^2. \tag{26}$$

To get the above result we have taken $N_f = 3$ in Eq. (23). We can see that the model gives the value of $\chi'(0)$ which is close to the estimate of Ref. [5]. The influence of the current quark masses on $\chi'(0)$ is expected to be small and the contribution of π and η mesons may be found from Eq. (8):

$$\chi'_{\pi,\eta}(0) \approx \left(28\ \mathrm{MeV}\right)^2,$$

and thus $\chi'_{\mathrm{tot}}(0) \approx (57\ \mathrm{MeV})^2$ for the total result.

The model prediction for the topological susceptibility is shown in figure. In the region of small and intermediate momenta our result is quantitatively close to the prediction of the QCD sum rules with the instanton effects included [1].

In the present talk we analyzed the correlation function of the singlet axial-vector currents within an effective nonlocal chiral quark model. By considering this correlator the topological susceptibility was found as a function of the Euclidean momentum and its first moment was estimated. We demonstrated that in realistic situation one always gets the inequality $\chi'(0) > \chi'_{\rm OZI}(0)$, thus discarding the mechanism explaining the «spin crisis» based on anomalous smallness of $\chi'(0)$. In addition, the fulfillment of the Crewther theorem was demonstrated. It would be interesting to verify the predictions given in Fig. 1 by modern lattice simulations.

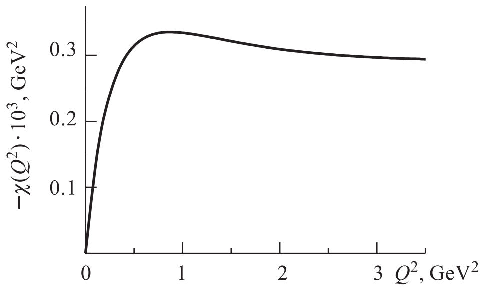

Fig. 1. Topological susceptibility versus Q^2 predicted by the model with $G' = 0.1\ G$, Eq. (20)

Acknowledgments. The author is grateful to W. Broniowski for fruitful cooperation and S. B. Gerasimov, H. Forkel, N. I. Kochelev, S. V. Mikhailov and O. V. Teryaev for useful discussions on the subject of the present work. This work is partially supported by RFBR grants (Nos. 02-02-16194, 03-02-17291) and INTAS-00-00-366.

REFERENCES

1. *Ioffe B. L.* hep-ph/9811217;
Ioffe B. L., Samsonov A. V. // Rus. J. Nucl. Phys. 2000. V. 63. P. 1448; Yad. Fiz. 2000. V. 63. P. 1527.
2. *Novikov V. A. et al.* // Phys. Lett. B. 1979. V. 86. P. 347.
3. *Veneziano G.* // Nucl. Phys. B. 1979. V. 15. P. 2139;
Di Vecchia P., Veneziano G. // Nucl. Phys. B. 1980. V. 171. P. 253.
4. *Crewther R. J.* // Phys. Lett. B. 1977. V. 70. P. 349.
5. *Ioffe B. L., Oganesian A. G.* // Phys. Rev. D. 1998. V. 57. P. 6590.
6. *Narison S., Shore G. M., Veneziano G.* // Nucl. Phys. B. 1999. V. 546. P. 235.
7. *Shore G. M., Veneziano G.* // Phys. Lett. B. 1990. V. 244. P. 75.
8. *Anselmino M., Efremov A., Leader E.* // Phys. Rep. 1995. V. 261. P. 1.
9. *Dorokhov A. E., Tomio L.* // Phys. Rev. D. 2000. V. 62. P. 014016;
Anikin I. V., Dorokhov A. E., Tomio L. // Part. Nucl. 2000. V. 31. P. 509; Fiz. Elem. Chast. Atom. Yadra. 2000. V. 31. P. 1023.
10. *Dorokhov A. E., Broniowski W.* // Eur. Phys. J. C. 2003. V. 32. P. 79.
11. *Dorokhov A. E., Esaibegian S. V., Mikhailov S. V.* // Phys. Rev. D. 1997. V. 56. P. 4062;
Dorokhov A. E. et al. // Eur. Phys. J. C. 2000. V. 13. P. 331.
12. *Forte S.* // Phys. Lett. B. 1989. V. 224. P. 189.
13. *Dorokhov A. E., Kochelev N. I.* // Mod. Phys. Lett. A. 1990. V. 5. P. 55.
14. *Dorokhov A. E., Kochelev N. I., Zubov Yu. A.* // Intern. J. Mod. Phys. A. 1993. V. 8. P. 603.

Received on January 22, 2004.