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MASS SPECTRA OF HEAVY-LIGHT MESONS AND DOUBLY HEAVY BARYONS

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Mass spectra of orbitally and radially excited heavy-light (B and D) mesons and doubly heavy baryons (Ξ_{cc} and Ξ_{bb}) are presented. The light-quark–heavy-diquark structure of the baryon is assumed. The expansion in only inverse powers of the heavy-(di)quark mass is carried out, while the light quark is treated completely relativistically. The relativistic treatment of the light quark plays an essential role. It is argued that P levels are inverted in the limit of infinitely heavy quark. The close similarity of the mass spectra of heavy-light mesons and doubly heavy baryons is revealed.

Представлены спектры масс орбитально и радиально возбужденных тяжело-легких (B и D) мезонов и дважды тяжелых барионов (Ξ_{cc} и Ξ_{bb}). Предполагается структура бариона в виде легкого кварка – тяжелого дикварка. Проводится разложение только по обратным степеням масс тяжелого (ди)кварка, в то время как легкий кварк рассматривается полностью релятивистским образом. Релятивистское рассмотрение легкого кварка играет важнейшую роль. Обосновывается, что P-уровни инвертируются в пределе бесконечно тяжелого кварка. Устанавливается близкое подобие спектров масс тяжело-легких мезонов и дважды тяжелых барионов.

In this paper, we present results of the mass spectrum calculations of heavy-light mesons ($q\bar{Q}$) and doubly heavy baryons (qQQ), where $Q = c, b$ are the heavy quarks and $q = u, d, s$ are the light quarks. Despite the different quark structure, these hadrons have some common features, which are explicitly revealed in the infinitely heavy quark limit. The two heavy quarks in the doubly heavy baryon compose a bound diquark system in the antitriplet colour state, which serves as a localized colour source. The light quark is orbiting around this heavy source at a distance larger ($\sim 1/m_q$) than the source size ($\sim 2/m_Q$). Thus the doubly heavy baryons look effectively like a two-body bound system and strongly resemble the heavy-light B and D mesons. Since the heavy-(di)quark spin and mass decouple in the infinitely heavy quark limit, all the properties of these hadrons are determined by the dynamics of light degrees of freedom, which is almost identical. The main distinction of the qQQ baryon from the $q\bar{Q}$ meson is that the QQ colour source, though being almost localized, still is a composite system bearing integer values of the total angular momentum $(0, 1, \dots)$. Hence, the interaction of the heavy diquark with the light quark is not point-like but is smeared out by the form factor expressed through the overlap of the diquark wave functions. Besides this, the diquark excitations contribute to the baryon excited states.

The light quark in heavy-light hadrons should be treated fully relativistically, since estimates of its velocity give $v/c \sim 0.7-0.8$. Therefore we use the relativistic quark model based

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on the quasi-potential approach. The quark interactions are described by the bound-state wave functions, which satisfy the relativistic equation of the Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_B(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_B(\mathbf{q}), \tag{1}$$

where μ_R is the relativistic reduced mass

$$\mu_R = \frac{M^4 - (m_q^2 - m_Q^2)^2}{4M^3}$$

and $b^2(M)$ is the c.m.s. relative momentum squared on mass shell

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}.$$

Here M is the heavy-hadron mass; m_Q and m_q are heavy-(di)quark and light-quark masses.

The kernel $V(\mathbf{p},\mathbf{q};M)$ is the quasi-potential operator of the quark–antiquark or quark–diquark interaction. Constructing this kernel, we adopted that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector with scalar linear confining potentials:

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_q(\mathbf{p})\bar{u}_Q(-\mathbf{p})\Bigg\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_q^{\mu}\gamma_Q^{\nu} + \\ + V_{\mathrm{conf}}^V(\mathbf{k})\Gamma_q^{\mu}\Gamma_{Q;\mu} + V_{\mathrm{conf}}^S(\mathbf{k})\Bigg\}u_q(\mathbf{q})u_Q(-\mathbf{q}), \tag{2}$$

where α_S is the QCD coupling constant; $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge; $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors, the effective long-range vector vertex contains the Pauli term

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \qquad k^\nu = (0,\mathbf{k}), \tag{3}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit are reduced to

$$V_{\mathrm{conf}}^V(r) = (1-\varepsilon)(Ar + B), \quad V_{\mathrm{conf}}^S(r) = \varepsilon(Ar + B), \tag{4}$$

reproducing

$$V_{\mathrm{conf}}(r) = V_{\mathrm{conf}}^S(r) + V_{\mathrm{conf}}^V(r) = Ar + B, \tag{5}$$

where ε is the mixing coefficient. All the parameters of the model such as quark masses, parameters of linear confining potential A and B, mixing coefficient ε, and anomalous chromomagnetic quark moment κ were fixed from the analysis of heavy quarkonium properties. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.30$ GeV have standard values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the charmonium radiative decays. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonium ${}^3P_J$ states. Note that the long-range magnetic contribution to the potential in the model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa = -1$. These values of the parameters ε and κ allow one to reproduce the results of heavy-quark effective theory [1].

The details of the mass spectra calculations of the heavy-light mesons and doubly heavy baryons are given in [1, 2]. We calculated the mass spectra first in the infinitely heavy quark mass limit and then with the inclusion of $1/m_Q$ corrections. The quasi-potential of the heavy-(di)quark interaction with light quark in the infinitely heavy quark limit can be presented in configuration space in the following form:

$$V_{m_Q\to\infty}(r) = \frac{E_q+m_q}{2E_q}\Bigg[V_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{(E_q+m_q)^2}\Bigg\{\mathbf{p}[V_{\rm Coul}(r)+ + V^V_{\rm conf}(r)-V^S_{\rm conf}(r)]\mathbf{p}-\frac{E_q+m_q}{2m_q}\Delta V^V_{\rm conf}(r)[1-(1+\kappa)]+ \frac{2}{r}\left(V'_{\rm Coul}(r)-V'^S_{\rm conf}(r)-V'^V_{\rm conf}(r)\left[\frac{E_q}{m_q}-2(1+\kappa)\frac{E_q+m_q}{2m_q}\right]\right)\mathbf{L}\mathbf{S}_q\Bigg\}\Bigg]. \quad (6)$$

Here $V_{\rm Coul}(r)$ is the Coulomb-like potential, the prime denotes differentiation with respect to r; $\mathbf{L}$ is the orbital momentum, and $\mathbf{S}_q$ is the spin operator of the light quark; $E_q = (M^2 - m_Q^2 + m_q^2)/(2M)$. We see that in this limit the heavy-(di)quark mass and spin decouple and the dynamics of the heavy hadron is determined by the light quark alone in accord with the heavy-quark symmetry. Thus the properties of heavy-light mesons and doubly heavy baryons are similar in the limit $m_Q \to \infty$. Inclusion of the first order $1/m_Q$ corrections breaks the heavy-quark symmetry and leads to different splittings in mesons and baryons [1, 2]. Note that in our calculations we treated the light quark completely relativistically without applying unjustified expansion in inverse powers of its mass.

In Tables 1–4, we present mass spectra of ground and excited states of the D, B mesons and Ξ_{cc}, Ξ_{bb} baryons. The corresponding level orderings are schematically shown in Figs. 1, 2. There we first show our predictions for D, B and Ξ_{cc}, Ξ_{bb} spectra in the infinitely heavy quark limit. In this limit the P-wave excitations of the light quark are inverted. It means that the mass of the state with higher light quark angular momentum $j = 3/2$ is smaller than the mass of the state with lower angular momentum $j = 1/2$. Next we include $1/m_Q$ corrections. This results in splitting of the degenerate states and mixing of states with different j, which have the same total angular momentum J and parity. The hyperfine splitting of P levels turns out to be of the same order of magnitude as the gap between $j = 1/2$ and $j = 3/2$ degenerate multiplets in the $m_Q \to \infty$ limit. The inclusion of $1/m_Q$ corrections leads also to the relative shifts of the hadron levels further decreasing this gap. As a result, some of the P levels from different (initially degenerate) multiplets overlap; however, centres of levels averaged over the heavy-(di)quark spin remain inverted.

The close similarity of the interaction of the light quark with the heavy antiquark and with the heavy diquark produces very simple relations between the ground state hyperfine

Table 1. **Mass spectrum of D mesons (in GeV)**

State	Particle	Theory	PDG(2002)	CLEO	DELPHI
$^1S_0$	D	1.875	1.8693(5)		
$^1S_1$	D^*	2.009	2.0100(5)		
$^1P_2$	D_2^*	2.459	2.459(4)		
$^1P_1$	D_1	2.414	2.4222(18)	2.425(3)	
$^1P_1$	$D_{1'}$	2.501		2.461(52)	
$^1P_0$	D_0^*	2.438			
$^2S_0$	D'	2.579			
$^2S_1$	$D^{*'}$	2.629			2.637(9)

Table 2. **Mass spectrum of B mesons (in GeV)**

State	Particle	Theory	PDG(2002)	OPAL	L3	DELPHI	CDF	ALEPH
$^1S_0$	B	5.285	5.2790(5)					
$^1S_1$	B^*	5.324	5.3250(6)					
$^1P_2$	B_2^*	5.733			5.768(8)	5.732(21)		5.739(13)
$^1P_1$	B_1	5.719		5.738(9)			5.71(2)	
$^1P_1$	B_1	5.757			5.670(16)			
$^1P_0$	B_0^*	5.738		5.839(14)				
$^2S_0$	B'	5.883						
$^2S_1$	$B^{*'}$	5.898				5.90(2)		

Table 3. **Mass spectrum of Ξ_{cc} baryons (in GeV)**

State	Mass		State	Mass	
$(n_dLn_ql)J^P$	This work	Ref. [3]	$(n_dLn_ql)J^P$	This work	Ref. [3]
$(1S1s)1/2^+$	3.620	3.478	$(1P1s)1/2^-$	3.838	3.702
$(1S1s)3/2^+$	3.727	3.61	$(1P1s)3/2^-$	3.959	3.834
$(1S1p)1/2^-$	4.053	3.927	$(2S1s)1/2^+$	3.910	3.812
$(1S1p)3/2^-$	4.101	4.039	$(2S1s)3/2^+$	4.027	3.944
$(1S1p)1/2'^-$	4.136	4.052	$(2P1s)1/2^-$	4.085	3.972
$(1S1p)5/2^-$	4.155	4.047	$(2P1s)3/2^-$	4.197	4.104
$(1S1p)3/2'^-$	4.196	4.034	$(3S1s)1/2^+$	4.154	4.072

splittings of the heavy-light mesons and the doubly heavy baryons

$$\Delta M(\Xi_{QQ}) \cong \frac{3}{4}\Delta M_{B,D}, \quad \Delta M(\Omega_{QQ}) \cong \frac{3}{4}\Delta M_{B_s,D_s}, \tag{7}$$

where the factor $3/4$ is just the product of the ratio of the baryon-to-meson spin matrix elements and the heavy quark–diquark mass ratio. Our results presented in Table 5 show that these relations are satisfied with a good accuracy [2].

We presented the results of the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons and doubly heavy baryons in quark–diquark approximation.

Table 4. **Mass spectrum of Ξ_{bb} baryons (in GeV)**

State	Mass		State	Mass	
$(n_d L n_q l) J^P$	This work	Ref. [3]	$(n_d L n_q l) J^P$	This work	Ref. [3]
$(1S1s)1/2^+$	10.202	10.093	$(2S1s)1/2^+$	10.441	10.373
$(1S1s)3/2^+$	10.237	10.133	$(2S1s)3/2^+$	10.482	10.413
$(1S1p)1/2^-$	10.632	10.541	$(2S1p)1/2^-$	10.873	
$(1S1p)3/2^-$	10.647	10.567	$(2S1p)3/2^-$	10.888	
$(1S1p)5/2^-$	10.661	10.580	$(2S1p)1/2'^-$	10.902	
$(1S1p)1/2'^-$	10.675	10.578	$(2S1p)5/2^-$	10.905	
$(1S1p)3/2'^-$	10.694	10.581	$(2S1p)3/2'^-$	10.920	
$(1S2s)1/2^+$	10.832		$(2P1s)1/2^-$	10.563	10.493
$(1S2s)3/2^+$	10.860		$(2P1s)3/2^-$	10.607	10.533
$(1P1s)1/2^-$	10.368	10.310	$(3S1s)1/2^+$	10.630	10.563
$(1P1s)3/2^-$	10.408	10.343	$(3S1s)3/2^+$	10.673	
$(1P1p)1/2^+$	10.763		$(3P1s)1/2^-$	10.744	
$(1P1p)3/2^+$	10.779		$(3P1s)3/2^-$	10.788	
$(1P1p)5/2^+$	10.786		$(4S1s)1/2^+$	10.812	
$(1P1p)1/2'^+$	10.838		$(4S1s)3/2^+$	10.856	
$(1P1p)3/2'^+$	10.856		$(4P1s)1/2^-$	10.900	

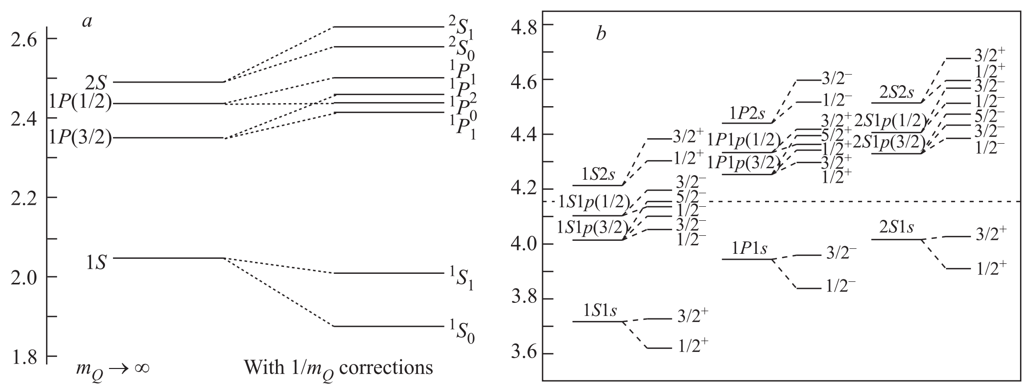

Fig. 1. Masses of D mesons (*a*) and Ξ_{cc} baryons (*b*) (in GeV)

The main advantage of the proposed approach consists in the fully relativistic treatment of the light quarks (u, d, s). We find that the P-wave levels of the light quark which correspond to heavy-(di)quark spin multiplets with $j = 1/2$ and $j = 3/2$ are inverted in the infinitely heavy (di)quark limit. The origin of this inversion is the following. The negative contribution of the confining potential to the spin-orbit term in (6) exceeds the positive one-gluon exchange contribution. Thus the sign before the spin-orbit term is negative, and the level inversion emerges. However, the $1/m_Q$ corrections, which produce the hyperfine splittings in these multiplets, are substantial. As a result the purely inverted pattern of P levels occurs only for B meson and Ξ_{bb}, Ω_{bb} baryons. For D, D_s, B_s mesons and Ξ_{cc}, Ω_{cc} baryons the levels from these multiplets overlap.

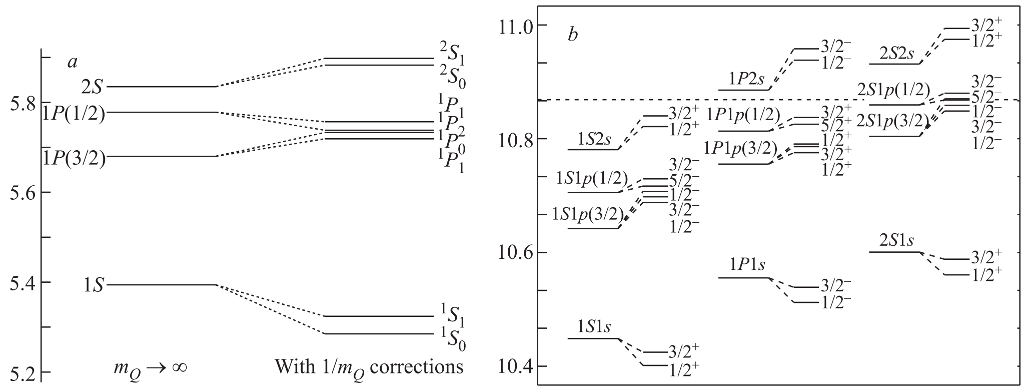

Fig. 2. Masses of B mesons (*a*) and Ξ_{bb} baryons (*b*) (in GeV)

Table 5. **Comparison of hyperfine splittings (in MeV) in doubly heavy baryons and heavy-light mesons**

$\Delta M(\Xi_{cc})$	$\frac{3}{4}\Delta M_D^{\rm exp}$	$\Delta M(\Xi_{bb})$	$\frac{3}{4}\Delta M_B^{\rm exp}$	$\Delta M(\Omega_{cc})$	$\frac{3}{4}\Delta M_{D_s}^{\rm exp}$	$\Delta M(\Omega_{bb})$	$\frac{3}{4}\Delta M_{B_s}^{\rm exp}$
107	106	35	34	94	108	30	35

Recently BaBar, CLEO and Belle collaborations [4] observed two new narrow states $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ decaying to $D_s^+\pi^0$ and $D_s^{*+}\pi^0$. The decays of these states indicate that they are likely to be 0^+ and 1^+. The measured masses of these new states are considerably lower than most of theoretical predictions (including our model) for corresponding P-wave states originating from $j = 1/2$ multiplet. Thus it is likely that these states are DK molecules, $D_s\pi$ atoms or four-quark states [5]. However more experimental and theoretical study of these states is needed.

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REFERENCES

1. *Ebert D., Galkin V. O., Faustov R. N.* // Phys. Rev. D. 1998. V. 57. P. 5663; 1999. V. 59. P. 019902(E).
2. *Ebert D. et al.* // Phys. Rev. D. 2002. V. 66. P. 014008.
3. *Gershtein S. S. et al.* // Phys. Rev. D. 2000. V. 60. P. 054021.
4. *Aubert B. et al. (BaBar Collab.)* // Phys. Rev. Lett. 2003. V. 90. P. 242001;
   Besson D. et al. (CLEO Collab.) // Phys. Rev. D. 2003. V. 68. P. 032002;
   Krokovny P. et al. (Belle Collab.). hep-ph/0308019.
5. *Barnes T., Close F. E., Lipkin H. J.* hep-ph/0305025;
   Szczepaniak A. P. // Phys. Lett. B. 2003. V. 567. P. 23;
   Browder T., Pakvasa S., Petrov A. A. hep-ph/0307054;
   Terasaki K. hep-ph/0309279.