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EXCLUSIVE ELECTRODISINTEGRATION OF THE DEUTERON IN THE BETHE–SALPETER APPROACH

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An exclusive process of the deuteron electrodisintegration is analyzed in the framework of Bethe–Salpeter formalism with a phenomenological Graz II rank-three separable interaction. The approximations made are the neglect of final-state interaction, two-body exchange currents, negative-energy components of the bound-state vertex function and the scattering T matrix. The comparison of the relativistic calculations of the exclusive cross section in the laboratory system with the experimental data is presented within different kinematic conditions.

Процесс эксклюзивного электрорасщепления дейтрона рассматривается в формализме Бете–Солпитера с феноменологическим сепарабельным ядром взаимодействия Graz II третьего ранга. Сделаны следующие приближения: не учитываются взаимодействие в конечном состоянии, релятивистские двухчастичные токи, а также компоненты вершинной функции связанного состояния и T-матрицы с отрицательной энергией. Приводится сравнение релятивистских расчетов эксклюзивного сечения реакции в лабораторной системе с экспериментальными данными в различных кинематических условиях.

INTRODUCTION

Disintegration of the deuteron (either by photons or electrons) has been and still is a rich source of information on the structure of electromagnetic current operators and nuclear dynamics. Previous and current experiments provide data both for inclusive and exclusive cases [1–4]. If the polarized beam and target are also used, the full experiment can be measured with all polarization observables. These experimental data give a powerful stimulation to investigate theoretically the exclusive process of the deuteron electrodisintegration.

There are several approaches to the theoretical description of the deuteron and, in particular, the deuteron break-up reaction. Usually the nonrelativistic treatment with solving the Schrödinger equation for the deuteron and final np pair is used. The two-body currents are also taken into account (see, for instance, [5, 6]). Some authors use a numerical solving of a relativistic wave equation for NN system based on a relativistic one-boson-exchange (OBE) model with one nucleon being on mass shell ([7, 8]). The other ones use a simple phenomenological approach with adding the lowest-order relativistic corrections to the nonrelativistic

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one-body current and including the kinematic wave function boost [9], or the model based on a direct evaluation of those Feynman diagrams that give the dominant contributions in the quasi-free region [10].

In the paper, the deuteron electrodisintegration within the covariant BS approach [11] with the separable Graz II interaction kernel is considered [12]. The exclusive differential cross section is calculated with the following approximations: the neglect of a final-state interaction, two-body exchange currents, negative-energy components of the bound-state vertex function and the scattering T matrix.

The main goal of the present work is to describe the deuteron electrodisintegration in a consistent relativistic approach with some approximations discussed below. This is done within the Bethe–Salpeter equation for the two-nucleon system. In this way it is possible to come to general conclusions about the amplitude of the process, which are not seen in the nonrelativistic approach. On the other hand, the nonrelativistic limit can be recovered and some links to the nonrelativistic corrections can be established.

The paper is organized as follows: in Sec. 1 the relativistic kinematics of the reaction and cross section are described; the deuteron problem in the BS formalism is considered in Sec. 2; Section 3 is devoted to the electromagnetic current of the hadron system; finally, results of numerical calculations are presented in Sec. 4.

1. KINEMATICS OF THE REACTION AND CROSS SECTION

Let us consider the relativistic kinematics of the exclusive deuteron break-up process. The initial electron $l = (E, \mathbf{l})$ collides with the deuteron in the rest frame $K = (M_d, \mathbf{0})$, where M_d is the deuteron mass. There are three particles in the final state: the final electron $l' = (E', \mathbf{l}')$, proton and neutron. Concerning the one-photon approximation and neglecting the electron mass, one can obtain the virtual photon momentum squared ($q = (\omega, \mathbf{q})$ is in the laboratory system (LS) frame):

$$q^2 = -Q^2 = (l - l')^2 = \omega^2 - \mathbf{q}^2 = -4|\mathbf{l}||\mathbf{l}'| \sin^2 \frac{\theta}{2}, \tag{1}$$

where θ is the electron scattering angle. The outgoing np pair is characterized by the invariant mass $s = P^2 = (p_p + p_n)^2$:

$$s = M_d^2 + 2M_d\omega + q^2. \tag{2}$$

The Lorentz-invariant matrix element of the reaction can be written in the following form (see Fig. 1):

$$\begin{aligned} M_{fi} = -\imath e^2 (2\pi)^4 \delta^{(4)}(K - P + q) \times \\ \times \langle l', s'_e | j^\mu | l, s_e \rangle \frac{1}{q^2} \langle np : (P, Sm_S) | J_\mu | d : (K, M) \rangle \end{aligned} \tag{3}$$

with $\langle l', s'_e | j^\mu | l, s_e \rangle = \bar{u}(l', s'_e)\gamma^\mu u(l, s_e)$ being the electron electromagnetic (EM) current. Dirac spinors $u(l, s_e)$, $(\bar{u}(l', s'_e))$ stand for initial (final) electrons, and $\langle np : (P, Sm_S)|J_\mu|d : (K, M)\rangle$ is an electromagnetic hadron current matrix element between the deuteron state $|d : (K, M)\rangle$ (M is a total momentum projection) and final np pair state $|np : (P, Sm_S)\rangle$ (S is a spin, and m_S is a spin projection).

Using the standard procedure the unpolarized exclusive differential cross section can be written as

$$\frac{d^5\sigma}{dE' d\Omega' d\Omega_{\mathbf{p}}} = \frac{\alpha^2}{8M_d(2\pi)^3} \frac{|\mathbf{l}'|}{|\mathbf{l}|} \frac{1}{q^4} \frac{R}{\sqrt{s}}\, l^{\mu\nu} W_{\mu\nu} \quad (4)$$

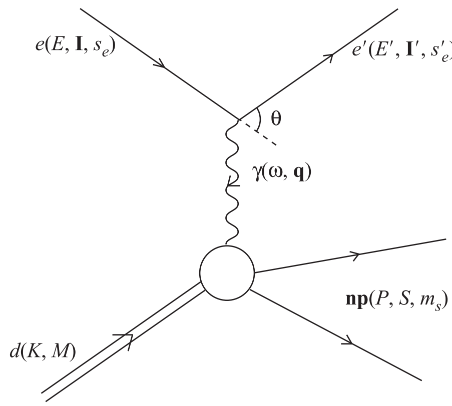

Fig. 1. The one-photon approximation

with the kinematical factor R describing the transition from a centre-of-mass system (CM) to a laboratory one in the form

$$R = \frac{\mathbf{p}^2}{\sqrt{1+\eta}|\mathbf{p}| - e_{\mathbf{p}}\sqrt{\eta}\, \cos\theta_{\mathbf{p}}}. \quad (5)$$

Three-momentum $\mathbf{p}$ concerns the final proton in LS, $e_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, $\theta_{\mathbf{p}}$ is an angle between the outgoing proton and Z axis, m is the nucleon mass, $\eta = \mathbf{q}^2/s$.

In Eq. (4) the unpolarized lepton

$$l^{\mu\nu} = \frac{1}{2} \sum_{s_e\, s'_e} \langle l', s'_e | j^{\mu\dagger} | l, s_e \rangle \langle l, s_e | j^\nu | l', s'_e \rangle = 2({l'}^\mu l^\nu + {l'}^\nu l^\mu) + g^{\mu\nu} q^2 \quad (6)$$

and hadron

$$W^{\mu\nu} = \frac{1}{3} \sum_{MSm_s} \langle d : (K, M)| J^{\mu\dagger} | np : (P, Sm_S)\rangle \langle np : (P, Sm_S)| J^\nu | d : (K, M)\rangle \quad (7)$$

tensors were introduced. Below the averaging of initial particles and summarizing of final ones were performed. Introducing helicity components of the tensors and using the lepton and hadron tensor Hermitian, one can write

$$\frac{d^5\sigma}{dE' d\Omega' d\Omega_{\mathbf{p}}} = \frac{\sigma_{\mathrm{Mott}}}{8M_d(2\pi)^3} \frac{R}{\sqrt{s}} \times$$
$$\times \left[l^0_{00} W_{00} + l^0_{++}(W_{++} + W_{--}) + l^0_{+-}\, 2\mathrm{Re}\, W_{+-} - l^0_{0+}\, 2\mathrm{Re}\,(W_{0+} - W_{0-}) \right] \quad (8)$$

with $\sigma_{\mathrm{Mott}} = \left(\alpha \cos\frac{\theta}{2} / 2E \sin^2\frac{\theta}{2} \right)^2$ being the Mott cross section and

$$l^0_{00} = \frac{Q^2}{\mathbf{q}^2}, \quad l^0_{0+} = \frac{Q}{|\mathbf{q}|\sqrt{2}} \sqrt{\frac{Q^2}{\mathbf{q}^2} + \tan^2\frac{\theta}{2}}, \quad l^0_{++} = \frac{1}{2}\tan^2\frac{\theta}{2} + \frac{Q^2}{4\mathbf{q}^2}, \quad l^0_{+-} = -\frac{Q^2}{2\mathbf{q}^2}. \quad (9)$$

Taking into account the conservation of the matrix element of the EM current of the hadron system in the form

$$q^{\mu}\left\langle np, Sm_S|J_{\mu}|D, M\right\rangle = 0, \tag{10}$$

one can exclude the zero-component of the hadron current and rewrite Eq. (8) in the following form:

$$\frac{d^5\sigma}{dE'd\Omega'd\Omega_{\mathbf{p}}} = \frac{\sigma_{\rm Mott}}{8M_d(2\pi)^3}\frac{R}{\sqrt{s}}\times$$
$$\times\left[\frac{Q^2}{\mathbf{q}^2}l^0_{00}W_{00} + l^0_{++}(W_{++}+W_{--}) + l^0_{+-}\,2{\rm Re}\,W_{+-} - \frac{Q}{|\mathbf{q}|}l^0_{0+}\,2{\rm Re}\,(W_{0+}-W_{0-})\right]. \tag{11}$$

It should be noted that in the last equation in the hadron tensor index, 0 means the longitudinal helicity instead of time component in Eq. (8). Expression (11) is gauge invariant by construction, although the hadron EM current matrix element is not.

2. BETHE–SALPETER AMPLITUDE FOR THE DEUTERON

The basic object describing the deuteron in the Bethe–Salpeter formalism is the amplitude $\Phi_M(k;K)$, which satisfies the homogeneous equation

$$\Phi_{M\,\alpha\beta}(k;K) = \imath S^{(1)}_{\alpha\eta}\left(\frac{K}{2}+k\right)S^{(2)}_{\beta\rho}\left(\frac{K}{2}-k\right)\int\frac{d^4k'}{(2\pi)^4}V_{\eta\rho,\epsilon\lambda}(k,k';K)\Phi_{M\,\epsilon\lambda}(k';K), \tag{12}$$

where $S^{(\ell)}(K/2-(-1)^{\ell}k)$ is a propagator of the ℓth nucleon; $V(k,k';K)$ is the kernel of a nucleon–nucleon (NN) interaction, and Greek letters denote the spinor indices. Considering the deuteron in the rest frame, one can use the partial-wave decomposition of the BS amplitude on the relativistic two-nucleon basic states $|aM\rangle \equiv |\pi,\, {}^{2S+1}L^{\rho}_{J}M\rangle$:

$$\Phi_M(k;K) = \sum_a \phi_a(k_0,|\mathbf{k}|)\mathcal{Y}_{aM}(\mathbf{k}), \tag{13}$$

where S stands for the total spin; L is the orbital angular momentum, and J ($J=1$ for the deuteron) is the total angular momentum with the projection M; relativistic quantum numbers ρ and π refer to the total energy-spin and relative-energy parity with respect to the change of sign of the relative energy, respectively. The function $\phi_a(k_0,|\mathbf{k}|)$ is a radial part, $\mathcal{Y}_{aM}(\mathbf{k})$ is a spin-angle part of the BS amplitude (for details, see [12, 13]).

In calculations it is convenient to use the BS vertex function of the deuteron defined as follows[1]:

$$\Phi_M(k;K) = S^{(1)}\left(\frac{K}{2}+k\right)S^{(2)}\left(\frac{K}{2}-k\right)\Gamma_M(k;K). \tag{14}$$

[1]For simplicity, the spinor indices are omitted.

The radial parts of the amplitude and the vertex function are connected in a way

$$\phi_a(k_0, |\mathbf{k}|) = \sum_b S_{ab}(k_0, |\mathbf{k}|; s) g_b(k_0, |\mathbf{k}|), \tag{15}$$

with S_{ab} being the decomposed two-nucleon propagator. In current calculations only positive-energy states of the deuteron amplitudes were taken into account ($^3S_1^+, ^3D_1^+$) and states with negative energy were omitted. In such a case the function S_{ab} is diagonal: $S_{++}(k_0, |\mathbf{k}|; s) = 1/\left(\sqrt{s}/2 + k_0 - e_{\mathbf{k}}\right)\left(\sqrt{s}/2 - k_0 - e_{\mathbf{k}}\right)$.

To solve Eq. (12), the rank-three covariant separable kernel of Graz II NN interaction is used (see details in [12]).

3. ELECTROMAGNETIC CURRENT OF THE HADRON SYSTEM

The Mandelstam technique [14, 15] provides the method to express the EM current matrix element of the hadron system in terms of the BS amplitudes and generalized (Mandelstam) current

$$\langle np : (P, Sm_S)|J_\mu|d : (K, M)\rangle = \\ = \imath \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \bar{\chi}_{Sm_s}(p; p^*, P)\, \Lambda_\mu(p, k; P, K)\, \Phi_M(k; K), \tag{16}$$

with p^* being the asymptotic relative four-momentum of the np pair: $Pp^* = 0$, which is connected with s as $\mathbf{p}^{*2} = s/4 - m^2$.

The Mandelstam current consists of one-body and two-body parts $\Lambda_\mu = \Lambda_\mu^{[1]} + \Lambda_\mu^{[2]}$. Taking into account the first term in the above formulae in the form

$$\Lambda_\mu^{[1]}(p, k; P, K) = \imath(2\pi)^4 \left\{ \delta^{(4)}\left(p - k - \frac{q}{2}\right) \Gamma_\mu^{(1)}\left(\frac{P}{2} + p, \frac{K}{2} + k\right) S^{(2)}\left(\frac{P}{2} - p\right)^{-1} + \right. \\ \left. + \delta^{(4)}\left(p - k + \frac{q}{2}\right) \Gamma_\mu^{(2)}\left(\frac{P}{2} - p, \frac{K}{2} - k\right) S^{(1)}\left(\frac{P}{2} + p\right)^{-1} \right\} \tag{17}$$

and neglecting the two-body current, one can obtain the relativistic impulse approximation (RIA) for the electromagnetic current matrix element of the reaction

$$\langle np : (P, Sm_S)|J_\mu|d : (K, M)\rangle = \imath \sum_{\ell=1,2} \int \frac{d^4p}{(2\pi)^4} \bar{\chi}_{Sm_S}(p; P) \times \\ \times \Gamma_\mu^{(\ell)}(q) S^{(\ell)}\left(\frac{P}{2} - (-1)^\ell p - q\right) \Gamma_M\left(p + (-1)^\ell \frac{q}{2}; K\right). \tag{18}$$

The γNN vertex is chosen in the on-mass-shell form:

$$\Gamma_\mu^{(\ell)}(q) = \gamma_\mu F_1^{(\ell)}(q^2) - \frac{1}{4m}\left(\gamma_\mu \hat{q} - \hat{q}\gamma_\mu\right) F_2^{(\ell)}(q^2), \tag{19}$$

where $F_1^{(\ell)}$ ($F_2^{(\ell)}$) is Dirac (Pauli) EM form factors of the nucleons with the following normalization conditions:

$$\begin{aligned} &F_1^{(1)}(0) = 1, \qquad F_2^{(1)}(0) = \varkappa_p, \\ &F_1^{(2)}(0) = 0, \qquad F_2^{(2)}(0) = \varkappa_n, \end{aligned} \tag{20}$$

and $\varkappa_p$ ($\varkappa_n$) being the proton (neutron) anomalous magnetic moments. In the calculations the dipole fit for nucleon form factors was used.

In the paper, the final-state interaction (FSI) between the outgoing nucleons is neglected, and the final np-pair BS amplitude is considered in the plane-wave approximation (PWA):

$$\begin{aligned} \bar{\chi}^{(0)}_{Sm_s}(p; p^*, P) &= (2\pi)^4\, \delta^{(4)}(p - p^*)\, \bar{\chi}^{(0)}_{Sm_S}(p^*, P) = \\ &= (2\pi)^4\, \delta^{(4)}(p - p^*) \sum_{m_1 m_2} C^{Sm_S}_{\frac{1}{2}m_1 \frac{1}{2}m_2}\, \bar{u}_{m_1}\left(\frac{P}{2} + p\right) \bar{u}_{m_2}\left(\frac{P}{2} - p\right). \end{aligned} \tag{21}$$

Substituting expression (21) for the np-pair amplitude into Eq. (18), one can perform an integration on the relative momentum p and obtain

$$\begin{aligned} \langle np : (P, Sm_S)|J_\mu|d : (K, M)\rangle = \imath \sum_{\ell=1,2} \bar{\chi}^{(0)}_{Sm_S}(p^*, P)\Gamma^{(\ell)}_\mu(q)\times \\ \times S^{(\ell)}\left(\frac{K}{2} - p^* - (-1)^\ell \frac{q}{2}\right) \Gamma_M\left(p^* + (-1)^\ell \frac{q}{2}; K\right). \end{aligned} \tag{22}$$

Equation (22) is the basic expression used in the calculations. To move ahead, one should transform the Dirac spinors in (21) to LS frame and perform the partial-wave decomposition.

4. CALCULATIONS AND RESULTS

The partial-wave decomposition of the deuteron and final np-pair BS amplitudes in Eq. (22) was performed by using the program written on the REDUCE analytic calculation language. The numeric calculations were performed with the help of the FORTRAN language programs.

Results of the calculations of the unpolarized exclusive cross section are shown in Fig. 2 (the experimental data are taken from [1,2]). The cross section is plotted as a function of the final neutron momentum (missing momentum) in LS frame. Three calculations differ by the initial and final electron energies and electron scattering angle (see the Table).

Different sets of kinematical conditions

	E, MeV/c	E', MeV/c	θ, $^\circ$	ω, MeV/c	$\lvert\mathbf{q}\rvert$, MeV/c	Q^2, GeV^2/c^2
Set I	500	395	59	105	450	0.191
Set II	500	352	44.4	148	350	0.101
Set III	560	360	25	200	278.9	0.038

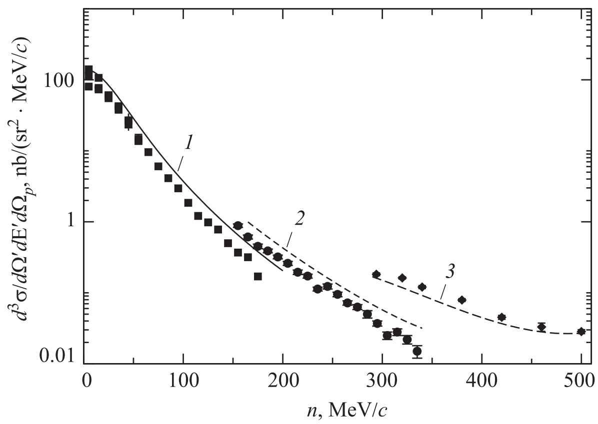

Fig. 2. Relativistic calculations of the exclusive differential cross section. Three different kinematic conditions: set I (*1*), set II (*2*) and set III (*3*). Experimental data from [1,2]: ■ — set I; ● — set II; ◆ — set III

As is seen from Fig. 2, the relativistic result is overestimated compared to the experimental data for set I and set II, and underestimated for set III. It indicates that the effects of the final-state interaction, two-body exchange currents, negative-energy components of the bound-state vertex function and the scattering T matrix should be taken into account. Such calculations are in progress.

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