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# CALCULATION OF SPIN ALIGNMENT OF DEUTERONS TRAVELING THROUGH MATTER

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Calculations of the spin alignment of the relativistic deuteron beam passing through matter are described. The tensor polarization is calculated within the framework of the Glauber multiple scattering theory. The calculation results are compared with the recent experimental data.

Описан расчет выстроенности по спину пучка релятивистских дейтронов, появляющейся при его прохождении через вещество. Тензорная поляризация вычисляется в рамках теории многократного рассеяния Глаубера. Результаты вычислений сравниваются с полученными недавно экспериментальными данными.

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**1.** It is known that the availability of a small quadruple moment of the deuteron gives rise to a number of polarization effects in the nuclear reactions involving the deuteron [1–5]. Recently the phenomenon of spin dichroism (production of a spin-aligned deuteron beam arising as initially unpolarized deuterons pass through matter) has been observed using an extracted unpolarized 5-GeV/$c$ deuteron beam of the Nuclotron [6]. In this paper the results of the calculation of the tensor polarization of the relativistic deuterons traveling through matter are given.

**2.** Before writing the expression for the total cross section of the deuteron-nucleus scattering, let us consider the nucleon-deuteron ($ND$) scattering. The total cross section of the $ND$ scattering for a definite spin state of the deuteron $\lambda$ may be represented in the form [7, 8]

$$\sigma_{ND}^{\mathrm{tot}(\lambda)} = 2\mathrm{Re} \int d^3 r_D \psi_D^{(\lambda)\dagger}(\mathbf{r_D}) \psi_D^{(\lambda)}(\mathbf{r}_D) \int d^2 b \gamma_{ND}(\mathbf{b}, \mathbf{s}). \tag{1}$$

Here $\mathbf{r}_D = (\mathbf{s}_D, z_D)$ is the distance between the nucleon and deuteron center of gravity; $\mathbf{s}_D$ and $z_D$ are the transversal and longitudinal parts of this distance, so that $d^3 r_D = d^2 s_D dz_D$.

The profile function $\gamma_N D(\mathbf{b}, \mathbf{s})$ can be expressed through the functions $\gamma_{Np}$ and $\gamma_{Nn}$ for the proton and the neutron, respectively:

$$\gamma_{ND}(\mathbf{b}, \mathbf{s}_D) = \left[\gamma_{Np}\left(\mathbf{b} - \frac{1}{2}\mathbf{s}_D\right) + \gamma_{Nn}\left(\mathbf{b} + \frac{1}{2}\mathbf{s}_D\right) - \right. \\ \left. - \gamma_{Np}\left(\mathbf{b} - \frac{1}{2}\mathbf{s}_D\right)\gamma_{Nn}\left(\mathbf{b} + \frac{1}{2}\mathbf{s}_D\right)\right]. \tag{2}$$

The nucleon profile functions are connected with the corresponding amplitudes of $NN$ scattering by the expressions

$$\begin{aligned}\gamma_{Np}\left(\mathbf{b}-\frac{1}{2}\mathbf{s}_D\right) &= \int d^2q f_{Np}(\mathbf{q})\exp\left[-i\left(\mathbf{b}-\frac{1}{2}\mathbf{s}_D\right)\mathbf{q}\right],\\ \gamma_{Nn}\left(\mathbf{b}+\frac{1}{2}\mathbf{s}_D\right) &= \int d^2q f_{Nn}(\mathbf{q})\exp\left[-i\left(\mathbf{b}+\frac{1}{2}\mathbf{s}_D\right)\mathbf{q}\right].\end{aligned} \tag{3}$$

If the amplitudes of the $NN$ scattering at high energies have the form

$$\begin{aligned}f_{Np}(\mathbf{q}) &= \frac{i}{4\pi}\sigma_{Np}^{\text{tot}}(1-i\alpha_{Np})\exp\left[-\frac{1}{2}Bq^2\right],\\ f_{Nn}(\mathbf{q}) &= \frac{i}{4\pi}\sigma_{Nn}^{\text{tot}}(1-i\alpha_{Nn})\exp\left[-\frac{1}{2}Bq^2\right],\end{aligned} \tag{4}$$

the profile functions may be written in the form

$$\begin{aligned}\gamma_{Np}\left(\mathbf{b}-\frac{1}{2}\mathbf{s}_D\right) &= \frac{1}{4\pi B}(1-i\alpha_{Np})\exp\left[-\frac{1}{2B}\left(\mathbf{b}-\frac{1}{2}\mathbf{s}_D\right)^2\right],\\ \gamma_{Nn}\left(\mathbf{b}+\frac{1}{2}\mathbf{s}_D\right) &= \frac{1}{4\pi B}(1-i\alpha_{Nn})\exp\left[-\frac{1}{2B}\left(\mathbf{b}+\frac{1}{2}\mathbf{s}_D\right)^2\right].\end{aligned} \tag{5}$$

The total cross section of $NN$ scattering $\sigma_{NN}$ and the ratio of the real to imaginary parts of the forward $NN$ scattering amplitude $\alpha$ averaged over the deuteron nucleons are

$$\sigma_{NN} = \frac{1}{2}(\sigma_{pp}+\sigma_{pn}), \quad \alpha = \frac{\sigma_{pp}\alpha_{pp}+\sigma_{pn}\alpha_{pn}}{\sigma_{pp}+\sigma_{pn}}. \tag{6}$$

The total cross section of the deuteron scattering on a nucleus is

$$\sigma_{DA}^{\text{tot}(\lambda)} = 2\text{Re}\int d^3r_D\psi_D^{(\lambda)\dagger}(\mathbf{r_D})\psi_D^{(\lambda)}(\mathbf{r}_D)\int d^2b \times \times\int|\Psi_A(\{r_i\})|^2\prod_{i=1}^{A}d^3r_i\,\Gamma_{AD}(\mathbf{b},\mathbf{s}_D;\{\mathbf{s}_i\}). \tag{7}$$

The nuclear profile function has the form

$$\Gamma_{AD} = 1-\prod_{i=1}^{A}\Big[1-\gamma_{ND}(\mathbf{b}-\mathbf{s}_i,\mathbf{s}_D)\Big]. \tag{8}$$

Here $\mathbf{b}$ is the impact parameter between the deuteron and the nucleus; $\mathbf{s}_i$ and $\mathbf{s}_D$ are the transversal parts of the distances of the nucleus nucleon from the center of gravity of the nucleus and the deuteron nucleon from the center of gravity of the deuteron, respectively.

The wave function of a nucleus $\Psi_A(\{\mathbf{r}_i\})$ is connected with the nuclear density $\rho(\mathbf{r}_i)$ by the relation

$$|\Psi_A(\{\mathbf{r}_i\})|^2 = \prod_{i=1}^{A}\rho(\mathbf{r}_i), \tag{9}$$

where

$$\int \rho(\mathbf{r}) d^3 r = 1, \quad \rho(\mathbf{r}) = \rho(\mathbf{s}, z). \tag{10}$$

The second integral in the expression for the total $DA$ cross section may be written in the form

$$\int |\Psi_A(\{r_i\})|^2 \prod_{i=1}^{A} d^3 r_i \, \Gamma_{AD}(\mathbf{b}, \mathbf{s}_D; \{\mathbf{s}_i\}) =$$
$$= 1 - \left[1 - \int \rho(\mathbf{s}, z) d^2 s dz \, \Gamma_{AD}(\mathbf{b}, \mathbf{s}_D; \mathbf{s})\right]^A. \tag{11}$$

The spin structure of the deuteron wave function is expressed in the following way:

$$\psi_D^{(\lambda)}(\mathbf{r}) = \left[\phi_s(\mathbf{r}) + \frac{1}{2\sqrt{2}}\big(3\, \boldsymbol{\sigma}_n \boldsymbol{\nu} \cdot \boldsymbol{\sigma}_p \boldsymbol{\nu} - 1\big)\phi_d(\mathbf{r})\right]\chi_D^{(\lambda)}, \tag{12}$$

where

$$\boldsymbol{\nu} = \frac{\mathbf{r}}{r}, \quad \int \big\{|\phi_s(\mathbf{r})|^2 + |\phi_d(\mathbf{r})|^2\big\}\, d^3 r = 1.$$

Here $\phi_s(\mathbf{r})$ and $\phi_d(\mathbf{r})$ are the wave functions of the $S$- and $D$-states of the deuteron; $\boldsymbol{\nu}$ is the orientation of the deuteron spin. The spin function of the deuteron $\chi_D^{(\lambda)}$ is connected with the nucleon spin functions by the relations

$$\begin{aligned} \chi_D^{(+1)} &= \chi_p^{(1/2)} \chi_n^{(1/2)}, \\ \chi_D^{(-1)} &= \chi_p^{-(1/2)} \chi_n^{-(1/2)}, \\ \chi_D^{(0)} &= \frac{1}{\sqrt{2}}\left[\chi_p^{(1/2)}\chi_n^{(-1/2)} + \chi_p^{(-1/2)}\chi_n^{(1/2)}\right]. \end{aligned} \tag{13}$$

**3.** If the deuteron wave functions $\phi_s$ and $\phi_d$ can be represented as the sums of the Gauss functions [10]

$$\phi_s = \sum_{i=1}^{5} A_i \exp(-\alpha_i q^2), \quad \phi_d = q^2 \sum_{i=1}^{5} B_i \exp(-\beta_i q^2), \tag{14}$$

and the nuclear density is chosen in the simple Gaussian form

$$\rho(\mathbf{r}) = \frac{1}{(\pi R_0^2)^{3/2}} \exp\left(-\frac{r^2}{R_0^2}\right), \tag{15}$$

the integrals of this problem can be taken analytically.

**4.** On the above assumptions in line with the multiple scattering theory, the difference of the total cross sections for the scattering of deuterons in different spin states $(\pm 1)$ and (0) from the nuclei may be written in the form

$$\Delta\sigma = \frac{1}{N_S + N_D} \sum_{N=1}^{A} (-1)^{N-1} \frac{A!}{(A-N)!} \Delta\sigma^{(N)}, \tag{16}$$

where the cross section difference for the $N$th collision is given by

$$\Delta\sigma^{(N)} = \pi R_1 R_2 \sum_{m=0}^{N} \sum_{n=0}^{N-m} \frac{\Delta_{m,n}^{(N)} a_1^{m+n} a_2^{N-m-n} \Omega_{m,n}^{(N)}}{[(m+n)R_2 + (N-m-n)R_1]\; n!\; m!\; (N-m-n)!}. \quad (17)$$

There the following notations were used:

$$\Delta_{m,n}^{(N)} = 3\sum_{i=1}^{5}\sum_{k=1}^{5} C_i D_k \left(\frac{\pi}{\tau_{i,k}}\right)^{3/2} \frac{\lambda_{m,n}^{(N)}}{(\lambda_{m,n}^{(N)} + \tau_{i,k})^2} + \\ + \frac{3}{2}\sum_{i=1}^{5}\sum_{k=1}^{5} D_i D_k \left(\frac{\pi}{\nu_{i,k}}\right)^{3/2} \frac{\lambda_{m,n}^{(N)}(3\lambda_{m,n}^{(N)} + 7\nu_{i,k})}{\nu_{i,k}(\lambda_{m,n}^{(N)} + \nu_{i,k})^3}$$

with

$$\lambda_{m,n}^{(N)} = \frac{1}{4}\left(\frac{N-m-n}{B} + \frac{4mnR_2^2 + (m+n)(N-m-n)R_1^2}{R_1\left[(m+n)R_2^2 + (N-m-n)R_1^2\right]}\right), \\ \Omega_{m,n}^{(N)} = \cos{(2N-m-n)\Phi}, \quad \Phi = \arctan\alpha. \quad (18)$$

The parameters $R_1,\ R_2,\ a_1$ and $a_2$ are expressed in terms of constants peculiar to this problem:

$$R_1^2 = R_0^2 + 2B, \quad R_2^2 = R_0^2 + B, \\ a_1 = \frac{\sigma_{NN}}{2\pi R_1^2}\sqrt{1+\alpha^2}, \quad a_2 = -\frac{\sigma_{NN}^2(1+\alpha^2)}{16\pi^2 B R_2^2}, \\ R_0^2 = \frac{2}{3}\left[\langle r_A^2\rangle - \langle r_p^2\rangle\right], \quad (19)$$

where $\langle r_A^2\rangle$ and $\langle r_p^2\rangle$ are the squares of the rms radii of the nucleus and the proton, respectively.

The effective numbers for the $S$- and $D$-states are

$$N_S = \sum_{i=1}^{5}\sum_{k=1}^{5} \frac{C_i C_k \pi^{3/2}}{(\rho_i + \rho_k)^{3/2}}, \quad N_D = \frac{15}{8}\sum_{i=1}^{5}\sum_{k=1}^{5} \frac{D_i D_k \pi^{3/2}}{(\omega_i + \omega_k)^{7/2}}, \quad (20)$$

where

$$C_i = A_i\,(2.5/\alpha_i)^{3/2}, \qquad D_i = \sqrt{2}B_i\,(2.5/\beta_i)^{7/2}, \\ \rho_i = 6.25/\alpha_i, \qquad \omega_i = 6.25/\beta_i, \\ \tau_{i,k} = \rho_i + \omega_k, \qquad \nu_{i,k} = \omega_i + \omega_k.$$

**5.** The following values of the parameters were used in the calculations: $\sigma_{NN} = 4.40$ fm$^2$, $\alpha_{NN} = -0.339$, $B = 0.297$ fm$^2$ [9]. The constants $a_i,\ b_i,\ \alpha_i$ and $\beta_i$ were taken from [10]; they correspond to the deuteron wave function of the Reid soft core potential. The rms radii used in the calculations are given in the second column of the table [11].

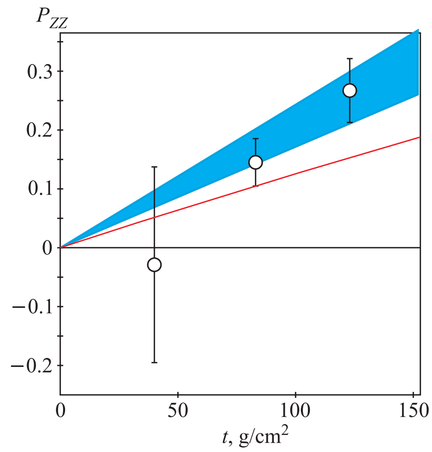
**Root-mean-square radii $R_A$, cross section differences $\Delta\sigma$, $P_{ZZ}$, and target thicknesses $x$ for different nuclei $A$ calculated under the assumption that the beam intensity behind the target falls down to 0.01 of the initial one**

| $A$ | $R_A$, fm | $\sigma_t$, mb | $\Delta\sigma$, fm$^2$ | $P_{ZZ}$ | $x$, cm |
|---|---|---|---|---|---|
| $^9$Be | 2.26 | 540 | 3.16 | 0.171 | 68.8 |
| $^{12}$C | 2.42 | 650 | 3.79 | 0.174 | 83.0 |
| $^{27}$Al | 3.06 | 1090 | 6.25 | 0.170 | 70.3 |
| $^{40}$Ca | 3.52 | 1400 | 7.78 | 0.165 | 141.2 |
| $^{64}$Cu | 3.88 | 1880 | 9.58 | 0.151 | 29.0 |
| $^{106}$Ag | 4.40 | 2630 | 11.91 | 0.135 | 30.0 |
| $^{197}$Au | 5.33 | 3850 | 15.56 | 0.121 | 20.3 |
| $^{207}$Pb | 5.42 | 3980 | 15.93 | 0.120 | 35.2 |

**6.** It can be shown that the tensor polarization of the deuteron beam arising from this cross section difference is

$$P_{ZZ} = \frac{1 - \exp(-N\Delta\sigma x)}{1 + \frac{1}{2}\exp(-N\Delta\sigma x)}, \tag{21}$$

where $N$ is the number of nuclei in cm$^3$ of matter $x$ cm thick. The calculation results are shown in the figure by solid curve together with the recent experimental data [6].




Tensor polarization of deuterons vs thickness of the carbon target [6]. The dashed region shows the error corridor, the solid curve is the calculation result

In the table the values of $P_{ZZ}$ for different nuclei were calculated under the assumption that the beam intensity behind the target falls down to 0.01 of the initial one. The total cross sections of the deuteron nucleus interaction $\sigma_t$ were calculated according to [12].

**7.** A formalism is elaborated to calculate the tensor polarization of an initially unpolarized deuteron beam arising as deuterons pass through matter. The effect is treated within the framework of the Glauber multiple scattering theory.

The results of the calculation are compared with the recent experimental data obtained with the extracted unpolarized 5-GeV/$c$ deuteron beam of the Nuclotron. The calculation results are in qualitative agreement with the experimental data.

The possibility of producing spin-aligned high-energy deuteron beams at the sacrifice of beam intensity is noted.

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