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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

CONTRIBUTIONS OF SEMI-HADRONIC STATES $P\gamma, S\gamma, \pi^+\pi^-\gamma$ TO AMM OF MUON IN THE FRAMEWORK OF THE NAMBU–JONA-LASINIO MODEL

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We calculate the contribution of semi-hadronic states with the pseudoscalar $P = \pi^0, \eta$ and scalar ($\sigma(550)$) meson accompanied with a real photon as an intermediate state of a heavy photon to the anomalous magnetic moment of the muon. We consider the intermediate states with π_0 and σ as hadrons in the framework of the Nambu–Jona-Lasinio (NJL) model. The contribution of the $\pi_0\gamma$ state is in agreement with results obtained in previous theoretical considerations as well as with the experimental data $a_\mu^{\pi_0\gamma} \approx 4.5 \cdot 10^{-10}$, besides we estimate $a_\mu^{\eta\gamma} = 0.7 \cdot 10^{-10}$, $a_\mu^{\sigma\gamma} \sim 1.5 \cdot 10^{-11}$, $a_\mu^{\pi^+\pi^-\gamma} \sim 3.2 \cdot 10^{-10}$. We also discuss the light-by-light (LbL) mechanism with $a_\mu^{\rm LbL} = 10.5 \cdot 10^{-10}$.

Вклад в аномальный магнитный момент мюона от полуадронных процессов — превращения виртуального фотона в реальный и систему адронов — рассчитан в рамках модели Намбу–Иона-Лазинио. Для случая $\gamma^* \to \pi^0\gamma; \eta\gamma; \sigma\gamma$ получены соответственно вклады в $(g-2)_\mu$: $4{,}5\cdot 10^{-10}$; $0{,}7\times 10^{-10}$; $0{,}15 \cdot 10^{-10}$. Для случая $\gamma^* \to \pi^+\pi^-\gamma$, кроме сечения в борновском приближении $e^+e^- \to \pi^+\pi^-\gamma$, рассчитанного Ю. Швингером, надо принимать во внимание квадрат формфактора пиона. Результат для вклада в $(g-2)_\mu$ $3{,}2 \cdot 10^{-10}$ оказывается в разумном согласии с экспериментальным значением $3{,}8\cdot 10^{-10}$. Мы также обсуждаем вклады в $(g-2)_\mu$ от полуадронного процесса $\gamma^* \to 3\gamma^*$ по механизму рассеяния света на свете с промежуточными состояниями адронов $\pi^0, \eta, \sigma, f_0, a_0$.

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One of modern precise tests of the Standard Model is the measurement of anomalous magnetic moment of muon (amm) a_μ. The SM contributions are usually split into three parts:

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadr}.$$

Contributions of hadrons associated with real photons (semi-hadrons ones) can be separated into two classes. One of them consists in diagrams of vertex type with heavy photon with insertion of hadronic vacuum polarization block (see Fig. 1). The other contains the light-by-light scattering block (LbL) and will be discussed below (Fig. 2). Using the dispersion

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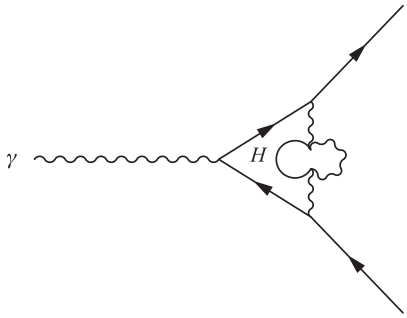

Fig. 1. Contributions from state $\gamma^* \to P; S; \pi^+\pi^-; \gamma$, where $H = \pi^0; \eta; \pi^+\pi^-$

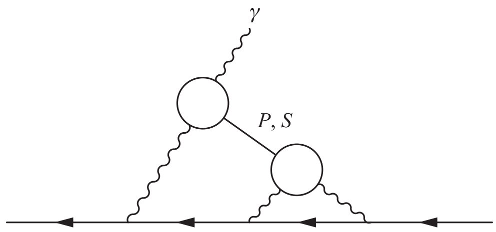

Fig. 2. Contributions of the LbL mechanism type with intermediate states $P; S$

approach, the first type of contributions can be written as

$$a_\mu^{P(S)\gamma} = \frac{1}{4\pi^3} \int\limits_{m_{P(S)}}^{\infty} ds \sigma^{e\bar{e}\to P(S)\gamma}(s) K\left(\frac{s}{M_\mu^2}\right). \quad (1)$$

The analytic form of the kernel $K(\rho)$ [1] is

$$K(\rho) = \int\limits_0^1 \frac{x^2(1-x)dx}{x^2 + (1-x)\rho};$$
$$K(\rho) = \frac{1}{2} - \rho + \frac{1}{2}\rho(\rho-2)\ln\rho - \frac{(\rho^2 - 4\rho + 2)\rho}{2\sqrt{\rho(\rho-4)}} \ln\frac{\sqrt{\rho}+\sqrt{\rho-4}}{\sqrt{\rho}-\sqrt{\rho-4}}; \quad (2)$$
$$\rho = \frac{s}{M_\mu^2}; \quad K^{(1)}(\rho)|_{\rho\gg 1} = \frac{1}{3\rho}.$$

The main part of contribution to a_μ^{hadr} of order 5004 (units 10^{-11} implied) arises from $\pi^+\pi^-$ channel annihilation of e^+e^- pair ($3\pi : 438 \cdot 10^{-11}$; $2K : 314 \cdot 10^{-11}$ [4]).

Below we will consider the annihilation channels

$$\begin{aligned} &e^+e^- \to \gamma^* \to P\gamma, S\gamma; \quad P = \pi_0, \eta; \quad S = \sigma, \\ &e^+e^- \to \gamma^* \to \pi^+\pi^-\gamma. \end{aligned} \quad (3)$$

During the recent years the papers with calculation of semi-hadron states were published [2–4]. Rather stable results were obtained for the $\pi_0\gamma$ state, whereas a contradictive result was obtained for contribution of $\sigma\gamma$ state [3]. Below we obtain these contributions in the framework of NJL model [9–11], both are consistent with modern experimental data [5].

The relevant part of chiral Lagrangian in $U(3) \times U(3)$ chiral NJL model is [6–9]

$$L = \bar{q}\Big[i\hat{\partial} + m - eQ\hat{A} + g_\pi(\lambda_3\pi_0 + \lambda_+\pi_+ + \lambda_-\pi_-)\gamma_5 + g_\sigma\sigma \cdot \lambda_3 + \\ + g_k(\lambda_+K_+ + \lambda_-K_-) + \frac{g_\rho}{2}(\lambda_3\hat{\rho}_0 + \lambda_4\hat{\omega})\Big]q, \quad (4)$$

where $\sigma = \lambda_u\sigma_u + \sigma_s\lambda_s$, $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ where u, d, s are the quark fields, $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$ is the quark charge matrix, $\lambda_4 = 1/\sqrt{3}(\sqrt{2}\lambda_0 + \lambda_8)$ where λ_i are Gell-Mann matrices and $\lambda_0 = \sqrt{2/3}\,\mathrm{diag}\,(1, 1, 1)$, $g_\rho = 5.95$ is the $\rho \to 2\pi$ coupling constant, $g_\sigma \approx 3$.

We will use the matrix element of subprocess $\gamma^*(q, \mu) \to P(p)\gamma(k, \nu)$

$$M^{\gamma^* \to P\gamma} = \frac{\alpha}{\pi f_\pi} F_P(q^2)\varepsilon_{\mu\nu\alpha\beta}\, q^\alpha\, k^\beta\, \epsilon^\mu(k)\, \epsilon^\nu(q), \quad f_\pi = 93 \text{ MeV}, \tag{5}$$

with condition $F(0) = 1$. We remind the current algebra expression for the pion decay width

$$\Gamma_{\text{exp}}^{\pi_0 \to 2\gamma} \approx 7.3 \text{ eV}.$$

The NJL result is

$$\Gamma_{\text{NJL}}^{\pi_0 \to 2\gamma} = \frac{\alpha^2 M_\pi^3}{(64\pi^3 f_\pi^2)} \approx 7.1 \text{ eV}.$$

The similar expression for $\gamma^*(q, \mu) \to S(p)\gamma(k)$ is

$$M^{\gamma^* \to S\gamma} = \frac{\alpha}{\pi f_\pi} F_S(q^2)(g_{\mu\nu} \cdot kq - q_\nu \cdot k_\mu)\epsilon^\mu(k)\epsilon^\nu(q). \tag{6}$$

Total cross sections of creation $P\gamma$, $S\gamma$ in electron–positron annihilation are

$$\sigma_{\text{theor}}^{e\bar{e} \to P\gamma} = \frac{8\pi\alpha}{3M_P^2}\Gamma_p^{\gamma\gamma}\left(1 - \frac{M_P^2}{s}\right)^3 \frac{M_\rho^4}{(s - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2}. \tag{7}$$

In the same may for scalar particles we obtain

$$\sigma_{\text{theor}}^{e\bar{e} \to S\gamma} = \frac{8\pi\alpha}{3M_S^2}\Gamma_S^{\gamma\gamma}\left(1 - \frac{M_S^2}{s}\right)^3 \frac{M_\omega^4}{(s - M_\omega^2)^2 + M_\omega^2\Gamma_\omega^2}. \tag{8}$$

The gauge invariant provides convergence of the loop momentum integral for a_μ so the application of such low-energy models as Nambu–Iona-Lasinio (NJL) one [8] for description of processes of conversion of a virtual photon to light mesons and in particular to mesons and a real photons, permits one to calculate constants of strong coupling g_π, g_ρ, g_σ.

Calculations lead to

$$(g{-}2)_\mu^{\pi_0\gamma} \approx 4.5 \cdot 10^{-10},$$
$$(g{-}2)_\mu^{\eta\gamma} \approx 0.7 \cdot 10^{-10},$$
$$(g{-}2)_\mu^{\sigma\gamma} \approx 0.15 \cdot 10^{-10}.$$

The contribution from the experimentally accessed region $0.6 < \sqrt{s} < 1.03$ GeV was obtained [5]:

$$\begin{aligned} a_\mu(\pi_0\gamma, 0.6 < \sqrt{s} < 1.03 \text{ GeV})^{\text{exp}} &= (4.5 \pm 0.15) \cdot 10^{-10}; \\ a_\mu(\eta\gamma, 0.69 < \sqrt{s} < 1.33 \text{ GeV})^{\text{exp}} &= (0.73 \pm 0.03) \cdot 10^{-10}. \end{aligned} \tag{9}$$

The contribution from the region below the experimentally accessible region is

$$a_\mu(\pi_0\gamma, \sqrt{s} < 0.6 \text{ GeV}) = (0.13 \pm 0.01) \cdot 10^{-10}. \tag{10}$$

The contribution of radiative processes with production of charged pions and two neutral ones was found [4, 5] to be

$$a_\mu(e^+e^- \to \pi^+\pi^-\gamma, \sqrt{s} < 1.2 \text{ GeV}) = (38.6 \pm 1.0) \cdot 10^{-11}. \tag{11}$$

Note that for $P\gamma$ we use only quark loops, whereas for $S\gamma$ besides quark loops the triangle loops with pions and kaons as well are relevant. Total contribution $\pi_0\gamma; \eta\gamma; 2\pi\gamma$ is

$$a_\mu^{\text{exp}}(e^+e^- \to \text{hadr.} + \gamma) = (93 \pm 1.0) \cdot 10^{-11}. \tag{12}$$

For process $e^+e^- \to \sigma\gamma$ S. Narison had obtained [3], starting from QCD sum rules, two different results, one of which is

$$a_\mu(\sigma(600)\gamma) = 0.1 \cdot 10^{-10}, \tag{13}$$

which is in agreement with our NJL approach.

As for $\gamma^* \to \rho \to \sigma\gamma$, the quark loops as well as loops with $\pi_\pm$, $K_\pm$ must be taken into account, and, besides the imaginary part of meson loops, amplitudes must be taken into account, whereas for quark loops only real part must be considered (naive confinement). Both components of σ meson $\sigma = \sigma_u \cos\alpha + \sigma_s \sin\alpha$ contribute, besides σ_s do not contain quarks and pion loops. Main contribution arises from σ_u. For the case $\sigma\gamma$ main contributions arise from light quarks and from the pion loop with constructive interference, resulting in $\Gamma_{\sigma\to 2\gamma} = 4.3$ keV [8, 9].

In NJL approach we obtain for $e^+e^- \to \sigma(550)\gamma$

$$a_\mu(\sigma(550)) = 0.16 \cdot 10^{-10}. \tag{14}$$

We use the σ-meson mass $m_\sigma = 550$ MeV as well calculated in [15] and agreement with experiment [16]. Calculating $\gamma^* \to \pi^+\pi^-\gamma$, we use Born approximation and the experimental pion form factor [19]:

$$(g-2)_\mu^{\pi^+\pi^-\gamma} = \frac{1}{4\pi^3} \int\limits_{4m_\pi^2}^{\infty} \sigma^{\pi^+\pi^-\gamma}(s) K(s)\, ds. \tag{15}$$

We use here [17, 18]

$$\begin{gathered}
\sigma^{e^+e^-\to\pi^+\pi^-\gamma}(s) = \frac{2\alpha}{\pi}\sigma_B(s)\cdot\Delta(s); \quad \sigma_B(s) = \frac{\pi\alpha^2\beta^3}{3s}|F_\pi(s)|^2; \quad \beta = \sqrt{1 - \frac{4m^2}{s}}; \\
\Delta = \frac{3}{4\beta^2}(1+\beta^2) - 2\ln\beta + 3\ln\frac{1+\beta}{2} + \\
+ \frac{1}{8\beta^3}(1-\beta)(-3-3\beta+7\beta^2-5\beta^3)L_\beta + \frac{1+\beta^2}{2\beta}F(\beta); \\
F(\beta) = -2\text{Li}(\beta) + 2\text{Li}(-\beta) - 2\text{Li}(1+\beta) + \\
+ 2\text{Li}(1-\beta) + 3\text{Li}\left(\frac{1+\beta}{2}\right) - 3\text{Li}\left(\frac{1-\beta}{2}\right) + 3\xi_2, \quad \xi_2 = \frac{\pi^2}{6}.
\end{gathered} \tag{16}$$

As a result, with $|F_\pi|^2 = 1$, we obtain $(g-2)_\mu^{\pi^+\pi^-\gamma} = 0.7 \cdot 10^{-10}$, but with real form factor [19], $(g-2)_\mu^{\pi^+\pi^-\gamma} = 3.13 \cdot 10^{-10}$, which is in agreement with contribution of nonresonance channel [5].

Analog of semi-hadronic contributions is the light-by-light (LbL) scattering mechanism with intermediate states with scalar and pseudo-scalar mesons (Fig. 2).

Convergence of different recent model calculations leads to the result [13] (see [14], A. Nyfeller talk and references therein):

$$a_\mu^{\rm LbL} = (10.5 \pm 2.6) \cdot 10^{-10}. \tag{17}$$

We put below the definite contributions (we follow the paper [13]):

$$\begin{gathered} \pi_0: \; 81.8 \cdot 10^{-11}; \quad \eta: 5.62 \cdot 10^{-11}; \quad \eta': (8 \pm 1.7) \cdot 10^{-11}; \\ \sigma(600): 11.67 \cdot 10^{-11}; \quad a_0(980): 0.62 \cdot 10^{-11}, \end{gathered} \tag{18}$$

with the total contribution

$$a_\mu^{\rm LbL} = (107.74 \pm 16.81) \cdot 10^{-11}. \tag{19}$$

In conclusion we note the results on $(g-2)^{\pi^0\gamma}$ and $(g-2)^{\rm LbL}$ obtained in paper [20]. The enhancement factor of vector-meson dominance was not taken into account for $(g-2)^{\pi^0\gamma}$. Besides, the result for $(g-2)^{\rm LbL}$ was obtained twice smaller than was mentioned above. In paper [21] contribution $(g-2)^{\rm LbL} \approx 6 \cdot 10^{-10}$ was obtained in the framework of the chiral nonlocal model.

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