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GEOMETRICAL PROPERTIES AND THE PHYSICAL MEANING OF SOME PARTICULAR RIEMANNIAN SUPERSPACES

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The geometrical meaning of a particularly simple metric in the superspace is elucidated and the possible connection with mechanisms of topological origin in high energy physics, i.e., the localization of the fields in a particular sector of supermanifold, is analyzed and discussed. The description and the analysis of some interesting aspects of the simplest Riemannian superspaces are presented from the point of view of the possible vacuum solutions.

Представлено толкование геометрического смысла сравнительно простой метрики в суперпространстве и возможная связь с механизмами топологического происхождения в физике высоких энергий, а именно проводится обсуждение и анализ локализации полей в выделенном секторе супермножества. С точки зрения возможных вакуумных решений рассматриваются описание и анализ некоторых интересных свойств простейших римановых суперпространств.

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1. INTRODUCTION AND MOTIVATION

Several attempts have been made by various groups to construct the theory of supergravity as the geometry of a superspace possessing nonzero curvature and torsion tensors without undesirable higher spin states [1, 2]. Only few years after those works, the consistent construction of the superfield supergravity was formulated in the pioneering papers independently by V. I. Ogievetsky and E. Sokatchev [3] and S. J. Gates and W. Siegel [4]. From these times in several areas of theoretical physics the description of different systems was given in the context of geometry of supermanifolds and superfields [5]: supergravity and d-branes models with warped supersymmetry [6], super-Landau systems [7], superbrane actions from nonlinearly realized supersymmetries [8], etc.

It is therefore of interest to study the geometry not only of the simplest superspaces, but also the more unusual or nonstandard ones and elucidate all the gauge degrees of freedom that they possess. This fact will clarify and expand the possibilities to construct more realistic physical models and new mathematically consistent theories of supergravity. On the other hand, the appearance of supergroups must draw attention to the study of the geometries of

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the homogeneous superspaces whose groups of motions they are. Another motivation of the study of these Riemannian superspaces is in order to establish some degree of uniqueness in the obtained supersymmetric solutions.

Motivated by the above-said, this work is devoted to study and analyze the simplest nontrivial supermetric given by Volkov and Pashnev in [9] from the point of view of the possible vacuum solutions:

$$ds^2 = \omega^\mu \omega_\mu + \mathbf{a}\omega^\alpha \omega_\alpha - \mathbf{a}^* \omega^{\dot\alpha} \omega_{\dot\alpha}.$$

This particular extended supermetric contains the complex parameters $\mathbf{a}$ and $\mathbf{a}^*$ that make it different of other more standard supermetrics. Then, our main task is to find the meaning and the role played by these complex parameters from the geometrical and physical point of view. To this end, we compare the solution of [10] that was computed in the extended four-dimensional superspace proposed in [9, 11], compactified to one dimension and restricted to the pure time-dependent case with:

i) the well-known solution described in references [12, 13] that was formulated in a superspace $(1 \mid 2)$;

ii) a multidimensional warped model described in [14], in this case with dependence on the four-dimensional coordinates.

Our goal is to show that, from the point of view of the obtained solutions, the complex parameters $\mathbf{a}$ *localize* the fields in a specific region of the bosonic part of this extended superspace, they explicitly *breakdown* the chiral symmetry when some conditions are required and all these very important properties *remain* although the supersymmetry of the model was completely broken. Also, besides all these highligts, we also show that the obtained vacuum states from the extended supermetric are very well defined in any Hilbert space.

The paper is organized as follows: in Sec. 2 we give a brief review, based on a previous work of the author [10], about the $N = 1$ extended four-dimensional superspace proposed by Volkov and Pashnev and its solution. Section 3 is devoted to analyze the relation of the supermetric under consideration with the superspace $(1 \mid 2)$ given explicitly under which conditions one is reduced to the other one from the point of view of the obtained vacuum solutions. In Sec. 4 a surprising connection between the extended supermetric and multidimensional warped gravity model solutions is shown and some hints of a possible *new mechanism* of the field localization are proposed, and finally in Sec. 5 the main results and concluding remarks are given.

2. THE EXTENDED FOUR-DIMENSIONAL $N = 1$ SUPERSPACE

The superspace $(1, 3 \mid 1)$ has four bosonic coordinates x^μ and one Majorana bispinor:

$$\left(t, x^i, \theta^\alpha, \overline{\theta}^{\dot\alpha}\right).$$

Two possible realizations for this superspace are $\rightarrow \begin{cases} osp\,(2,2) \rightarrow \text{Bosonic}-\text{Fermionic} \\ osp\,(1/2, \mathbb{R}) \rightarrow \text{Bosonic} \end{cases}$
with the following group structure for the bosonic–fermionic realization:

$$\begin{pmatrix} SU(1,1) & Q \\ Q & SU(1,1) \end{pmatrix}.$$

We will concentrate our analysis on the superspace $(1,3 \mid 1)$ with extended line element as in [9, 11]

$$ds^2 = \omega^{\mu}\omega_{\mu} + \mathbf{a}\omega^{\alpha}\omega_{\alpha} - \mathbf{a}^{*}\omega^{\dot{\alpha}}\omega_{\dot{\alpha}} \tag{1}$$

invariant to the following supersymmetric transformations:

$$x'_{\mu} = x_{\mu} + i\left(\theta^{\alpha}(\sigma)_{\alpha\dot{\beta}}\overline{\xi}^{\dot{\beta}} - \xi^{\alpha}(\sigma)_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\right); \quad \theta'^{\alpha} = \theta^{\alpha} + \xi^{\alpha}; \quad \overline{\theta'}^{\dot{\alpha}} = \overline{\theta}^{\dot{\alpha}} + \overline{\xi}^{\dot{\alpha}},$$

where the Cartan forms of the group of the supersymmetry are

$$\omega_{\mu} = dx_{\mu} - i\left(d\theta\ \sigma_{\mu}\overline{\theta} - \theta\ \sigma_{\mu}d\overline{\theta}\right); \quad \omega^{\alpha} = d\theta^{\alpha}; \quad \omega^{\dot{\alpha}} = \overline{d\theta}^{\dot{\alpha}}.$$

The spinorial indices are related as follows (the dotted indices are similarly related, as usual):

$$\theta^{\alpha} = \varepsilon^{\alpha\beta}\theta_{\beta}; \quad \theta_{\alpha} = \theta^{\beta}\varepsilon_{\beta\alpha}; \quad \varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}; \quad \varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}; \quad \varepsilon_{12} = \varepsilon^{12} = 1.$$

The complex constants $\mathbf{a}$ and $\mathbf{a}^{*}$ in the extended line element are arbitrary. This arbitrarity for the choice of $\mathbf{a}$ and $\mathbf{a}^{*}$ is constrained by the invariance and reality of the interval (1). The solution for the metric in the time-dependent case with 3 spatial dimensions compactified (i.e., $\mathbf{R}^1 \otimes S^3$, [15]) takes the form [10]

$$g_{ab}(t) = \mathrm{e}^{A(t)+\xi\varrho(t)} g_{ab}(0) \tag{2}$$

with the following superfield solution:

$$\varrho(t) = \phi_{\alpha} + \overline{\chi}_{\dot{\alpha}}$$

(i.e., chiral plus antichiral parts). The system of equations for $A(t)$ and $\varrho(t)$ that we are looking for was given in [10], and is the following:

$$|\mathbf{a}|^2 \ddot{A} + m^2 = 0, \quad \ddot{\overline{\chi}}_{\dot{\alpha}} - i\frac{\omega}{2}\left(\sigma^0\right)^{\alpha}_{\ \dot{\alpha}}\ \phi_{\alpha} = 0, \quad -\dot{\phi}_{\alpha} + i\frac{\omega}{2}\left(\sigma^0\right)_{\alpha}^{\ \dot{\beta}}\ \overline{\chi}_{\dot{\beta}} = 0. \tag{3}$$

The above system can be solved exactly with the following result:

$$A = -\left(\frac{m}{|\mathbf{a}|}\right)^2 t^2 + c_1 t + c_2; \quad c_1, c_2 \in \mathbb{C} \tag{4}$$

and

$$\phi_{\alpha} = \overset{\circ}{\phi}_{\alpha}\left(\alpha\,\mathrm{e}^{i\omega t/2} + \beta\,\mathrm{e}^{-i\omega t/2}\right) + \frac{2i}{\omega}\left(\sigma^0\right)_{\alpha}^{\ \dot{\beta}}\ \overline{Z}_{\dot{\beta}}, \tag{5}$$

$$\overline{\chi}_{\dot{\alpha}} = \left(\sigma^0\right)^{\alpha}_{\ \dot{\alpha}}\ \overset{\circ}{\phi}_{\alpha}\left(\alpha\,\mathrm{e}^{i\omega t/2} - \beta\,\mathrm{e}^{-i\omega t/2}\right) + \frac{2i}{\omega}\left(\sigma^0\right)^{\alpha}_{\ \dot{\alpha}}\ Z_{\alpha}, \tag{6}$$

where $\overset{\circ}{\phi}_{\alpha}$, Z_{α} and $\overline{Z}_{\dot{\beta}}$ are constant spinors and the frequency goes as: $\omega^2 \sim 4/|a|^2$. The superfield solution for the fields (see the «square states» of [10, 11]) that we are looking for, has the following form:

$$g_{ab}(t) = \exp\left[-\left(\frac{m}{|\mathbf{a}|}\right)^2 t^2 + c_1 t + c_2\right] \mathrm{e}^{\xi\varrho(t)} g_{ab}(0) \tag{7}$$

with

$$\varrho(t) = \overset{\circ}{\phi}_\alpha \left[\left(\alpha\, e^{i\omega t/2} + \beta\, e^{-i\omega t/2} \right) - \left(\sigma^0\right)^\alpha_{\ \dot\alpha} \left(\alpha\, e^{i\omega t/2} - \beta\, e^{-i\omega t/2} \right) \right] + \\ + \frac{2i}{\omega} \left[\left(\sigma^0\right)_\alpha^{\ \dot\beta}\ \overline{Z}_{\dot\beta} + \left(\sigma^0\right)^\alpha_{\ \dot\alpha}\ Z_\alpha \right] \quad (8)$$

and

$$g_{ab}(0) = \langle \Psi(0) | \, L_{ab} \, | \Psi(0) \rangle \quad (9)$$

that is nothing else than the «square» of the state Ψ^1 ($L_{ab} = \begin{pmatrix} a \\ a^+ \end{pmatrix}_{ab}$ with a and a^+ being the standard creation and annihilation operators). The meaning of expression (9) was given by the authors in [10] and can be resumed as:

i) it is the «square» of the state Ψ and it is the *fundamental solution* of the square root of the interval (1), precisely describing a *trajectory* in the superspace [9–11];

ii) for these states Ψ the *zero component of the current is not positively definite* given explicitly by

$$j_0(x) = 2E\Psi^\dagger\Psi$$

but for the states g_{ab}

$$j_0(x) = 2E^2 g^{ab} g_{ab}$$

then, $j_0(x)$ for the states g_{ab} is positively definite (the energy E appears squared);

iii) from ii), such states Ψ are related to *ordinary physical observables* only through their «square» g_{ab} in the sense of expressions as (9), and this fact is very important in order to explain the reason why these fractional spin states are not easy to see or to detect in the nature under ordinary conditions (for all the details we recommend the reader to see paper [10] and references therein);

iv) and fundamentally we will take under consideration here only the particular case of spin 2 because for this state the Hilbert space is dense and these states lead a *thermal* spectrum [10, 16] (g_{ab} in expression (9) has $s = 2$: each state Ψ contributes to a spin weight equal to one). Other interesting possibilities given by this type of coherent states solutions and their physical meaning, that can give some theoretical evidence of more degrees of freedom for the graviton in the sense of [29], will be analyzed in detail in a separate paper [16].

The g_{ab} at time t is given by the following expression [10, Appendix]:

$$g_{ab}(t) = \exp\left[-\left(\frac{m}{|\mathbf{a}|} \right)^2 t^2 + c_1' t + c_2' \right] e^{\xi \varrho(t)} \, |f(\xi)|^2 \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}_{ab}, \quad (10)$$

where α and α^* are the respective eigenvalues of the creation-annihilation operators a and a^+. And the dynamics for Ψ becomes now to

$$\Psi_\lambda(t) = \exp\left\{ -\frac{1}{2} \left[\left(\frac{m}{|\mathbf{a}|} \right)^2 t^2 + c_1' t + c_2' \right] \right\} \exp\left[\frac{\xi \varrho(t)}{2} \right] |f(\xi)| \begin{pmatrix} \alpha^{1/2} \\ \alpha^{*1/2} \end{pmatrix}_\lambda . \quad (11)$$

[1] This particular realization was initially introduced in [28] between the fundamental states $|\Psi\rangle$ in the initial time, where the subalgebra is the Heisenberg–Weyl algebra (with generators a, a^+ and $(n + 1/2)$).

3. RELATION WITH THE $(1 \mid 2)$ SUPERSPACE

We pass now to the description of the superspaces under consideration from the uniqueness of the possible solutions for the metric components, the supergroup structures defined by the possible group of motions and the possible physical interpretation of these results. The superspace $(1, 2)$ has one bosonic coordinate t and two Majorana spinors: $x^\mu \equiv \left(t, \theta^1, \theta^2\right)$ (we use similar notation as in [12, 13]). The big group in which this superspace is contained is $OSP\,(3, 2)$, schematically as

$$\begin{pmatrix} O\,(3) & Q \\ Q & SP\,(2) \end{pmatrix}.$$

The solution for the metric in this case is given by [12, 13]

$$\overline{g}_{ab} = g_{ab}\, \mathrm{e}^{2\sigma(t,\theta)}, \tag{12}$$

where the following superfield was introduced:

$$\sigma\,(t, \theta) = A\,(t) + \theta^\beta B_\beta + \theta^\alpha \theta_\alpha F\,(t)\,.$$

From the Einstein equations for the $(1 \mid 2)$ superspace we obtain the following set:

$$\begin{cases} \dot{B}_\alpha + b_\alpha^\beta B_\beta - \dot{A} B_\alpha = 0, \\ \ddot{A} - \dfrac{1}{2}\dot{A}^2 + \dfrac{1}{2} B^\gamma B_\gamma = \dfrac{\lambda}{4}\left(\mathrm{e}^{2A} - 1\right), \end{cases} \tag{13}$$

where $b_{\alpha\beta} = b_{\beta\alpha}$ is an arbitrary symmetric matrix. Making a suitable transformation in the first of above equations the explicit form of the B_γ field that we are looking for is

$$B^\gamma B_\gamma = \nu^\alpha \nu_\alpha\, \mathrm{e}^{2A}. \tag{14}$$

ν_α is a constant spinor and $\sqrt{b}$ was associated in [13] with the mass. Inserting (14) in the second equation of system (13) leads to the following new equation:

$$\ddot{A}' - \frac{1}{2}\dot{A}'^2 = \frac{\lambda}{4}\left(\mathrm{e}^{2A'} - 1\right), \tag{15}$$

where the transformation $A' = A - \dfrac{\nu^\alpha \nu_\alpha}{\lambda}$ was used. Notice that in [13] the derivation of the solution of Eq. (15) was not explicitly explained, but however it is easy to see that it can be reduced to the following expression:

$$\left(\dot{W}\right)^2 = \frac{\lambda}{4}\left(W^2 + \frac{1}{2W^2}\right) + C \tag{16}$$

with $W = \exp\left(-\dfrac{A'}{2}\right)$ and C is an arbitrary constant. When $C = 0$, Eq. (16) is the equation of motion for a supersymmetric oscillator in the potential of the form $k\left(X^2 + \dfrac{1}{X^2}\right)$, for which the group $O(3)$ is a dynamic symmetry group. Notice that from the point of view of a potential it is possible to redefine it in order that C disappears, but the conservation of C is crucial for the determination of the families of solutions of the problem. This type of equations of motion for an oscillator with conformal symmetry was considered earlier in the non-supersymmetric case in [17]. The solutions for the possible values of constant C are

$$\begin{gathered} C = 0 \to \mathrm{e}^{-A} = \frac{\sqrt{2}}{2}\sinh\left(\sqrt{\lambda}t + \varphi_0\right), \quad \varphi_0 = \sqrt{\lambda}t_0, \\ \frac{8C^2}{\lambda^2} < 1 \to \mathrm{e}^{-A} = \frac{\sqrt{2}}{2}\left[\sinh\left(\sqrt{\lambda}t + \varphi_0\right)\sqrt{1-\varkappa^2} - \varkappa\right], \quad \varkappa = \frac{2\sqrt{2}C}{\lambda}, \\ \frac{8C^2}{\lambda^2} = 1 \to \mathrm{e}^{-A} = \frac{\sqrt{2}}{2}\left[\frac{\mathrm{e}^{\left(\sqrt{\lambda}t+\varphi_0\right)}}{\sqrt{2}} - \varkappa\right], \\ \sigma(t,\theta) = A(t) + \theta^\alpha B_\alpha. \end{gathered} \tag{17}$$

Notice that λ takes the place of the cosmological constant and is related to b by $b = -\lambda/2$. The superfield solution (17) $N = 2$ (chiral or antichiral two-components spinors) has conformal symmetry in the case $C = 0$ and is not unique: as was pointed out in [12, 13, 18] there exists a larger class of vacuum solutions. The dynamics of the solution is very simple as is easy to see from Eqs. (17), that is not the case in the superspace $(1, 3 \mid 1)$ as we showed in the previous Section.

With the description of both superspaces above, we pass now to compare them in order to establish if a one-to-one mapping exists between these superspaces. By simple inspection we can see that the fermionic part of the superspace solutions (2) and (17) is mapped one to one, explicitly (for the $(1 \mid 2)$ superspace indexes 1 and 2 for α and β are understood):

$$\nu_\alpha = -2\beta\overset{\circ}{\phi}_\alpha, \quad 2\sqrt{b} = \omega, \quad \theta^1 \leftrightarrow \overline{\theta}^{\dot{\alpha}}, \quad \theta^2 \leftrightarrow \theta^\alpha, \quad \left(\sigma^0\right)^\alpha_{\ \dot{\alpha}} \leftrightarrow b_1^2$$

if the following conditions over the four-dimensional solution hold

$$\alpha = \beta, \quad Z_\alpha = \overline{Z}_{\dot{\beta}} = 0.$$

For the bosonic part of the superfield solutions (17) and (4) no direct relation exists between them. Only taking the limit of the constants $|\mathbf{a}| \to \infty$ of the extended superspace $(1, 3 \mid 1)$ (i.e., going to the standard $(1, 3 \mid 1)$ superspace) the Gaussian solution (7) goes to the same type as described in (17) for the $(1 \mid 2)$ superspace, with $c_1 \approx \sqrt{\lambda}$ and $c_2 \approx \varphi_0$. And this fact is nontrivial because the chirality is explicitly restored in this limit as we can easily see from Eqs. (3) when $|\mathbf{a}| \to \infty$, $\omega^2 \to 0$. It is clear that the solution coming from four-dimensional extended superspace is a physical one because it represents a semiclassical (Gaussian) state of the Husimi's type [10, 19]. The important role played by constants $\mathbf{a}$ and $\mathbf{a}^*$ in the extended line element (1) is to localize the physical state in a precise region of the space–time, as is easily seen from expression (7). This fact can give some hints in order to explain and to treat from the mathematical point of view the mechanism of confinement, spontaneous compactification and other problems in high energy particle physics that can have a topological origin [16].

4. RELATION WITH «WARPED» GRAVITY MODELS

It is well known that large extra dimensions offer an opportunity for a new solution to the hierarchy problem [20]. Field theoretical localization mechanisms for scalar and fermions [21] as well as for gauge bosons [22] were found. The crucial ingredient of this scenario is a brane on which standard model particles are localized. In string theory, fields can naturally be localized on D-branes due to the open strings ending on them [23]. Up until recently, extra dimensions had to be compactified, since the localization mechanism for gravity was not known. It was suggested in [24] that gravitational interactions between particles on a brane in uncompactified five-dimensional space could have the correct four-dimensional Newtonian behaviour, provided that the bulk cosmological constant and the brane tension are related. Recently, it was found by Randall and Sundrum that gravitons can be localized on a brane which separates two patches of AdS_5 space–time [25]. The necessary requirement for the four-dimensional brane Universe to be static is that the tension of the brane is fine-tuned to the bulk cosmological constant [24, 25]. On the other hand, recent papers present an interesting model in which the extra dimensions are used only as a mathematical tool taking advantage of the AdS/CFT correspondence that claims that the 5D warped dimension is related to a strongly coupled 4D theory [26].

A remarkable property of the solution given by expression (7) is that the physical state $g_{ab}(x)$ is localized in a particular position of the space–time. The supermetric coefficients $\mathbf{a}$ and $\mathbf{a}^*$ play the important role of localization of the fields in the bosonic part of the superspace in similar and suggestive form as the well-known «warp factors» in multidimensional gravity [14] for a positive (or negative) tension brane. But the essential difference is that, because of the $\mathbb{C}$-constants $\mathbf{a}$ and $\mathbf{a}^*$ coming from the $B_{L,0}$(even) fermionic part of the superspace under consideration, not additional and/or topological structures that break the symmetries of the model (i.e., reflection Z_2-symmetry) are required: the natural structure of the superspace produces this effect.

Also it is interesting to remark here that the Gaussian-type solution (7) is a very well defined physical state in a Hilbert space [10, 19] from the mathematical point of view, contrary to the case $u(y) = c\,e^{-H|y|}$ given in [14] that, although it was possible to find a manner to include it in any Hilbert space, it is strongly needed to take special mathematical and physical particular assumptions whose meaning is obscure. The comparison with the case of five-dimensional gravity plus cosmological constant [14] is given in the table[1].

Notice the following important observations:

i) that for the solution in the five-dimensional gravity plus Λ case, the explicit presence of the cosmological term is necessary for the consistency of the model: the «fine-tuning» $H \equiv \sqrt{-\dfrac{2\Lambda}{3}} = \dfrac{|T|}{M^3}$, where T is the tension of the brane and M^3 is the constant of the Einstein–Hilbert $+\Lambda$ action;

ii) about the localization of the fields given by the particular superspace treated here, the Z_2 symmetry is noncompatible with the solution that clearly is not chiral or antichiral. This fact is consistent with the analysis given for a similar superspace as considered in [10, 27],

[1]The extended superspace solution in the case contains all the four-dimensional coordinates: $x \equiv (t, \overline{x})$, $c'_1 x \equiv c'_{1\mu} x^{\mu}$ and c'_2 scalar.

Spacetime	$(5-D)$ gravity $+\Lambda$	Superspace $(1,3 \mid 1)$
Interval	$ds^2 = A\,(y)\,dx^2_{3+1} - dy$	$ds^2 = \omega^{\mu}\omega_{\mu} + \mathbf{a}\omega^{\alpha}\omega_{\alpha} - \mathbf{a}^{*}\omega^{\dot{\alpha}}\omega_{\dot{\alpha}}$
Equation	$\Big[-\partial^2_y - m^2\,\mathrm{e}^{H\lvert y\rvert} +$ $+H^2 - 2H\delta(y)\Big]u(y) = 0$	$\Big[\lvert a\rvert^2\left(\partial^2_0 - \partial^2_i\right) + \frac{1}{4}\left(\partial_\eta - \partial_\xi + i\,\partial_\mu\,(\sigma^\mu)\,\xi\right)^2 -$ $-\frac{1}{4}\left(\partial_\eta + \partial_\xi + i\,\partial_\mu\,(\sigma^\mu)\,\xi\right)^2 + m^2\Big]^{ab}_{cd} g_{ab} = 0$
Solution	$u(y) = c\,\mathrm{e}^{-H\lvert y\rvert}$, $H \equiv \sqrt{-\frac{2\Lambda}{3}} = \frac{\lvert T\rvert}{M^3}$	$g_{ab}(x) = \exp\left[-\left(\frac{m}{\lvert\mathbf{a}\rvert}\right)^2 x^2 + c'_1 x + c'_2\right] \times$ $\times \mathrm{e}^{\xi\varrho(x)}\,\lvert f(\xi)\rvert^2 \begin{pmatrix} \alpha \\ \alpha^{*} \end{pmatrix}_{ab}$

where the consistent solutions are superprojected in a sector of the physical states that are not chiral or antichiral;

iii) the field equation in final form for the five-dimensional gravity depends on the extra dimension, that in the model proposed here it depends on all superspace coordinates but the field solution is attached in the $3+1$ space–time.

From the points discussed above and the «state of the art» of the problem, we see the importance of proposing new mechanisms and alternative models that can help us to understand and to handle the problem. Then, it is not difficult to think to promove the extended supermetric under study to build a strongly coupled 4D model, using this particular $N=1$ toy superspace. We will treat this issue in a further work [16].

5. CONCLUDING REMARKS

In the present paper from the point of view of the symmetries and the obtained vacuum solutions we have analyzed the superspace $N=1$ extended metric proposed by Volkov and Pashnev in [9]. This particular model, in spite of its high simplicity, presents a much richer structure than the other degenerate standard superspaces because it contains the complex parameters $\mathbf{a}$ and $\mathbf{a}^*$ that make it different. The role played by the complex parameters $\mathbf{a}$ and $\mathbf{a}^*$ can be resumed as follows:

i) the $\mathbb{C}$-parameters $\mathbf{a}$ and $\mathbf{a}^*$ fix the field in a specific sector of the even part ($B_{L,0}$) of the supermanifold;

ii) these parameters, that are responsible for the nontrivial part of the model, break the chiral symmetry of the field solution. The chiral symmetry is restored when the metric in question degenerates in the limit $\lvert\mathbf{a}\rvert \to \infty$ (with all other parameters of the model fixed);

iii) the fields remain attached in a specific region of the space–time when the supersymmetry of the model is completely broken even if all fermions are switched off.

iv) we have analyzed and compared from the point of view of the obtained solutions the superspace $(1\,|\,2)$ with the particular superspace $(1,3\,|\,1)$ proposed by Volkov and Pashnev [9, 11], compactified to one dimension and restricted to the pure time-dependent case. The possibility that the nondegenerate superspace $(1,3\,|\,1)$ with extended line element is reduced to the superspace $(1\,|\,2)$ is subject to the condition $\lvert\mathbf{a}\rvert \to \infty$. The fermionic part of both superspaces is mapped one to one by means of a suitable definition of the fermionic variables and coefficients.

In comparison with the five-dimensional gravity plus cosmological constant of [14], the simple supersymmetric model under analysis here has the following advantages:

v) the mechanism of localization of the fields in the bosonic four-dimensional part of the supermanifold does not depend on the cosmological constant;

vi) the fields attached are Gaussian-type solutions (7), very well defined physical state in a Hilbert space from the mathematical point of view, contrary to the case $u(y) = c\,\mathrm{e}^{-H|y|}$ given in [16];

vii) not additional and/or topological structures that break the symmetries of the model (i.e., reflection Z_2-symmetry) are required to attach the fields: the natural structure of the superspace produces this effect through the $\mathbb{C}$-parameters $\mathbf{a}$ and $\mathbf{a}^*$.

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APPENDIX

The dynamics of the $|\Psi\rangle$ fields, in the representation that we are interested in, can be simplified considering these fields as coherent states in the sense that they are eigenstates of a^2 [19]

$$\begin{aligned}\left|\Psi_{1/4}\left(0,\xi,q\right)\right\rangle &= \sum_{k=0}^{+\infty} f_{2k}\left(0,\xi\right)\left|2k\right\rangle = \sum_{k=0}^{+\infty} f_{2k}\left(0,\xi\right)\frac{\left(a^{\dagger}\right)^{2k}}{\sqrt{(2k)!}}\left|0\right\rangle, \\ \left|\Psi_{3/4}\left(0,\xi,q\right)\right\rangle &= \sum_{k=0}^{+\infty} f_{2k+1}\left(0,\xi\right)\left|2k+1\right\rangle = \sum_{k=0}^{+\infty} f_{2k+1}\left(0,\xi\right)\frac{\left(a^{\dagger}\right)^{2k+1}}{\sqrt{(2k+1)!}}\left|0\right\rangle.\end{aligned} \tag{A1}$$

From a technical point of view these states are one-mode squeezed states constructed by the action of the generators of the $SU(1,1)$ group over the vacuum. For simplicity, we will take all normalization and fermionic dependence or possible CS fermionic realization into the functions $f(\xi)$. Explicitly at $t = 0$

$$\begin{aligned}\left|\Psi_{1/4}\left(0,\xi,q\right)\right\rangle &= f\left(\xi\right)\left|\alpha_{+}\right\rangle, \\ \left|\Psi_{3/4}\left(0,\xi,q\right)\right\rangle &= f\left(\xi\right)\left|\alpha_{-}\right\rangle,\end{aligned} \tag{A2}$$

where $|\alpha_{\pm}\rangle$ are the CS basic states in the subspaces $\lambda = 1/4$ and $\lambda = 3/4$ of the full Hilbert space. In the case of the physical state that we are interested in, we used the HW realization for the states Ψ

$$|\Psi\rangle = \frac{f\left(\xi\right)}{2}\left(\left|\alpha_{+}\right\rangle + \left|\alpha_{-}\right\rangle\right) = f\left(\xi\right)\left|\alpha\right\rangle, \tag{A3}$$

where, however, the linear combination of the states $|\alpha_+\rangle$ and $|\alpha_-\rangle$ spans now the full Hilbert space (dense) corresponding λ to the CS basis. The «square» states at $t = 0$ are

$$g_{ab}(0) = \langle\Psi(0)|\, L_{ab}\, |\Psi(0)\rangle = \langle\Psi(0)| \begin{pmatrix} a \\ a^+ \end{pmatrix}_{ab} |\Psi(0)\rangle = f^*(\xi)\, f(\xi) \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}_{ab}. \quad \text{(A4)}$$

The algebra (topological information of the group manifold) is «mapped» over the spinors solutions through the eigenvalues α and α^*. Notice that the constants c_1' and c_2' in the exponential functions in expressions (10) and (11) can be easily determined as functions of frequency ω as in [19] for the Schrödinger equation.

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