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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

QUANTUM INFORMATION TRANSMISSION BETWEEN TWO QUBITS THROUGH AN INTERMEDIARY PHOTON GAS

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The theory of the quantum information transmission between two semiconductor two-level quantum dots as two qubits through an intermediary photon gas in a cavity is presented. The reduced density matrix of each two-level quantum dot is the quantum information encoded into this qubit. The quantum information exchange between two distant qubits imbedded in the photon gas is performed in the form of the mutual dependence of their reduced density matrices due to the interaction between the electrons in the qubits and the photon gas. The system of rate equations for the reduced density matrix of the two-qubit system is derived. From the solution of this system of equations follows the mutual dependence of the reduced density matrices of two distant qubits.

Представлена теория передачи квантовой информации между двумя полупроводниковыми двухуровневыми квантовыми точками как двумя кубитами через промежуточный фотонный газ в полости. Приведенная матрица плотности каждой двухуровневой квантовой точки является квантовой информацией, закодированной в этом кубите. Обмен квантовой информацией между двумя отстоящими кубитами, погруженными в фотонный газ, выполняется в форме взаимной зависимости их приведенных матриц плотности благодаря взаимодействию между электронами в кубитах и фотонным газом. Получена система уравнений для скорости приведенной матрицы плотности двухкубитной системы. Из решения этой системы уравнений следует зависимость приведенных матриц плотности двух отстоящих кубитов.

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INTRODUCTION

The quantum state transfer between two interacting two-level quantum systems was widely investigated as the physical mechanism of the quantum information transfer between two neighbour qubits due to the interaction between electrons in different two-level systems. Beside of this type of the local quantum information transfers, there exists also another type of the long-distance quantum state transfers between distant two-level quantum systems due

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to the photon exchange between them or through some intermediary media [1–8]. They may be considered as the physical mechanisms of the transmission of the quantum information between the distant qubits — the quantum communication. In this report we present the theory of the quantum state transmission between two two-level semiconductor quantum dots as two qubits resonantly interacting with the monoenergetic photons of an intermediary photon gas in a cavity — the photonic quantum bus.

Consider a complex system consisting of a monoenergetic photon gas in a cavity and two identical two-level semiconductor quantum dots (QDs) placed at a distance inside this cavity, the bulk material of the QDs being a direct band gap semiconductor with the allowed dipole radiative transition between the conduction and valence bands, the upper discrete energy level being in the conduction band while the lower one being in the valence band. We chose to work with the electrons having definite spin projections as well as with the photons in the corresponding polarization state such that the dipole radiative transitions between two levels are allowed, and omit all electron spin projection and photon polarization indices for simplicity. Then the system of interacting electrons and photons has the following total Hamiltonian:

$$H = \sum_{i=1,2} E_i a_i^+ a_i + \sum_{\mu=1,2} E_\mu b_\mu^+ b_\mu + \omega\gamma^+\gamma + g[\gamma^+(a_2^+ a_1 + b_2^+ b_1) + (a_1^+ a_2 + b_1^+ b_2)\gamma], \quad (1)$$

where a_i and $a_i^+, i = 1, 2$, are the destruction and creation operators for the electrons at two energy levels E_i ($i = 1$ for the upper level and $i = 2$ for the lower one, $E_1 > E_2$) in the QD «a», b_μ and b_μ^+, $\mu = 1, 2$, are similar operators for the QD «b» at two energy levels E_μ, γ and γ^+ are the photon destruction and creation operators, ω is the photon energy, and g is the effective constant of the electron–photon interaction.

The reduced density matrix of each QD is the quantum information encoded into this qubit, and the mutual dependence of the reduced density matrices of two QDs due to their interaction with the photon gas in the cavity can be considered as some form of the quantum information exchange between these two qubits — the quantum communication. On the basis of the equations of motion for the quantum operators derived from the total Hamiltonian (1) the mutual dependence of the reduced density matrices of two qubits will be established.

In Sec. 1 the system of rate equations for the (reduced) density matrix of the system of two electrons in two QDs is derived in the mean-field approximation. On the basis of the solution to this system of linear first-order differential equations in some special simple case, the mutual dependence between the reduced density matrices of two qubits is discussed in Sec. 2. It is a form of the quantum communication between two qubits.

1. RATE EQUATIONS

Consider the Hilbert subspace of the states with one electron in each QD. Then the matrix elements of the reduced density matrix of the two-electron four-level system — the trace of the total density matrix with respect to the states of the photon gas can be represented in the form

$$\rho_{(i\mu)(j\nu)} = \langle a_j^+ a_i b_\nu^+ b_\mu \rangle, \quad (2)$$

where $\langle \ldots \rangle$ denotes the equilibrium statistical average at a given temperature. By using the equation of motion for the quantum operators a_i, a_i^+, b_μ, b_μ^+ and γ, γ^+ with the Hamiltonian (1) we derive the following differential equation:

$$
\begin{aligned}
i\frac{d\rho_{(i\mu)(j\nu)}}{dt} &= (E_i + E_\mu - E_j - E_\nu)\rho_{(i\mu)(j\nu)} + \\
&+ \sum_k \{g_{ik}[X_{(k\mu)(j\nu)} + Y_{(k\mu)(j\nu)}] - [X_{(i\mu)(k\nu)} + Y_{(i\mu)(k\nu)}]g_{kj}\} + \\
&+ \sum_\sigma \{g_{\mu\sigma}[X_{(i\sigma)(j\nu)} + Y_{(i\sigma)(j\nu)}] - [X_{(i\mu)(j\sigma)} + Y_{(i\mu)(j\sigma)}]g_{\sigma\nu}\}, \quad (3)
\end{aligned}
$$

where $g_{11} = g_{22} = 0$, $g_{12} = g_{21} = g$ and

$$
\begin{aligned}
X_{(i\mu)(j\nu)} &= \langle \gamma a_j^+ a_i b_\nu^+ b_\mu \rangle, \\
Y_{(i\mu)(j\nu)} &= \langle \gamma^+ a_j^+ a_i b_\nu^+ b_\mu \rangle.
\end{aligned} \quad (4)
$$

The differential equations for $X_{(i\mu)(j\nu)}$ and $Y_{(i\mu)(j\nu)}$ contain the statistical averages of products of six operators $\gamma\gamma a_j^+ a_i b_\nu^+ b_\mu$, $\gamma\gamma^+ a_j^+ a_i b_\nu^+ b_\mu$, $\gamma^+\gamma a_j^+ a_i b_\nu^+ b_\mu$, $\gamma^+\gamma^+ a_j^+ a_i b_\nu^+ b_\mu$. We apply the mean-field approximation to the statistical averages of these products of six operators and have

$$
\begin{aligned}
\langle \gamma\gamma a_j^+ a_i b_\nu^+ b_\mu \rangle &\approx \langle \gamma\gamma \rangle \rho_{(i\mu)(j\nu)}, \\
\langle \gamma\gamma^+ a_j^+ a_i b_\nu^+ b_\mu \rangle &\approx \langle \gamma\gamma^+ \rangle \rho_{(i\mu)(j\nu)}, \\
\langle \gamma^+\gamma a_j^+ a_i b_\nu^+ b_\mu \rangle &\approx \langle \gamma^+\gamma \rangle \rho_{(i\mu)(j\nu)}, \\
\langle \gamma^+\gamma^+ a_j^+ a_i b_\nu^+ b_\mu \rangle &\approx \langle \gamma^+\gamma^+ \rangle \rho_{(i\mu)(j\nu)}.
\end{aligned}
$$

In order to study the density matrix (2) in the second-order approximation with respect to the coupling constant g we need to have the differential equations for the quantities (4) only in the first-order approximation and therefore we can use for the averages $\langle\gamma\gamma\rangle$, $\langle\gamma\gamma^+\rangle$, $\langle\gamma^+\gamma\rangle$ and $\langle\gamma^+\gamma^+\rangle$ their values in the case of a free photon gas. Denote by n the photon density in the photon gas at the given temperature. We obtain the following differential equations:

$$
\begin{aligned}
i\frac{dX_{(i\mu)(j\nu)}}{dt} &= (E_i + E_\mu - E_j - E_\nu + \omega)X_{(i\mu)(j\nu)} - \\
&- n\left[\sum_k \rho_{(i\mu)(k\nu)} g_{kj} + \sum_\sigma \rho_{(i\mu)(j\sigma)} g_{\sigma\nu}\right] + \\
&+ (1+n)\left[\sum_l g_{il}\rho_{(l\mu)(j\nu)} + \sum_\tau g_{\mu\tau}\rho_{(i\tau)(j\nu)}\right], \quad (5)
\end{aligned}
$$

$$
\begin{aligned}
i\frac{dY_{(i\mu)(j\nu)}}{dt} &= (E_i + E_\mu - E_j - E_\nu - \omega)Y_{(i\mu)(j\nu)} - \\
&- (1+n)\left[\sum_k \rho_{(i\mu)(k\nu)} g_{kj} + \sum_\sigma \rho_{(i\mu)(j\sigma)} g_{\sigma\nu}\right] + \\
&+ n\left[\sum_l g_{il}\rho_{(l\mu)(j\nu)} + \sum_\tau g_{\mu\tau}\rho_{(i\tau)(j\nu)}\right]. \quad (6)
\end{aligned}
$$

Thus, we have derived the rate equations (3), (5) and (6) for the quantum communication between two qubits through a photon gas in a cavity.

2. QUANTUM INFORMATION EXCHANGE BETWEEN TWO QUBITS

The quantum states of the qubits «a» and «b» are described by their reduced density matrices $\rho^{(a)}$ and $\rho^{(b)}$, respectively, with the matrix elements

$$\rho^{(a)}_{(ij)}(t) = \sum_{\mu} \rho_{(i\mu)(j\mu)}(t), \qquad \rho^{(b)}_{(\mu\nu)}(t) = \sum_{i} \rho_{(i\mu)(i\nu)}(t). \tag{7}$$

Each density matrix, $\rho^{(a)}$ or $\rho^{(b)}$, is the quantum information encoded into the corresponding qubit. Suppose that at the initial time moment $t = 0$ the quantum states of two qubits are independent and described by their density matrices $\rho^{(a)}(0)$ and $\rho^{(b)}(0)$. The density matrix of the two-qubit system at this initial time moment $t = 0$ is the tensor product of $\rho^{(a)}(0)$ and $\rho^{(b)}(0)$:

$$\rho_{(i\mu)(j\nu)}(0) = \rho^{(a)}_{ij}(0)\rho^{(b)}_{\mu\nu}(0). \tag{8}$$

Due to the interaction with the intermediary photon gas in the cavity there arises the following mutual dependence between two reduced density matrices: $\rho^{(b)}_{\mu\nu}(t)$ at any $t > 0$ depend not only on their initial values, but also on $\rho^{(a)}_{ij}(0)$ and vice versa. This means that the «output» quantum information encoded in the qubit «b» at any $t > 0$ depends on the «input» quantum information encoded in the qubit «a» at the initial time moment $t = 0$.

Consider a simple example with the «input» quantum information

$$\rho^{(a)}(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{9a}$$

and the initial condition for the qubit «b»

$$\rho^{(a)}(0) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{9b}$$

in the system with the resonant radiative transition $\omega = E$ and the vanishing photon density in the cavity. Then in the nonvanishing lowest-order approximation with respect to the small coupling constant g the reduced density matrices have the following matrix elements:

$$\begin{aligned} \rho^{(a)}_{11}(t) &= 1 - \frac{1}{2}(1 - \cos\sqrt{2}gt) + \frac{1}{4}(1 - \cos 2gt), \\ \rho^{(a)}_{22}(t) &= \frac{1}{2}(1 - \cos\sqrt{2}gt) - \frac{1}{4}(1 - \cos 2gt), \\ \rho^{(a)}_{12}(t) &= \rho^{(a)}_{21}(t) = 0 \end{aligned} \tag{10a}$$

for the qubit «a» and

$$\begin{aligned} \rho^{(b)}_{11}(t) &= \frac{1}{2}(1 - \cos\sqrt{2}gt) + \frac{1}{4}(1 - \cos 2gt), \\ \rho^{(b)}_{22}(t) &= 1 - \frac{1}{2}(1 - \cos\sqrt{2}gt) - \frac{1}{4}(1 - \cos 2gt), \\ \rho^{(b)}_{12}(t) &= \rho^{(b)}_{21}(t) = 0 \end{aligned} \tag{10b}$$

for the qubit «b». Note that the oscillation frequencies ($2g$ and $\sqrt{2}g$) are proportional to the coupling constant g, but the amplitudes of the harmonic oscillations in the r.h.s. of the formulae (10a) and (10b) do not depend on g. It is the consequence of the resonance in the radiation emission and absorption processes. This phenomenon is similar to the polariton effect due to the photon–exciton transition in semiconductors.

For the comparison consider the opposite limiting case with the nonresonant interaction $\omega \approx 0$. With the same «input» quantum information (9a) and initial condition (9b) at a later time moment $t > 0$ we have

$$\begin{aligned}
\rho_{11}^{(b)}(t) &= \rho_{22}^{(a)}(t) = 10\frac{g^2}{E^2}(1-\cos Et) - 2\frac{g^2}{E^2}(1-\cos 2Et),\\
\rho_{11}^{(a)}(t) &= \rho_{22}^{(b)}(t) = 1 - 10\frac{g^2}{E^2}(1-\cos Et) + 2\frac{g^2}{E^2}(1-\cos 2Et),\\
\rho_{12}^{(a)}(t) &= \rho_{21}^{(a)}(t) = \rho_{12}^{(b)}(t) = \rho_{21}^{(b)}(t) = 0.
\end{aligned} \tag{11}$$

The amplitudes of the harmonic oscillations in the r.h.s. of formulae (11) are proportional to the small ratio g^2/E^2.

The investigation of the dependence of the matrix elements of the reduced density matrices $\rho^{(a)}(t)$ and $\rho^{(b)}(t)$ on the «input» quantum information $\rho^{(a)}(0)$ and the initial condition $\rho^{(b)}(0)$ as well as on the physical parameters E, ω, g^2 and n by means of the numerical calculations is in progress.

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REFERENCES

1. *Cirac J. I. et al.* Quantum State Transfer and Entanglement Distribution among Distant Nodes in a Quantum Network // Phys. Rev. Lett. 1997. V. 78. P. 3221.
2. *Imamoglu A. et al.* Quantum Information Processing Using Quantum Dot Spins and Cavity QED // Phys. Rev. Lett. 1999. V. 83. P. 4204.
3. *Lu-Ming Duan et al.* Quantum Communication between Atomic Ensembles Using Coherent Light // Phys. Rev. Lett. 2000. V. 85. P. 5643.
4. *Browne D. E. et al.* Robust Creation of Entanglement between Ions in Spatially Separate Cavities // Phys. Rev. Lett. 2003. V. 91. P. 067901.
5. *Childress L. et al.* Fault-Tolerant Quantum Communication Based on Solid-State Photon Emitters // Phys. Rev. Lett. 2006. V. 96. P. 070504.
6. *van Loock P. et al.* Hybrid Quantum Repeater Using Bright Coherent Light // Ibid. P. 240501.
7. *Press D. et al.* Photon Antibunching from a Single Quantum-Dot-Microcavity System in the Strong Coupling Regime // Phys. Rev. Lett. 2007. V. 98. P. 117402.
8. *Clark S. M. et al.* Quantum Computers Based on Electron Spins Controlled by Ultrafast Off-Resonant Single Optical Pulses // Ibid. V. 99. P. 040501.