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SELF-ORGANIZING PHYSICAL FIELDS AND GRAVITY

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Самоорганизующиеся физические поля и гравитация

Показано, что теория самоорганизующихся физических полей обеспечивает адекватное и самосогласованное рассмотрение гравитационных явлений. Общий вывод состоит в том, что сущностью гравидинамики является новая полевая концепция времени и общековариантный закон сохранения энергии, из которого, в частности, следует, что темная энергия является не чем иным, как энергией гравитационного поля. Из естественных геометрических законов гравидинамики выведены динамические уравнения гравитационного поля. Найдены два точных решения этих уравнений. Одно из них представляет ударную гравитационную волну, а другое описывает вселенную, заполненную только гравитационной энергией. Эти решения сравниваются с решениями Шварцшильда и Фридмана в общей теории относительности Эйнштейна.

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Self-Organizing Physical Fields and Gravity

It is shown that the Theory of Self-Organizing Physical Fields provides the adequate and consistent consideration of the gravitational phenomena. The general conclusion lies in the fact that the essence of gravidynamics is the new field concept of time and the general covariant law of energy conservation which in particular means that dark energy is simply the energy of the gravitational field. From the natural geometrical laws of gravidynamics the dynamical equations of the gravitational field are derived. Two exact solutions of these equations are obtained. One of them represents a shock gravitational wave and the other represents the Universe filled up with the gravitational energy only. These solutions are compared with the Schwarzschild and Friedmann solutions in the Einstein general theory of relativity.

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1. INTRODUCTION

It is evident that coordinates have no physical meaning in both General Theory of Relativity and Quantum Mechanics (notion of trajectory disappears in quantum theory of particles, uncertainty principle [1], principle of general invariance [2]). On this background and keeping in mind that reality is greatly accord and similar in itself, we can put forward the principle of self-organization: the origin and nature of the fundamental physical concepts and laws consist in their absolute independence of any outer and a priori conditions. The Theory of Self-Organization represents a careful application of the known mathematical laws for definition and description of the Self-Organizing Physical Fields, and bypassing an observer and artefacts it permits one to recognize space, time, internal symmetry, spin, charge as attributes of the self-organizing field system. The Theory of Self-Organization is absolutely independent of any outer and a priori conditions and provides absolute knowledge about reality. Measurements can be only incomplete and relative but their statistical properties should be reproduced with time.

It is no doubt that the concept of physical space is the basic and especially important attribute of the Self-Organizing Fields since there are very strong evidences that the essential properties of physical space are tightly connected with the most fundamental laws governing the behavior of physical systems and, hence, these properties predetermine the objective physical regularities. Euclid put us on the truth track by taking space as the primary concept of science. In accordance with the principle of self-organization all details of this intimate relationship can be expressed in the form of exact mathematical laws and the Theory of Self-Organizing Fields emerges as an integral and logically coherent system. The observer himself can use the Theory of Self-Organization as a unique instrument to learn more objective information about the physical phenomena themselves and find new creative approaches and innovative possibilities for his own goals (the calculations and any activity can be fruitful only on the reliable basis of understanding that the Theory of Self-Organization provides in full measure).

The development of the Theory of Self-Organization was originated by the creation of the natural, argued, and novel concept of time [3]. Time by itself exists in the form of the scalar temporal field and is the cornerstone of any dynamical theory and, hence, the dynamical theory of gravity as well. The coming-into-being of the Theory of Self-Organization was completed with formulation of the theory of spin as manifestation of the geometrical structure of physical space in the form of spin symmetry with its bipolar structure [4, 5]. From the new concept of time it follows directly that there are phenomena outside the time (in this case a temporal field is simply absent). In the Theory of Self-Organization this takes the form of two divisions: the static (timeless) Theory of Self-Organization and the dynamical Theory of Self-Organization. The timeless Theory of Self-Organization of the physical fields represents entirely a new division of the field theory and involves the following chapters: spinstatics and gefstatics (the theory of the general electromagnetic field outside the time). Gravistatics has not an independent status since its equations with the absence of the other fields have only a trivial solution. In the dynamical Theory of Self-Organization we correspondingly distinguish gravidynamics (a subject of the present consideration), spindynamics, and the dynamical theory of general electromagnetic field (gefdynamics). The equations of gefdynamics in the geometrical form were established in [6, 7]. Gefdynamics involves the Maxwell theory as the theory of a singlet state of the general electromagnetic field and describes the so-called dark matter [7]. The connection between statics and dynamics is very simple. We consider the 3-dimensional physical space in the static Theory of Self-Organization as an initial space cross section of 4-dimensional physical space in the dynamical Theory of Self-Organization. Due to this, the problem of initial singularity disappears. The so-called «Big Bang» can be considered as a transition of the self-organizing physical system from a timeless state to a dynamical state (as a release of the internal potential energy of a timeless system).

The paper is organized as follows. In Sec. 2, the concept of physical space is introduced and the fundamental role of the positive-definite Riemann metric is recognized. In the Theory of Self-Organization the physical space is the strict realization of the abstract notion of manifold. In Sec. 3, the concepts of a really geometrical quantity and geometrical internal symmetry are introduced. The set of the really geometrical quantities is very restricted. With geometrical internal symmetry we have the real understanding of the mysterious internal symmetries of modern physics (without the introduction of the artificial «charged spaces»). Self-Organization presupposes that the physical space and internal symmetry are tightly connected and kept inseparable. There are only three realizations of the general concept of geometrical internal symmetry that define the Self-Organizing Physical Fields and equations of these fields. Section 4 represents the novel concept of time and the discrete symmetries in the geometric (coordinate-independent) form with bilateral (left-right) symmetry as the main topic. The bilateral symmetry is the fundamental realization of the general concept of geometrical internal symmetry and has especially important meaning providing the natural and non-trivial introduction of the causal structure into the equations of Self-Organizing

Fields. In Sec. 5, we establish equations of gravidynamics in the geometrical form. Section 6 is devoted to the derivation of dynamical equations of the gravitational field. In Sec. 7 the exact solutions of these equations are found. One of them represents the shock spherical gravitational wave and the other represents the Universe with the homogeneous and isotropic distribution of the gravitational energy only. These solutions are compared with the Schwarzschild and Friedmann solutions in General Theory of Relativity. Summary is given in Sec. 8.

2. CONCEPT OF PHYSICAL SPACE

The concept of abstract differential manifold is basic in modern differential geometry. In both Geometry and General Relativity a manifold is a priori an element and on the given manifold one considers the different metrics and other structures. This is not convenient for the Theory of Self-Organization since it is absolutely free from any external and a priori conditions. The abstract theory of manifolds learns a given manifold itself. In this case, a process of coming-into-being cannot be considered. Thus, this abstract structure from the physical point of view is not the most interesting aspect of the theory of manifolds. It is more important to know where and how a manifold of a physical system emerges. After Whitney, we know that the class of abstract differential manifolds is not wider than the class of submanifolds of Euclidean spaces and we use this result to show that the realization of abstract manifold as a surface in Euclidean space of a fairly large number of dimensions permits one to consider a manifold as physical space, i.e., as the inner element of the absolute Self-Organization. Of course, the abstract theory of manifolds conserves its meaning as the unique tool that permits one to solve problems otherwise inaccessible.

The algebraic model of the familiar Euclidean space provides a natural generalization. The n-dimensional Euclidean space R^n is the linear structure in the set of n-tuples (vectors)

$$\mathbf{x} = (x^1, \cdots, x^n),$$

which is defined by the natural rules

$$\lambda \mathbf{x} = (\lambda x^1,\, \lambda x^2, \cdots \lambda x^n),$$

$$\mathbf{x} + \mathbf{y} = (x^1 + y^1, \cdots, x^n + y^n),$$

where λ is a real number and x^a are independent real variables (the coordinates of points in R^n). Every n-tuple $\mathbf{x} = (x^1, \cdots, x^n)$ corresponds to a definite position in R^n with the known visualization of this position in the dimensions $n = 1, 2, 3$. The distance function is given by

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(x^1 - y^1)^2 + \cdots + (x^n - y^n)^2}.$$

From the known formula $d^2(\mathbf{y},\mathbf{z}) = d^2(\mathbf{x},\mathbf{y}) + d^2(\mathbf{x},\mathbf{z}) - 2d(\mathbf{x},\mathbf{y})d(\mathbf{x},\mathbf{z})\cos\varphi$ one can derive the algebraic representation for the $\cos\varphi$ and scalar product

$$(\mathbf{x}\cdot\mathbf{y}) = x^1y^1 + \cdots + x^ny^n.$$

We give the following definition of physical space which is only possible in the theory based on the principle of self-organization. The physical space as a whole is the 3-dimensional manifold (in the timeless theory of Self-Organization) and the 4-dimensional manifold (in the dynamical theory of Self-Organization). There is a deep mathematical reason for this choice. A smooth manifold consists of a topological manifold and differential structure defined on it. It is known [8] that a topological manifold always admits differential structure if and only if its dimension is not larger than four and, hence, the case of general situation is realized for the dimension $n = 2,\ 3,\ 4$ only. The physical space is designed by the self-organizing fields as follows. The region of 4-dimensional physical space (as a surface in the imbedding Euclidean space of a fairly large number of dimensions) is analytically defined by the equations

$$x^a = F^a(u^1, u^2, u^3, u^4),$$

where the functions $F^a(u^1, u^2, u^3, u^4)$ of four independent parameters u^1, u^2, u^3, u^4 (the Gauss coordinates) are the solution of the characteristic system of non-linear equations in partial derivatives

$$\delta_{ab}\frac{\partial F^a}{\partial u^i}\frac{\partial F^b}{\partial u^j} = g_{ij}(u^1, u^2, u^3, u^4), \quad a,\, b = 1, \cdots, 4+k,\ k \geqslant 0. \tag{1}$$

The known functions $g_{ij}(u^1, u^2, u^3, u^4)$ in the right-hand side of equations (1) represent the positive-definite Riemann metric (a metric of principal type)

$$ds^2 = g_{ij}du^idu^j, \tag{2}$$

which we put in correspondence to the gravitational field. The Einstein idea gets here a new mathematical content but conserves its fundamental physical meaning. The functions $g_{ij}(u)$ are the solution of the controlling system of equations which connects the gravitational field $g_{ij}(u)$ (positive-definite Riemann metric) with other Self-Organizing Physical Fields (the general electromagnetic field and the spinning field). The controlling system of equations is the essence of the Theory of Self-Organization and involves equations of gravidynamics (subject of the present consideration), equations of spinstatics and spindynamics, and equations of gefstatics and gefdynamics. Solving the controlling system of equations we find the functions $g_{ij}(u^1, u^2, u^3, u^4)$, and with this result we can design local physical space and find a minimal dimension of imbedding Euclidean space as a result of solution of the characteristic system of equations (1). It should be noted that one cannot exclude the Riemann metric g_{ij} as an independent element and works with functions $F^a(u^1, \cdots u^4)$ only, since in this case the dimension of the imbedding Euclidean space should be fixed a priori but this is in contradiction with the principle of self-organization that theory should be absolutely independent of any outer and a priori conditions. A region of 3-dimensional physical space of the static Theory of Self-Organization is defined quite analogously.

It should be noted that the physical space itself does not determine singly its parametric representation, the parameters u^1, u^2, u^3, u^4 may, indeed, be subjected to any arbitrary continuous transformation (reparametrization invariance). Equations of gravidynamics are determined not only by a reparametrization invariance but mainly by geometrical internal symmetry (bilateral symmetry).

In the Theory of Self-Organizing Physical Fields the physical space and a minimal dimension of the imbedding Euclidean space arises as a result of solution of a characteristic system of differential equations (1). A choice of geometry adequate to the physical situation is realized by the self-organizing physical system itself. The general conclusion lies in the fact that the physical space of a self-organizing system is generated by the system itself and, hence, they are interdependent. The so-defined physical space is the basic notion. This means that all other definitions, notions and laws should be connected with the geometrical structure of physical space to be geometrical and physical. The Galileo and Etvesh experiments demonstrate that the «gravitational forces» are inertial in nature and, hence, they are originated by constraints. The equations $x^a = F^a(u^1, \cdots u^4)$ represent these constraints. The field equations provide no means to rule out either multiply-connected physical spaces, or physical spaces which are nonorientable.

3. CONCEPT OF REALLY GEOMETRICAL QUANTITY

Self-Organization presupposes the following guide principle of geometrization: the geometrical structure of physical space (points, curves, congruences of curves, families of curves) determines a very restricted set of really geometrical quantities (fields) and along with that geometrical internal symmetry that makes these quantities variable and defines their status as self-organizing physical fields.

There is an infinite set of tensor fields [9] connected with the coordinate covering of a manifold. However, only some types of the tensor fields are connected with the geometrical structure of a manifold and, hence, can be considered as really geometrical quantities. In the Theory of Self-Organization we extract fundamental physical fields out of a very restricted set of really geometrical quantities and this is evidently the geometrization of physics in the strict sense. The points, curves and families of curves (submanifolds) put together the geometrical structure of a manifold and, hence, a scalar field is the simplest really geometrical quantity which can be considered as a map that puts into correspondence to any point of a manifold a definite number. An important class of really geometrical quantities is defined by the functionals on the set of curves and submanifolds. Any curve on an n-dimensional manifold is a geometrical locus that is defined by the equations

$$\gamma : u^i = \varphi^i(t).$$

The families of curves (submanifolds S_p) are defined by the equations

$$S_p : u^i = \varphi^i(t_1, t_2, \cdots t_p) \quad (p = 2, 3, ...n).$$

After Gauss and Riemann, the functional

$$l(\gamma, g) = \int\limits_{t_0}^{t_1} \sqrt{g_{ij} \frac{du^i}{dt} \frac{du^j}{dt}} dt$$

on a manifold is called the length of the curve γ. Thus, the really geometrical quantity g_{ij} is a field introduced at first by Riemann as the positive-definite metric on a manifold and, as it was explained above, this field plays the fundamental role in the Theory of Self-Organization.

The important classes of the functionals on a manifold give the line integral

$$\int a_i du^i = \int a_i \frac{du^i}{dt} dt$$

and its generalization on the families of curves in the form of the iterated integrals

$$\int a_{i_1 \cdots i_p} \frac{\partial u^{i_1}}{\partial t^1} \cdots \frac{\partial u^{i_p}}{\partial t^p} dt^1 \cdots dt^p,$$

where a_i are components of a covector field and $a_{i_1 \cdots i_p}$ are components of an antisymmetrical tensor field of the rank p. Thus, we see (with Stoke's theorem as an additional argument) that the covariant antisymmetric tensor fields (including a scalar field and a covariant vector field) are the quantities connected with the geometrical structure of a manifold and, hence, they all are the really geometrical quantities.

A congruence of curves (a stream on a manifold) is given by a system of regular differential equations

$$\frac{du^i}{dt} = v^i(u^1(t), u^2(t), \cdots, u^n(t)).$$

The right-hand side of this system of equations is called a vector field on a manifold and, hence, the vector field belongs to the set of really geometrical quantities.

The equations of a parallel displacement for a vector field along the given curve γ

$$\frac{dv^i}{dt} + P^i_{jk} v^k \frac{du^j}{dt} = 0$$

give one more really geometrical quantity with the components P^i_{jk}, known as affine (linear) connection. If the curve γ belongs to the stream of the vector field v^i and hence $du^i/dt = v^i$, then the linear connection defines a geodesic stream on a manifold by the equations

$$\frac{d^2u^i}{dt^2} + P^i_{jk} \frac{du^j}{dt} \frac{du^k}{dt} = 0.$$

Thus, the set of quantities connected with the geometrical structure of a manifold is very restricted. Let us enumerate the really geometrical quantities:
1) the positive-definite Reimann metric g_{ij};
2) the vector field v^i;
3) the affine (linear) connection P^i_{jk};
4) the scalar and covariant vector fields, antisymmetric covariant tensor fields that can be shown as the 2^n-tuple

$$(a,\, a_i,\, a_{ij},\, \cdots a_{ijk\cdots l}).$$

This very small zoo of the really geometrical quantities is sufficient for understanding of everything. The symmetry principles made their appearance in the twentieth century physics with identification of the invariance group of the Maxwell equations. With this as a precedent, symmetries took on a character in physicists minds as a priori principles of universal validity. The natural generalization appears as the fundamental principle that basic laws should be defined by the widest possible groups of transformations. So the principle of general invariance (covariance) resulted above all in the Einstein theory of the gravitational field. This principle proved insufficient to reach the goal at which field physics is aimed: a unified field theory deriving all laws from one common structure of the world. The Theory of Self-Organization gives this structure in the form of geometrical internal symmetry.

By definition, geometrical internal transformations come in after the introduction of the really geometrical quantities to make these really geometrical quantities variable. An example of geometrical internal symmetry was discovered at first in Weyl's work [10], where he investigated the process of re-calibration in which a metric g_{ij} is replaced by $\overline{g}_{ij} = \lambda g_{ij}$ in which λ is an arbitrary positive function of position. We formulate the general principle that outlines the foundational role of the geometrical internal symmetry.

The geometrical internal symmetry makes the really geometrical quantities variable changing their geometrical status and being broken it leaves the trace *in the form of the differential equations for these quantities that describe all phenomena connected with the geometrical internal symmetry* (the equations of spinstatics and spindynamics, the equations of gefstatics and gefdynamics). Our main goal here is gravidynamics and for this reason we restrict our consideration of the geometrical internal symmetry by the only bilateral symmetry defined below.

4. CONCEPT OF TEMPORAL FIELD AND DISCRETE SYMMETRIES

We start with the intrinsic (and coordinate-independent) representation of dynamics. All dynamical laws have the following general form: *the rate of change with time of certain quantity equals to the result of action of some operator on this quantity*. The rate of change with time is the operator of evolution which defines causality in the field theory. The geometric and coordinate-independent definition of this foundational notion is the key point of any dynamical theory and cannot be given without the creation of the new concept of time. Indeed, in the Theory of Self-Organization the coordinates have no physical sense. Hence, it is quite obvious that time by itself can be represented only as a really geometrical quantity in physical space. The idea was put forward [3] that *time by itself is a scalar field* suggesting, by way of justification, a self-consistent dynamical theory of self-organizing fields which does not depend on any outer and a priori conditions (the dynamical division of the Theory of Self-Organization). Thus, dynamics is first of all the dynamical equations with the operator of evolution as the manifestation of the temporal structure defined by the temporal field.

In the Theory of Self-Organizing Physical Fields properties of time and physical space are not defined by the properties of devices and by the methods of measurements which are the aspects of the human being. This is the intrinsic matter of the physical system itself. The intrinsic temporal field (together with other fields) designs physical space, as it was explained above, but it has also other very important functions in the dynamical theory of self-organizing physical fields. It is our goal here to represent shortly these fundamental properties inherent in the very nature of time.

The temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 in the region U of physical space is denoted by

$$f(u) = f(u^1, u^2, u^3, u^4).$$

The space cross sections of physical space are defined by the temporal field. For the real number t, the space cross section is given by the equation

$$f(u^1, u^2, u^3, u^4) = t. \tag{3}$$

Since the temporal field is a scalar field, the partial derivatives define the covector field $t_i = \partial_i f$. The gradient of the temporal field (or the stream of time) is the vector field $\mathbf{t}$ with the components

$$t^i = (\nabla f)^i = g^{ij} \frac{\partial f}{\partial u^j} = g^{ij} \partial_j f = g^{ij} t_j,$$

where g^{ij} are the contravariant components of the positive-definite Riemann metric (2). The gradient of the field of time defines the direction of the most rapid increase (decrease) of the field of time. The rate of change with time of some quantity is the Lie derivative or the covariant derivative in the direction of the gradient of the field of time. The symbols $D_{\mathbf{t}}$ and $\nabla_{\mathbf{t}} = t^i \nabla_i$ denote these operations, where ∇_i is the covariant derivative with respect to the connection that belongs to the Riemann metric g_{ij}.

The rate of change with time of the temporal field itself is given by the formula $D_{\mathbf{t}} f = \nabla_{\mathbf{t}} f = t^i \partial_i f = g^{ij} \partial_i f \, \partial_j f$. The temporal field obeys the fundamental equation

$$D_{\mathbf{t}} f = g^{ij} \frac{\partial f}{\partial u^j} \frac{\partial f}{\partial u^j} = 1, \tag{4}$$

expressing that time flows equably. It is interesting to compare this equation with the definition of time given by Newton: «Absolute, true and mathematical time of itself, and from its own nature, flows equably without relation to anything external». The rate of change with time of the gravitational potential g_{ij} is given by the expression

$$D_{\mathbf{t}} g_{ij} = t^k \frac{\partial g_{ij}}{\partial u^k} + g_{kj} \frac{\partial t^k}{\partial u^i} + g_{ik} \frac{\partial t^k}{\partial u^j}.$$

For the antisymmetric covariant tensor field $a_{ij\cdots k}$ of rank p we have

$$D_{\mathbf{t}} a_{ij\cdots l} = t^k \partial_k a_{ij\cdots l} + p a_{k[j\cdots l} \partial_{i]} t^k.$$

Similar formulae can be presented for any geometrical quantities. The operator $\nabla_{\mathbf{t}}$ has no sense for the cases of the gravitational field and the general electromagnetic field since $\nabla_{\mathbf{t}} g_{ij} = 0$ identically and the potential of the general electromagnetic field is themselves a linear connection (from the geometrical point of view) [6, 7]. The operator $\nabla_{\mathbf{t}}$ is very important in spindynamics.

Now we shall introduce in the geometrical (coordinate-independent) form the following discrete symmetries: time reversal and bilateral symmetry. The bilateral symmetry plays the foundational role in the Theory of Self-Organization in general and as regards the gravidynamics as well. First of all, let us consider a very important notion of time reversal and invariance with respect to this symmetry. In the geometrical form the time reversal invariance means that theory is invariant with respect to the transformations

$$T: \quad t^i \to -t^i.$$

It is clear that theory will be time reversal invariant if the gradient of the temporal field appears in all formulae only as an even number of times, like $t^i t^j$. This definition will be explained in more detail after the definition of the intrinsic coordinates.

Now let us consider how to introduce the foundational notion of right and left in the framework of the Theory of Self-Organization. It is very important to recognize that the symmetry of the right and left (bilateral symmetry) is the realization of the concept of geometrical internal symmetry and is tightly connected with the stream of time. A pair of the vector fields $\mathbf{v}$ and $\overline{\mathbf{v}}$ has the bilateral symmetry with respect to the stream of time if the sum of these fields is collinear to the gradient of the temporal field and their difference is orthogonal to it,

$$\overline{\mathbf{v}} + \mathbf{v} = \lambda \mathbf{t}, \quad (\overline{\mathbf{v}}, \mathbf{t}) = (\mathbf{v}, \mathbf{t}),$$

where $(\mathbf{v}, \mathbf{w}) = g_{ij} v^i w^j = v^i w_i$ is the scalar product. In components, we have

$$\overline{v}^i + v^i = \lambda t^i, \quad (\overline{v}^i - v^i) t_i = 0$$

and, hence,

$$\overline{v}^i = 2(\mathbf{v}, \mathbf{n})\, n^i - v^i = (2 n^i n_j - \delta^i_j) v^j, \quad v^i = (2 n^i n_j - \delta^i_j) \overline{v}^j, \quad n^i = \frac{t^i}{\sqrt{(\mathbf{t}, \mathbf{t})}}.$$

Under the derivation of the field equations we have to consider equation (4) as a constraint and that is why we introduce the normalized vector field n^i. If the vector field v^i is right-sided (lies to the right from the stream of time), then the vector field $\overline{v}^i$ will be left-sided (lies to the left from the stream of time). The choice as to which is right or left is determined arbitrarily. We see that if $(\mathbf{v}, \mathbf{t}) = 0$, then $\overline{v}^i = -v^i$. Symmetry of right and left $\overline{v}^i = 2(\mathbf{v}, \mathbf{n})\, n^i - v^i$ may be presented in the form of the linear transformation $\overline{v}^i = R^i_j v^j$, where

$$R^i_j = 2 n^i n_j - \delta^i_j, \quad R^i_k R^k_j = \delta^i_j \quad \mathrm{Det}(R^i_j) = -1.$$

It is natural to call this transformation the reflection. Thus, for the other really geometrical quantities the bilateral symmetry may be realized as a representation of the reflection. Since $\overline{v}^i = R^i_j v^j$, for the metric tensor we have $\overline{g}_{ij} = g_{kl} R^k_i R^l_j = g_{ij}$ and, hence, g_{ij} is invariant with respect to the reflection. The same holds for the stream of time. Thus, with the temporal field not only evolution is tightly connected but the notion of right and left as well. The equivalence of the right and left can be considered as an important principle. The scalar product is invariant with respect to the reflection. Indeed, $(\overline{\mathbf{v}}, \overline{\mathbf{w}}) = (\mathbf{v}, \mathbf{w})$ since g_{ij} is invariant with respect to the reflection.

Here it is convenient to make a general remark about the so-called pseudo-Riemann geometry. This title is connected with the quadratic form

$$\overline{\varphi} = \overline{g}_{ij} v^i v^j$$

that is not positive definite. Let

$$\varphi = g_{ij} v^i v^j$$

be the positive definite quadratic form. Then a tensor $\overline{g}_{ij}$ can be represented as follows: $\overline{g}_{ij} = g_{ik} S^k_j$. We get

$$\overline{\varphi} = \overline{g}_{ij} v^i v^j = g_{ik} v^i v^j S^k_j = g_{ik} v^i w^k,$$

where $w^k = v^j S^k_j$. It is evident that the pseudo-Riemann quadratic form has the geometric meaning of the angle between two directions. Thus, one can avoid using the pseudo-geometrical terminology and concentrate on recognizing the geometrical meaning of the operator S^i_j in the framework of the genuine Riemann geometry defined by the positive-definite Riemann metric. We should like to remind that the Euclidean geometry underlies geometry and Euclid put us on the truth track by taking space as the primary concept of science.

The associated scalar product defined by the bilateral symmetry has the form

$$\langle \mathbf{v}, \mathbf{w} \rangle = (\overline{\mathbf{v}}, \mathbf{w}).$$

It is not difficult to show that the associated scalar product is also invariant with respect to the reflection: $\langle \overline{\mathbf{v}}, \overline{\mathbf{w}} \rangle = \langle \mathbf{v}, \mathbf{w} \rangle$. Let us consider the physical meaning of the associated scalar product $\langle \mathbf{v}, \mathbf{w} \rangle$. Since

$$\langle \mathbf{v}, \mathbf{v} \rangle = (\overline{\mathbf{v}}, \mathbf{v}) = 2(\mathbf{v}, \mathbf{n})^2 - (\mathbf{v}, \mathbf{v}) = |\mathbf{v}|^2 (2\cos^2\varphi - 1) = |\mathbf{v}|^2 \cos 2\varphi,$$

where φ is the angle between the vectors v^i and t^i, the associated scalar product is indefinite and can be positive, negative or equal to zero, according to the value of the angle φ. In particular, $(\overline{\mathbf{v}}, \mathbf{v}) = 0$, if $\varphi = \pi/4$. Thus, the associated scalar product $\langle \mathbf{v}, \mathbf{w} \rangle$ is time reversal invariant and permits one to classify all vectors depending on what angle they form with the stream of time. As we see from the formula $(\overline{\mathbf{v}}, \mathbf{v}) = \overline{g}_{ij} v^i v^j$, where

$$\overline{g}_{ij} = 2n_i n_j - g_{ij}, \tag{5}$$

the associated scalar product may be formalized with the help of the auxiliary metric $\overline{g}_{ij}$ as the metric of the normal hyperbolic type, which is defined by the temporal field and the bilateral symmetry. The contravariant components of the tensor field $\overline{g}_{ij}$ are $\overline{g}^{ij} = 2n^i n^j - g^{ij}$. Hence, the bilateral symmetry defines the natural causal structure on the physical space and can be identified with it. The conclusion lies in the fact that with the temporal field we get full understanding of the Lorentzian metric on the basis of the positive-definite metric and the bilateral symmetry. From different points of view it is very important to recognize that in General Theory of Relativity time was eliminated and this is the reason of the insurmountable conceptual difficulties of this theory. Since the bilateral symmetry is the realization of the concept of the geometrical internal symmetry, we conclude that equations of gravidynamics are defined by the internal symmetry because the simplest passage from statics to dynamics can be realized as the increasing of the dimension of physical space and the exchange of the positive definite metric by the auxiliary metric (5). Let us pay a bit of attention to this point. The bilateral symmetry gives an evident method of introduction of the temporal field into Lagrangians of the self-organizing physical fields. The geometrical representation of the dynamical laws of the gravitational field can be defined by the Lagrangian $L = \overline{R}$, where $\overline{R}$ is the scalar that is constructed from the auxiliary metric (5) following the formulae of the Reimann geometry. Since the auxiliary metric (5) is defined by the two fields connected by equation (4), it is necessary to pay special attention when deriving the equations of field. A standard method is to incorporate the constraint (4) via a Lagrange multiplier $\varepsilon = \varepsilon(u)$, rewrite the action density for a gravity field in the form $L_g = \overline{R} + \varepsilon(g^{ij}t_i t_j - 1)$ and treat the components of the fields g_{ij} and f as independent variables. Thus, one needs to vary the action

$$A = \frac{1}{2}\int \overline{R}\sqrt{g}\, d^4u + \int L_m(\overline{g}, F)\sqrt{g}\, d^4u + \frac{1}{2}\int \varepsilon\, (g^{ij}t_i t_j - 1)\sqrt{g}\, d^4u, \quad (6)$$

where $g = \mathrm{Det}(g_{ij}) > 0$ and $L_m(\overline{g}, F)$ is the Lagrangian density of the other self-organizing fields F which incorporate time through the auxiliary metric (5) in the conventional form (Lagrangian $L_m(\overline{g}, F)$ depends only on the values of $\overline{g}_{ij}$ in point). It is a very important condition since the minimal interaction of the fields with gravity is supposed. The concept of potential field (introduced in [7]) is the realization of this principle. It is interesting that the concept of the potential field and the concept of the really geometrical quantity actually outline the same set of fundamental fields.

The method of introduction of causality with the help of the auxiliary metric (5) does not require special explanation but in general, it is not evident how to derive the dynamical laws (the form of which was defined earlier) from their subsidiary geometrical representation. This problem will be solved here in gravidynamics. As the first stage our strategy is to derive equations of gravidynamics in the form most close to the Einstein equations.

5. EQUATIONS OF GRAVIDYNAMICS IN THE GEOMETRICAL FORM

In the Theory of Self-Organization the concept of time is the cornerstone and provides the energy conservation. The energy conservation is the consequence of the concept of time and means that the rate of change with time of the total energy density of the gravity field and all other self-organizing fields is equal to zero and, hence, the energy density represents the first integral of the system. To prove this statement, we derive equations of gravidynamics from the action (6).

Since the auxiliary metric (5) is the function of g_{ij} and f, it is convenient to use the chain rule. We have $\delta\overline{R} = \overline{g}^{ij}\delta\overline{R}_{ij} + \overline{R}_{ij}\delta\overline{g}^{ij}$. Further we denote by Γ^i_{jk} and $\overline{\Gamma}^i_{jk}$ the Christoffel symbols belonging to the fields g_{ij} and $\overline{g}_{ij}$, respectively. From (5), after some calculations, we obtain

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + H^i_{jk}, \tag{7}$$

where

$$H^i_{jk} = n^i(\nabla_j n_k + \nabla_k n_j) + (2n^i n^l - g^{il})(n_j\delta^m_k + n_k\delta^m_j)(\nabla_m n_l - \nabla_l n_m).$$

In the last formula ∇_j is a covariant derivative with respect to the connection Γ^i_{jk}. It is easy to derive from (7) that $\overline{\Gamma}^j_{jk} = \Gamma^j_{jk}$ and, hence, $\overline{g}^{ij}\delta\overline{R}_{ij}$ can be omitted as a perfect differential. Varying now $\overline{g}^{ij}$ we get

$$\delta\overline{g}^{ij} = \delta g^{ij} + P^{ij}_{kl}\delta g^{kl} + Q^{ijk}\partial_k\delta f,$$

where

$$P^{ij}_{kl} = 2n^i n^j n_k n_l - n^i(n_k\delta^j_l + n_l\delta^j_k) - n^j(n_k\delta^i_l + n_l\delta^i_k),$$
$$Q^{ijk} = \frac{2}{\sqrt{(t,t)}}(2n^i n^j n^k - n^i g^{jk} - n^j g^{ik}).$$

A tensor field P^{ij}_{kl} is symmetrical in covariant and contravariant indices.

Let $G_{ij} = \overline{R}_{ij} - \frac{1}{2}\overline{g}_{ij}\overline{R}$ be the Einstein tensor. Further we put $\delta L_F = \frac{1}{2}M_{ij}\delta\overline{g}^{ij}$ and introduce in the standard way the energy-momentum tensor $T_{ij} = M_{ij} - \overline{g}_{ij}L_F$. Observing that

$$\overline{g}_{ij} + \overline{g}_{kl}P^{kl}_{ij} = g_{ij}, \quad g_{ij}Q^{ijk} = n_i n_j Q^{ijk} = 0,$$

it is easy to verify that a total variation of the action (with neglect of a perfect differential) can be represented in the following form:

$$\delta A = \frac{1}{2}\int \left(A_{ij}\delta g^{ij} + B\delta f + \delta\varepsilon\left(g^{ij}t_i t_j - 1\right)\right)\sqrt{g}d^4u, \tag{8}$$

where

$$A_{ij} = G_{ij} + G_{kl}P^{kl}_{ij} + T_{ij} + T_{kl}P^{kl}_{ij} + \varepsilon\, t_i t_j - \frac{1}{2}\varepsilon\,(g^{kl}t_k t_l - 1)g_{ij},$$

$$B = -\nabla_k\left((G_{ij} + T_{ij} - \varepsilon t_i t_j)Q^{ijk}\right) - 2\nabla_k(\varepsilon t^k).$$

One can consider the tensor P^{kl}_{ij} as an operator P acting in the space of symmetrical tensor fields. The characteristic equation of this operator has the form $P^2 + 2P = 0$ and, hence, $(P+1)^2 = 1$. Thus, the operator $P+1$ is inverse to itself. Since $t_i t_j + t_k t_l P^{kl}_{ij} = -t_i t_j$, from (8) it follows that the equations of gravidynamics can be written in the following form:

$$G_{ij} + T_{ij} = \varepsilon\partial_i f\,\partial_j f, \quad g^{ij}\partial_i f\,\partial_j f = 1, \tag{9}$$

$$\nabla_k(\varepsilon t^k) = 0. \tag{10}$$

Equations (9) constitute the full system of equations of gravidynamics in the geometrical form and, as it is shown above, emerge from the principle of self-organization. Equation (10) expresses the law of energy conservation in the Theory of Self-Organizing Physical Fields which, evidently, is general covariant. To make sure that we indeed deal with conservation of energy, it is sufficient to figure out that action (6) is invariant with respect to the transformation $f \to f + a$, where a is constant. Thus, equation (10) results also from the Noether theorem. It is also clear that the Lagrange multiplier ε has a physical meaning of the energy density of the system in question. From Eqs. (9) it follows that

$$\varepsilon = G_{ij}t^i t^j + T_{ij}t^i t^j = \varepsilon_g + \varepsilon_m, \tag{11}$$

where $\varepsilon_g = G_{ij}t^i t^j = \frac{1}{2}\overline{R}_{ij}g^{ij}$ is the energy density of the gravitational field and $\varepsilon_m = T_{ij}t^i t^j$ is the energy density of other fields in this representation. Consider the law of energy conservation from the various points of view. First of all we consider the link between (9) and (10).

The so-called local energy conservation is written as follows: $\overline{\nabla}_i T^{ij} = 0$, where $T^{ij} = T_{kl}\overline{g}^{ik}\overline{g}^{jl}$ and $\overline{\nabla}_i$ is a covariant derivative with respect to $\overline{\Gamma}^i_{jk}$. These equations are fulfilled on the solutions of equations for the fields F. Since $\overline{\nabla}_i G^{ij} = 0$ identically, from field equations (9) it follows that

$$\overline{\nabla}_i T^{ij} = \varepsilon t^i(\overline{\nabla}_i t^j) + t^j\overline{\nabla}_i(\varepsilon t^i).$$

From (4) and (7) we get $\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + 2t^i\nabla_j t_k$ and finally we have

$$\overline{\nabla}_i T^{ij} = t^j\nabla_i(\varepsilon t^i).$$

In view of this the energy conservation law can be treated as the condition of compatibility of the field equations. In this sense, the law of energy conservation is analogous to the law of charge conservation.

Show that the rate of change with time of the energy density $D_t(\sqrt{g}\varepsilon)$ equals to zero and, hence, this quantity is the first integral of the system in question. We have $D_t(\sqrt{g}\varepsilon) = t^i\partial_i(\sqrt{g}\varepsilon) + \sqrt{g}\varepsilon\partial_i t^i = \partial_i(\sqrt{g}\varepsilon t^i)$. Since $\sqrt{g}\nabla_k(\varepsilon t^k) = \partial_i(\sqrt{g}\varepsilon t^i)$, from the law of energy conservation (10) it follows that the energy density is the first integral of the system

$$D_t(\sqrt{g}\varepsilon) = 0. \tag{12}$$

In the natural coordinates x^1, x^2, x^3, t, defined below, $\mathbf{t} = (0,0,0,1)$ and this equation has a more customary form

$$\frac{\partial}{\partial t}(\sqrt{g}\varepsilon) = 0.$$

Let us compare the law of energy conservation with the law of charge conservation which can be written in a general covariant form as follows: $\nabla_i J^i = 0$. Putting $J^i = t^i(t,J) + J^i - t^i(t,J) = \rho t^i + P^i$ we find that $D_t(\sqrt{g}\rho) + \partial_i(\sqrt{g}P^i) = 0$. Thus, we see that the charge density is not in general the first integral of the system while the energy density always is. In order to understand this phenomenon, we give the derivation of the equations of gravidynamics in the Hamiltonian form introducing the fundamental notion of momentum of the gravitational field.

6. THE EQUATIONS OF GRAVIDYNAMICS IN HAMILTONIAN FORM

It is clear that momentum of the gravitational field should be connected with the rate of change with time of the gravitational potential. Consider a tensor field of the type (1,1)

$$P^i_j = \frac{1}{2}g^{ik}D_t g_{jk} = \frac{1}{2}g^{ik}(\nabla_j t_k + \nabla_k t_j) = g^{ik}\nabla_j t_k = \nabla_j t^i.$$

A tensor field so defined is the momentum of the gravitational field or simply the momentum of the field. Give some important relations for the momentum:

$$t_i P^i_j = 0, \quad t^j P^i_j = 0, \quad g^{ik}P^l_k g_{lj} = P^i_j.$$

$$D_t P^i_j = t^l \nabla_l P^i_j, \quad \nabla_i P^k_j - \nabla_j P^k_i = R_{ijl}{}^k t^l.$$

$$D_t \Gamma^i_{jk} = \nabla_j P^i_k + \nabla_k P^i_j - g^{il}g_{km}\nabla_l P^m_j, \quad D_t g = g g^{ik} D_t g_{ik} = 2gP^i_i.$$

As the following step in the required direction we write equations (9) in the following form:

$$\overline{R}_{ij} + T_{ij} - \frac{1}{2}\overline{g}_{ij}T = \frac{1}{2}g_{ij}\,\varepsilon, \tag{13}$$

where $T = T_{ij}\overline{g}^{ij}$. If we start with equations (13), we get by contraction (11) and $\overline{R} = \varepsilon + T$, and so we can get back to (9). We may use either (9) or (13) as the basic equations.

With (4) and (7) we get $\overline{R}_{ij} = R_{ij} + \nabla_l(t^l D_t g_{ij})$ and, hence, $g^{ik}\overline{R}_{jk} = R^i_j + 2D_t P^i_j + 2P^k_k P^i_j$. Introduce the tensor fields

$$B^i_j = h^i_k R^k_l h^l_j + D_t P^i_j + P^k_k P^i_j, \quad N^i_j = h^i_k T^k_l h^l_j - \frac{1}{2}T h^i_j,$$

where $h^i_j = \delta^i_j - t^i t_j$ is the projection operator, $h^i_k h^k_j = h^i_j$. We next define the energy flow vector of the gravitational field

$$\mathrm{G}_i = \nabla_k P^k_i - \partial_i P^k_k + t_i(P^k_l P^l_k + D_t P^k_k)$$

and the energy flow vector of other fields

$$\Pi_i = \varepsilon_m\, t_i - T_{ik}t^k.$$

It is evident that $(\mathbf{t}, \mathbf{G}) = (\mathbf{t}, \mathbf{\Pi}) = 0$. With this we can derive from equations (13) the following system of equations:

$$D_t P^i_j + P^k_k P^i_j + B^i_j + N^i_j = \frac{1}{2}\varepsilon\, h^i_j, \tag{14}$$

$$\mathrm{G}_i = \Pi_i, \tag{15}$$

and vice versa. Our statement is that equations (14) and (15) represent in general covariant Hamiltonian form the physical laws of the gravitational field. We write equations (14) in the form

$$\frac{1}{\sqrt{g}}D_t(\sqrt{g}P^i_j) + B^i_j + N^i_j = \frac{1}{2}\varepsilon\, h^i_j$$

and add that from (14) it follows that

$$D_t(\mathrm{G}_i - \Pi_i) + P^k_k(\mathrm{G}_i - \Pi_i) = \frac{1}{\sqrt{g}}D_t(\sqrt{g}(\mathrm{G}_i - \Pi_i)) = 0.$$

The last equation means that equations (14) and (15) are compatible.

Equation (15) has a very simple physical sense that the energy flow vector of the gravitational field is exactly equal to the energy flow vector of other fields and we can see why the total energy density is the first integral. Thus, the fluxes of the gravitational and other forms of energy flow in the opposite directions and are exactly equal to each other. We know that our sun is a surprisingly powerful source of the radiant energy. Now it is evident that if there is a flow of gravitational energy in the direction to the sun, then we can say that our sun is the transducer of the gravitational energy into the electromagnetic one.

For the energy density of the gravitational field we have $\varepsilon_g = \frac{1}{2}\overline{R}_{ij}g^{ij} = \frac{1}{2}R + (P^i_i)^2 + D_t P^i_i = T + U$, where

$$T = \frac{1}{2}(P^i_i)^2 - \frac{1}{2}P^i_j P^j_i \tag{16}$$

is the density of kinetic energy and

$$U = \frac{1}{2}R + \frac{1}{2}(P^i_i)^2 + \frac{1}{2}P^i_j P^j_i + D_t P^i_i \tag{17}$$

is a density of potential energy of the gravitational field.

For better understanding of the last constructions we consider the general covariant dynamical laws of the gravitational field in the intrinsic (or natural) coordinates. To this end we give the definition of natural coordinates and show that in these coordinates the causal structure (the gradient of the temporal field) has the canonical form $\mathbf{t} = (0, 0, 0, 1)$.

Geometrically, the stream of time is defined as a congruence of lines (lines of time) on the manifold. Analytically, the lines of time are defined as the solutions of the autonomous system of differential equations

$$\frac{du^i}{dt} = g^{ij}\frac{\partial f}{\partial u^j} = g^{ij}\partial_j f = (\nabla f)^i, \quad (i = 1, 2, 3, 4). \tag{18}$$

Let

$$u^i(t) = \varphi^i(u_0^1,\, u_0^2,\, u_0^3,\, u_0^4, t) = \varphi^i(u_0, t) \tag{19}$$

be the solution to equations (18) with the initial data $\varphi^i(u_0, t_0) = u_0^i$. Substituting $u^i(t) = \varphi^i(u_0, t)$ into the function $f(u^1,\, u^2,\, u^3,\, u^4)$ we obtain $p(t) = f(\varphi(u_0, t))$. By virtue of (4) and (18), one finds

$$\frac{dp(t)}{dt} = \frac{\partial f}{\partial u^i}\frac{du^i}{dt} = g^{ij}\frac{\partial f}{\partial u^i}\frac{\partial f}{\partial u^j} = 1.$$

It leads to $f(\varphi(u_0, t)) = t - t_0 + f(u_0)$. Suppose that all initial data belong to the space section $f(u_0^1,\, u_0^2,\, u_0^3,\, u_0^4) = t_0$. Rewriting this relation in the parametric form $u_0^i = \psi^i(x^1, x^2, x^3)$, Eqs. (19) can be written as the system of relations

$$u^i = \phi^i(x^1, x^2, x^3, t). \tag{20}$$

The functions (20) have continuous partial derivatives with respect to the parameters x^1, x^2, x^3, t and their functional determinant is not equal to zero. Since in the coordinates x^1, x^2, x^3, t temporal field has a simple form $f(x^1, x^2, x^3, t) = f(\phi(x,t)) = t$, in these coordinates $t_i = \partial_i f = (0,0,0,1)$. Since the coordinates x^1, x^2, x^3 do not vary along the lines of time,

$$\frac{dx^\mu}{dt} = g^{\mu 4} = 0, \quad (\mu = 1,2,3) \quad \frac{dt}{dt} = g^{44} = 1.$$

Hence the functions $\phi^i(x^1, x^2, x^3, t)$ design natural coordinates of the dynamical system in question. Thus, in the natural coordinates

$$t_i = (0,0,0,1), \quad g^{\mu 4} = 0, \quad g^{44} = 1, \quad t^i = g^{ij} t_j = g^{i4} = (0,0,0,1),$$

which is what we had to prove. Therefore, it follows that in the natural coordinates x^1, x^2, x^3, t metric (2) takes the form

$$ds^2 = g_{\mu\nu}(x^1,\, x^2,\, x^3,\, t) dx^\mu dx^\nu + (dt)^2, \quad \mu, \nu = 1,2,3, \tag{21}$$

since $t_i = g_{ij} t^j = g_{i4}$.

In the natural coordinates the field equations have the simplest form in the sense that all components of the gradient of the temporal field and four components of the gravitational potential take numerical values. It should be noted that the natural coordinates are important since they are similar to the Darboux coordinates in the theory of symplectic manifolds which is a geometrical basis for the Hamilton mechanics. What is more, there are transformations of coordinates which conserve the causal structure and these transformations are analogous to the canonical transformations in Hamiltonian mechanics. We see that causal structure is analogous to the symplectic structure of the Hamiltonian mechanics.

From the above consideration it also follows that variable t, parametrizing the line of time, can be considered as a coordinate of time of the physical system in question. This name is justified particularly by the fact that the rate of change with time of any field is equal to the partial derivative with respect to t, i.e., $D_t = \partial/\partial t$ in the natural coordinates. This is a very important point since we see that the natural coordinates give the possibility to write all equations in the form analogous to the canonical form of the Hamilton equations.

If we reverse time setting $\tilde{t}^i = -t^i$, then the lines of time are parametrized by the new variable $\tilde{t}$. It is not difficult to show that there is a one-to-one and mutually continuous correspondence between the parameters t and $\tilde{t}$ given by the relation $\widetilde{t} = -t$. From here it is clear that in the system of coordinates defined by the time reversal, the variable $-t$ will be the coordinate of time. Thus, the geometric (coordinate-independent) definition of the time reversal is adjusted to the familiar definition that is connected with the transformation of coordinates.

With respect to the coordinate transformations that conserve causal structure the quantity $g_{\mu\nu}(t, x^1, x^2, x^3), \quad (\mu, \nu = 1,2,3)$ from the metric (21) transforms as the symmetrical tensor and $dl^2 = g_{\mu\nu}(x^1,\, x^2,\, x^3,\, t)dx^\mu dx^\nu$ may be considered as the metrics of the space sections of the physical manifold. For this one parametric set of three-dimensional metrics we can consider the Christoffel symbols $L^\mu_{\nu\sigma}$ and covariant derivative ∇_μ with respect to the $L^\mu_{\nu\sigma}$, Riemannian tensor curvature $S_{\mu\nu\sigma}{}^\tau$, tensor Ricci $S_{\nu\sigma} = S_{\mu\nu\sigma}{}^\mu$ and scalar curvature $S = g^{\mu\nu}S_{\mu\nu} = S^\mu_\mu$ as usual. In the natural coordinates we have $L^\mu_{\nu\sigma} = \Gamma^\mu_{\nu\sigma}$, $P^4_i = P^i_4 = 0$, $B^4_i = B^i_4 = 0$, $(i = 1, 2, 3, 4)$ and what is more important that

$$B^\mu_\nu = S^\mu_\nu, \quad R + (P^i_i)^2 + P^i_j P^j_i + 2D_t P^i_i = S.$$

Thus, out of (16) and (17) we get for the densities of the kinetic and potential energy of the gravitational field the following representation:

$$T = \frac{1}{2}(P^\sigma_\sigma)^2 - \frac{1}{2}P^\mu_\nu P^\nu_\mu, \quad U = \frac{1}{2}S.$$

Now we write equations (14) and (15) in the natural coordinates under the assumption that T_{ij} is trivial. We have

$$\dot{P}^\mu_\nu + P^\sigma_\sigma P^\mu_\nu + S^\mu_\nu = \frac{1}{2}\varepsilon\, \delta^\mu_\nu, \tag{22}$$

$$G_\mu = \nabla_\nu P^\nu_\mu - \partial_\mu P^\sigma_\sigma = 0, \tag{23}$$

where dot stands for partial derivative with respect to the variable t. Since

$$\dot{P}^\mu_\nu + P^\sigma_\sigma P^\mu_\nu = \frac{1}{\sqrt{g}}\frac{\partial}{\partial t}(\sqrt{g}\, P^\mu_\nu)$$

equations (22) can be written in the following form:

$$\frac{\partial}{\partial t}(\sqrt{g}\, P^\mu_\nu) + \sqrt{g}S^\mu_\nu = \frac{1}{2}\sqrt{g}\,\varepsilon\, \delta^\mu_\nu.$$

With this and

$$\frac{\partial}{\partial t}(\sqrt{g}\varepsilon) = 0$$

we see that

$$\frac{\partial^2}{\partial t^2}(\sqrt{g}\, P^\mu_\nu) = -\frac{\partial}{\partial t}(\sqrt{g}\, S^\mu_\nu).$$

The last relation shows that the Cauchy problem for the gravitational field is not as difficult as for the electromagnetic field.

The general conclusion lies in the fact that the essence of gravity is the law of energy conservation and the so-called dark energy is simply the gravitational energy.

7. TWO EXACT SOLUTIONS

Now we consider the question of exact solutions of the gravidynamics equations (22) and (23). The starting point of our consideration is the geometrical (general covariant) definition of the static gravitational field.

Gravitational field is *static* if the rate of change with time of the gravitational potential is equal to zero

$$D_{\mathbf{t}} g_{ij} = t^k \frac{\partial g_{ij}}{\partial u^k} + g_{kj} \frac{\partial t^k}{\partial u^i} + g_{ik} \frac{\partial t^k}{\partial u^j} = 0.$$

It is evident that the definition of the static gravitational fields is general covariant. For the static gravitational field the momentum of this field is equal to zero and this can be considered as the equivalent definition of the static gravitational field. Since

$$D_t g_{ij} = \nabla_i t_j + \nabla_j t_i, \quad \nabla_i t_j - \nabla_j t_i = 0,$$

one can consider the equations

$$\nabla_i t_j = 0$$

as else one definition of the static gravitational field.

From equations (14) and (15) it is not difficult to derive that the nontrivial gravitational field itself cannot be static. This is the motivation to use the empirical (noninternal) representations about the spherical symmetry and spherical gravitational wave. We can suggest that in the absence of other physical fields the equations of gravidynamics may have solution in the form of a shock gravitational wave. In the natural coordinates this wave can be represented as follows:

$$dl^2 = A^2 dr^2 + B^2 (d\theta^2 + \sin^2\theta \, d\phi^2),$$

where $A = A(r \pm t), \quad B = B(r \pm t)$. For the nonzero components of the Ricci tensor of this one parametric set of metric we find the following expressions:

$$S_{11} = \frac{2}{AB}(A^{'}B^{'} - AB^{''}), \quad S_{22} = \frac{B}{A^3}(A^{'}B^{'} - AB^{''}) + 1 - \frac{B^{'2}}{A^2}$$

and $S_{33} = S_{22} \sin^2\theta$, where prime stands for a derivative with respect to the variable r. Hence, for the potential energy of this configuration we have

$$U = \frac{1}{2} S = \frac{2}{A^3 B}(A^{'}B^{'} - AB^{''}) + \frac{1}{B^2}\left(1 - \frac{B^{'2}}{A^2}\right).$$

To know where $U = 0$, we immediately put $B^{'} = A$.

For the nontrivial components of the momentum we have

$$P_1^1 = \frac{\dot{A}}{A}, \quad P_2^2 = P_3^3 = \frac{\dot{B}}{B},$$

where dot denotes a partial derivative with respect to the variable t. The energy flow vector of the gravitational field has only one nonzero component

$$\mathrm{G}_1 = \frac{2}{AB}(\dot{A}B^{'} - A\dot{B}^{'})$$

and in accordance with the relation $A = B^{'}$, the equation $\mathrm{G}_i = 0$ is fulfilled. Thus, we have two equations

$$2B\ddot{B} + \dot{B}^2 = 0, \quad \ddot{A}B^2 + \dot{A}\dot{B}B = \frac{1}{2}A\dot{B}^2.$$

Since $A = A(r \pm t), \quad B = B(r \pm t)$, only from the equation for B we find that $B\dot{B}^2 = m$, where m is a constant of integration and hence

$$B = m\left[\frac{3}{2}\frac{(r \pm t)}{m}\right]^{\frac{2}{3}}.$$

Since $A = B^{'}$,

$$A^{-1} = \left[\frac{3}{2}\frac{(r \pm t)}{m}\right]^{\frac{1}{3}},$$

and it is not difficult to see that the second equation for A and B is fulfilled automatically. Thus, the solution of the equations of gravidynamics invariant with respect to time reversal can be represented in the following form:

$$ds^2 = dt^2 + \frac{dr^2}{\left[\frac{3}{2}\frac{(r-t)}{m}\right]^{\frac{2}{3}}} + \left[\frac{3}{2}\frac{(r-t)}{m}\right]^{\frac{4}{3}} m^2(d\theta^2 + \sin^2\theta d\phi^2)$$

for $t \geqslant 0$ and

$$ds^2 = dt^2 + \frac{dr^2}{\left[\frac{3}{2}\frac{(r+t)}{m}\right]^{\frac{2}{3}}} + \left[\frac{3}{2}\frac{(r+t)}{m}\right]^{\frac{4}{3}} m^2(d\theta^2 + \sin^2\theta d\phi^2)$$

for $t \leqslant 0$.

Since the kinetic energy $T = 0$ for $r \pm t \neq 0$, we can conclude that this solution has a singularity on the wave front set where energy of the shock gravitational wave is concentrated. To compare the solution obtained with the Schwarzschild solution in General Theory of Relativity, we can use the transformation of coordinates [11]. We see that the shock gravitational wave is considered as the static solution in General Theory of Relativity; however, the general covariant definition of the static field is absent here and that is evidently a contradictory situation. *Thus, from the physical point of view, this «stationary» picture hides the real dynamical process and investigations should be oriented toward the experimental verification of this physical situation.*

Since the so-called dark energy is the simply gravitational energy this makes it possible to consider another very simple but important physical situation. The idea of Universe with a homogeneous and isotropic distribution of the gravitational energy may be realized in the natural coordinates by the one parametric set of metrics

$$dl^2 = a^2(t)d\sigma^2,$$

where $d\sigma^2$ is the metric of the unit 3D sphere, which in the four-dimensional spherical coordinates has the form

$$d\sigma^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2).$$

Now the problem is to find a function $a(t)$. We have

$$S^\mu_\nu = \frac{2}{a^2}\delta^\mu_\nu, \quad P^\mu_\nu = \frac{\dot{a}}{a}\delta^\mu_\nu,$$

and hence for the density of the gravitational energy we get

$$\varepsilon_g = \frac{3}{a^2}(\dot{a}^2 + 1).$$

We know that $\partial(\sqrt{g}\,\varepsilon_g)/\partial t = 0$. Since $\sqrt{g} = a^3\sin^2\psi\sin\theta$, then by integrating over the variables ψ, θ, ϕ we get the following equation for $a(t)$ from the law of energy conservation

$$a(\dot{a}^2 + 1) = 2a_0 = \mathrm{const}.$$

With respect to the new variable η, such that $dt = ad\eta$ the solution of this equation can be presented in the following form $a = a_0(1 - \cos\eta)$. Thus, we get that the evolution of the Universe with a homogeneous and isotropic distribution of the gravitational energy is described in parametric form as follows:

$$a = a_0(1 - \cos\eta), \quad t = a_0(\eta - \sin\eta)$$

and this solution is invariant with respect to time reversal if $-\frac{\pi}{2} \leqslant \eta < 0$, $0 < \eta \leqslant \frac{\pi}{2}$.

The second solution presupposes that the Universe is filled up the gravitational energy («dark energy») only. Formally, the same solution in General Theory of

Relativity arises as a result of filling the Universe up a dust of unknown nature and is known as the Friedmann solution. *From our consideration it follows that «matter» has a field nature and the Universe with only «dark energy» is a good approximation from different points of view. We have a singularity at $\eta = 0$ and conclude that the problem of initial state is an open question that cannot be solved without consideration of the other forms of energy in the Universe. In the framework of the Theory of Self-Organizing Physical Fields the energy of the general electromagnetic field (dark matter) is the natural and actually unique candidate*.

8. SUMMARY

The main concepts and principles of the gravidynamics (as a chapter of the Theory of Self-Organizing Physical Fields) are formulated. The absolute form of the energy conservation is discovered, the fundamental physical meaning of the energy flows is recognized, and the real possibility appeares to seize the energy of the gravitational field. In the Theory of Self-Organization we deal with a redistribution of the different forms of energy and this is the general and most fundamental characteristic of the self-organizing system. Identifying the gravitational energy with dark energy and dark matter with the general electromagnetic field («heavy light») the Theory of Self-Organizing Physical Fields gives the solution of the problem of matter and thus opens the possibility to develop the theoretical cosmology on the completely new grounds.

Since from the novel concept of time underlying the Theory of Self-Organization it follows that the flows of gravitational energy are quite usual and universal phenomena, it is interesting to look for simple methods of their detection and usage. Of course, experimental investigations are needed for these purposes; here it is important to point out that the freedom of choosing the coordinates (a practical simulation of a real physical situation) and the law of energy conservation are the natural instruments for the gravitational physics. In view of this it is interesting to systematize and analyze all evidences that can be interpreted as flows of the gravitational energy.

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