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EXTRAPOLATION OF THE ENERGIES OF THE 2_1^+, 4_1^+, 6_1^+ STATES IN THE SUPERHEAVY EVEN–EVEN NUCLEI

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Экстраполяция энергий 2_1^+-, 4_1^+-, 6_1^+-состояний в сверхтяжелых четно-четных ядрах

Исходя из имеющейся корреляции энергии деформации и нижайшей энергии возбуждений были получены оценки для энергий 2_1^+-состояний. Рассмотренная систематика отношений энергий в ротационной полосе позволила также получить оценки для 4_1^+- и 6_1^+-состояний. Проведено сравнение с результатами других работ.

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Extrapolation of the Energies of the 2_1^+, 4_1^+, 6_1^+ States in the Superheavy Even–Even Nuclei

Based on the existing correlation between the deformation energy and the lowest excitation energy, estimates were obtained for energy of the 2_1^+ states. The considered systematics of the energy ratios in the rotational band also made it possible to obtain estimates for the 4_1^+ and 6_1^+ states. A comparison with the results of other works was carried out.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

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INTRODUCTION

Advances in the synthesis of the atomic nuclei of superheavy elements [1–3] made it possible to realize research in the field of atomic nuclei up to $Z = 118$ and to plan experiments to synthesize atomic nuclei with $Z = 119, 120$ and more. This area is unique in terms of studying the structure of atomic nuclei, since superheavy nuclei exist due to quantum mechanical effects that stabilize these nuclei. The role of stabilizing shell effects in the stability of superheavy nuclei is demonstrated [1–3] by comparing experimental results with theoretical calculations and empirical systematics. For study of the structure of atomic nuclei of superheavy elements, it is extremely useful to have information about the energies of at least a few of the lowest levels. Experimental data on the energy levels in the region of superheavy nuclei are extremely poor, and theoretical calculations are pretty ambiguous. Nevertheless, in the experimental study of superheavy nuclei, it is useful to get an idea of the energy of the excited nuclear states in advance. Such data are especially significant in study of the β and γ decays of high-spin isomers in superheavy nuclei [4]. Therefore, the purpose of this work was to obtain estimates for the energies of a number of the lowest levels in superheavy nuclei. In this paper, the estimation of the energy of excited states in the region of nuclei under consideration is based on the correlation of the relative energy of the first excited level $E(2_1^+)$ and the deformation energy in even–even nuclei, demonstrated in [5].

The deformation energy $E_{\rm def}$ is defined as the difference between the energy of a nucleus in its deformed equilibrium and spherical shapes [6]:

$$E_{\rm def} = E(\beta) - E(0). \qquad (1)$$

Estimates for this energy were taken from two independent papers [7] and [8]. It should be noted that although the deformation energy estimates in [7] are overestimated compared to those given in [6], the same is observed in the recent paper [8]. However, it can be assumed that if the trend of $E_{\rm def}$ dependence on the mass number is conveyed correctly, then the correlation of $E_{\rm def}$ with the energy 2_1^+ levels allows one to get the correct $E(2_1^+)$ values. For determination of the correlation curve parameters, the known experimental data on $E(2_1^+)$ values must be used for different set of calculated $E_{\rm def}$ energies.

1. DEPENDENCE OF $E(2_1^+)$ ON THE DEFORMATION ENERGY

In [5], the systematization of the energy of collective states was carried out by using the values of the deformation energy obtained in [7] in terms of the Hartree–Fock–Bogolyubov approximation, taking into account the realistic Gonya forces. The experimentally known energies of the lowest 2_1^+ states in even–even nuclei from $_{90}$Th up to $_{104}$Rf and theoretical deformation energies are applied to establish a correspondence between them. Figure 1 shows the correlations between the first excitation energies $E(2_1^+)$ and $E_{\rm def}$ taken from [7], and in Fig. 2, $E_{\rm def}$ are taken from [8].

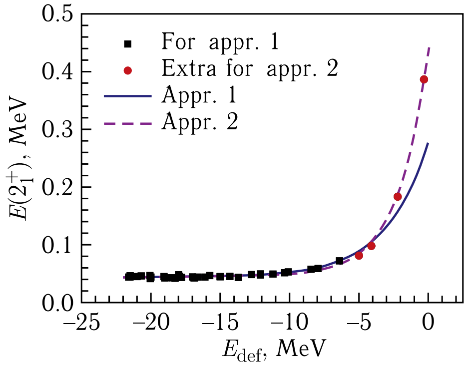

Fig. 1. Dependence of experimental energies $E(2_1^+)$ on calculated deformation energies obtained in [7] for even–even isotopes from Th to Rf: "for appr. 1" means the points, along which the first approximation passed; "extra for appr. 2" — those points that were additionally taken into account at the second approximation; "appr. 1" and "appr. 2" — approximate values

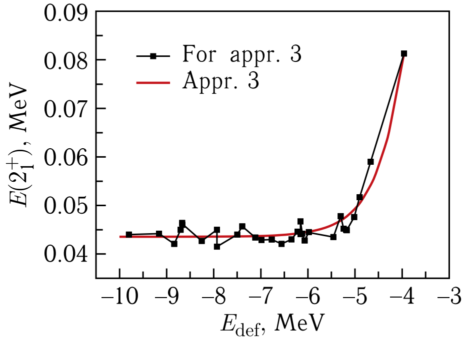

Fig. 2. Dependence of experimental energies $E(2_1^+)$ on calculated deformation energies obtained in [8] for even–even isotopes from U to Rf: "for appr. 3" means the points, on which the approximation passes; "appr. 3" — approximate values

The presence of a long plateau in these figures at $E_{\rm def} < -10$ MeV made it possible to use the dependence of the moment of inertia J on the deformation energy by analogy with the mean field potential dependence on the radius in the Woods–Saxon representation. This made it possible to use the following parameterization [5]:

$$E(2_1^+) = \frac{3}{J}; \quad J = \frac{J_0}{1 + \exp\left(\frac{E_{\rm def}+V_1}{\zeta V_0}\right)}. \tag{2}$$

Keeping a similar functional dependence, we will use the parameterization for the energy of the first excitation in the form

$$E(2_1^+) = b_1 + b_2\, e^{aE_{\rm def}}. \tag{3}$$

The parameters in formula (3) are defined using three data options. Data used to define parameters in the first variant in this equation are given in Table 1, as well as in Fig. 1. Table 2 shows the nuclei and their characteristics, which were additionally taken into account for the second set of parameters. The data from Table 2 are also displayed in Fig. 1. The third version of the data on the deformation energy, unlike the first two options, was taken from [8] and is shown in Fig. 2. In the last variant, in accordance with the fact that in [8] the deformation energies are presented starting from U isotopes, we do not use those points where the energies of the first excitation are rather large, namely, only for two nuclei such energies are more than 60 keV and do not exceed 85 keV.

Table 1. **Data used to determine the approximation parameters. Deformation energies (in MeV) are taken from** [7], 2_1^+ **energies (in MeV) — from** [9]

$E_{\rm def}$	$E(2_1^+)$	Nucleus	$E_{\rm def}$	$E(2_1^+)$	Nucleus	$E_{\rm def}$	$E(2_1^+)$	Nucleus
−21.6	0.044	^{256}Rf	−18.45	0.0429	^{246}Cm	−15.0	0.0449	^{238}U
−21.505	0.0442	^{254}No	−18.25	0.0421	^{242}Cm	−15.0	0.0446	^{236}Pu
−21.495	0.0464	^{252}No	−17.95	0.0457	^{252}Cf	−14.6	0.0478	^{242}U
−21.0	0.045	^{250}Fm	−17.8	0.0434	^{248}Cm	−14.3	0.0452	^{236}U
−20.7	0.046	^{248}Fm	−17.0	0.0445	^{242}Pu	−13.7	0.0435	^{234}U
−20.07	0.0421	^{252}Fm	−16.9	0.043	^{250}Cm	−12.75	0.0484	^{236}Th
−20.05	0.0415	^{248}Cf	−16.85	0.0428	^{240}Pu	−12.1	0.0476	^{232}U
−20.0	0.044	^{246}Cf	−16.65	0.0442	^{244}Pu	−12.1	0.0496	^{234}Th
−19.05	0.045	^{254}Fm	−16.1	0.0441	^{238}Pu	−11.2	0.0494	^{232}Th
−19.0	0.0427	^{250}Cf	−15.75	0.0467	^{246}Pu	−10.35	0.0517	^{230}U
−18.9	0.042965	^{244}Cm	−15.0	0.045	^{240}U	−10.1	0.0532	^{230}Th
						−8.45	0.0578	^{228}Th
						−7.95	0.059	^{228}U
						−6.4	0.0722	^{226}Th

T a b l e 2. **Additional data to those given in the previous table, included in the second (appr. 2) version of the approximation procedure.** E_{def} **and** $E(2_1^+)$ **are in MeV**

E_{def}	$E(2_1^+)$	Nucleus
−4.1	0.0981	^{224}Th
−2.2	0.1833	^{222}Th
−5.0	0.0813	^{226}U
−0.3	0.3865	^{220}Th

As a result of optimization for the first variant, the following values of the parameters were obtained:

$$a = 0.33\ \mathrm{MeV}^{-1};\quad b_1 = 0.04349\ \mathrm{MeV};\quad b_2 = 0.23322\ \mathrm{MeV}. \tag{4}$$

For the second option, we have got

$$a = 0.467\ \mathrm{MeV}^{-1};\quad b_1 = 0.04494\ \mathrm{MeV};\quad b_2 = 0.39179\ \mathrm{MeV}, \tag{5}$$

and for the third option, the parameters of which were determined by deformation energies from [8], we have got

$$a = 1.85\ \mathrm{MeV}^{-1};\quad b_1 = 0.04355\ \mathrm{MeV};\quad b_2 = 58.319\ \mathrm{MeV}. \tag{6}$$

The first excitation energies $E(2_1^+)$ were estimated from the deformation energies taken from [7] as follows. If the deformation energy, whose values are given in Table 3, is less than −5.5 MeV, then the energy estimates are

T a b l e 3. **Deformation energies** E_{def} **(in MeV) taken from** [7]**. Excitation energy for these nuclei is unknown**

N	$Z = 104$	$Z = 106$	$Z = 108$	$Z = 110$
146	−19.50			
148	−20.70	−19.5		
150	−21.40	−20.4	−18.95	
152	−21.6	−20.95	−19.3	
154	−21.02	−20.3	−19.4	
156	−20.02	−19.7	−19.75	
158	−18.75	−18.8	−18.9	−19.2
160	−17.5	−17.75	−18.0	−18.3
162	−16.1	−16.5	−17.15	−17.1
164	−12.9	−13.4	−13.5	−12.95
166	−10.2	−10.7	−10.4	−9.7
168	−7.6	−8.0	−7.7	−7.6
170	−5.6	−5.5	−6.0	−5.3
172	−4.4	−4.2	−4.75	−4.2
174	−2.95	−3.0	−3.2	−2.9
176	−2.1	−2.2	−2.1	−2.4
178			−1.55	−0.9
180				−0.5

made in accordance with the first option, otherwise, with the second one. Such a combined definition of excitation energies will be called variant 1–2 (ext. 1–2). The third option was applied for all deformation energies taken from [8].

For a number of synthesized even–even nuclei, the half-lives are known (Table 4).

Tables 5–8 show the results of approximations (appr. 1–2, appr. 3) obtained both by using the deformation energies from [7] and [8], and also energy estimations obtained in [6] for $Z = 98-110$. For $_{96}$Cm isotopes (Table 5), the obtained results are based on the deformation energies from [7] and [8], and we have got the estimation of the energy values close to those from [6]. This can be related to the fact that for extra variant 3, the deformation energies for Cm isotopes change quite dynamically up to -3.84 MeV for ^{262}Cm.

For $_{98}$Cf with A from 238 to 256, the approximations give close results and the predictions from [6], starting from $A = 254$, have definitely larger energy values. The discrepancies in the considered approximations are due to the fact that $|E_{\rm def}|$ decreases only to 4.47 MeV. This discrepancy increases for the elements following those starting from $N = 160$.

For $_{100}$Fm (Table 6), if for $A = 240-260$ approximations give close values, then starting from $A = 260$ ($N = 160$), the energy differences grow, and this is also due to the slight drop in $|E_{\rm def}|$ in [8] with increasing mass number.

T a b l e 4. **Known half-lives of superheavy even–even nuclei with $Z \geqslant 104$**

Isotope	$T_{1/2}$	Isotope	$T_{1/2}$
$^{254}_{104}$Rf	23(3) μs	$^{258}_{106}$Sg	2.9^{+13}_{-7} ms
^{256}Rf	6.67(10) ms	^{260}Sg	4.95(33) ms
^{258}Rf	14.7^{+12}_{-1} ms	^{262}Sg	6.9^{+38}_{-18} μs
^{260}Rf	21(1) ms	^{264}Sg	37^{+27}_{-11} ms
^{262}Rf	2.3(4) s	^{266}Sg	21^{+20}_{-12} s
$^{264}_{108}$Hs	≈ 0.8 ms	$^{268}_{110}$Ds	—
^{266}Hs	2.3^{+13}_{-6} ms	^{270}Ds	0.10^{+14}_{-4} ms
^{268}Hs	0.4^{+18}_{-2} s	^{272}Ds	—
$^{276}_{112}$Cn	—	$^{284}_{114}$Fl	2.5^{+18}_{-8} ms
^{278}Cn	—	^{286}Fl	0.16^{+7}_{-3} s
^{280}Cn	—	^{288}Fl	0.52^{+22}_{-13} s
^{282}Cn	0.50^{+33}_{-1} ms	^{290}Fl	—
^{284}Cn	101^{+41}_{-22} ms	$^{286}_{116}$Lv	—
^{286}Cn	—	^{288}Lv	—
^{288}Cn	—	^{290}Lv	15^{+26}_{-6} ms
		$^{294}_{118}$Og	0.69 ms

Table 5. **Estimates of the level energies for isotopes with $Z = 96, 98$. E_{def} are given in MeV, $E(2_1^+)$ — in keV**

Z	N	A	Appr. 1–2		Appr. 3		Data from [6]	
			E_{def}	$E(2_1^+)$	E_{def}	$E(2_1^+)$	$E(2_1^+)$	$E(4_1^+)$
96	138	234	−12.8	46.9	−5.35	46.5		
	140	236	−15.2	45.0	−5.88	44.6		
	142	238	−16.3	44.6	−6.39	44		
	144	240	−17.7	44.2	−6.63	43.8	44.2	
	146	242	−18.3	44	−6.56	43.9	42.9	
	148	244	−19	43.9	−6.77	43.8	43.4	
	150	246	−18.5	44	−6.99	43.7	45.1	
	152	248	−18	44.1	−7.12	43.7	45.3	
	154	250	−16.9	44.4	−6.35	44	46.9	
	156	252	−15.4	45	−5.62	45.3		
	158	254	−14	45.8	−5.16	47.7		
	160	256	−12.6	47.1	−5.08	48.4		
	162	258	−11.1	49.5	−5.32	46.7		
	164	260	−8.6	57.1	−4.34	62.6		
	166	262	−5.6	77	−3.84	92		
	168	264	−4.8	87	−3.94	83.4		
	170	266	−4.6	91				
	172	268	−2.3	179				
	174	270	−1.6	230				
98	140	238	−15.5	44.9	−6.44	44		
	142	240	−16.9	44.4	−7.02	43.7		
	144	242	−18.2	44.1	−7.33	43.6		
	146	244	−19.3	43.9	−7.13	43.7	43.5	
	148	246	−20	43.8	−7.50	43.6	43.7	
	150	248	−20.05	43.8	−7.93	43.6	44.5	
	152	250	−19	43.9	−8.26	43.6	43.6	
	154	252	−18	44.1	−7.40	43.6	45.1	
	156	254	−17	44.3	−6.74	43.8	48.2	
	158	256	−15.6	44.8	−6.28	44.1	50.8	
	160	258	−14.05	45.8	−6.22	44.1		
	162	260	−12.5	47.3	−6.45	44		
	164	262	−9.9	52.4	−5.33	46.6		
	166	264	−7.4	63.8	−4.70	53.3		
	168	266	−5.9	72	−4.47	58.5		
	170	268	−4.05	104	−4.57	55.9		
	172	270	−3	142				
	174	272	−2	199				
	176	274	−1.4	249				

For $_{102}$No, approximations give close results for $A = 242 - 260$. Starting from $A = 262$ ($N = 160$), the discrepancy grows. Moreover, the trends in the $E(2_1^+)$ energy changes in appr. 1–2 and in accordance with the estimates

Table 6. **The same as in Table 5, but for isotopes with $Z = 100, 102$. $E_{\rm def}$ are given in MeV, $E(I_1^+)$ — in keV**

Z	N	A	Appr. 1–2		Appr. 3		Data from [6]	
			$E_{\rm def}$	$E(2_1^+)$	$E_{\rm def}$	$E(2_1^+)$	$E(2_1^+)$	$E(4_1^+)$
100	140	240	−15.5	44.9				
	142	242	−16.5	44.5	−7.33	43.6		
	144	244	−18.3	44	−7.58	43.6		
	146	246	−19.7	43.8	−7.58	43.6		
	148	248	−20.3	43.8	−8.67	43.6		
	150	250	−20.95	43.7	−8.70	43.6	43.9	
	152	252	−20	43.8	−8.84	43.6	42.0	
	154	254	−19	43.9	−7.93	43.6	43.4	
	156	256	−17.95	44.1	−7.25	43.6	46.4	
	158	258	−16.95	44.4	−7.14	43.7	48.9	
	160	260	−15.1	45.1	−7.16	43.7	50.3	
	162	262	−13.5	46.2	−7.51	43.6		
	164	264	−11.3	49.1	−6.56	43.9		
	166	266	−9.2	54.7	−5.55	45.6		
	168	268	−6.1	72	−5.24	47.2		
	170	270	−5.1	81.1	−4.98	49.4		
	172	272	−2.3	179	−4.91	50.2		
	174	274	−2.2	185				
	176	276	−1.7	222				
102	140	242	−14.9	45.2				
	142	244	−15.6	44.8				
	144	246	−18.4	44	−7.24	43.6		
	146	248	−19.7	43.8	−7.37	43.6	46.1	154
	148	250	−20.2	43.8	−7.93	43.6	45.7	152.7
	150	252	−21.3	43.7	−8.67	43.6	44.5	148.5
	152	254	−21.4	43.7	−9.16	43.6	41.6	138.6
	154	256	−20.75	43.7	−8.48	43.6	43.1	144.1
	156	258	−19.6	43.8	−8	43.6	45.8	152.8
	158	260	−18.05	44.1	−7.79	43.6	47.9	159.9
	160	262	−16.9	44.4	−7.95	43.6	48.9	162.9
	162	264	−15.1	45.1	−8.53	43.6	46.2	154.2
	164	266	−13.6	46.1	−7.66	43.6	51.2	170.2
	166	268	−9.55	53.5	−6.6	43.8		
	168	270	−7.1	65.9	−6.10	44.3		
	170	272	−5.3	78	−5.68	45.1		
	172	274	−4	105	−5.57	45.5		
	174	276	−2.8	151	−5.65	45.2		
	176	278	−1.7	222				
	178	280	−1.5	239				

from [6] with increasing mass number are close, although their estimates in [6] are overestimated. Similarly, for ${}_{104}$Rf (Table 7), the difference in the

Table 7. **The same as in Table 5, but for isotopes with $Z = 104, 106$. The only known experimental data refer to ^{256}Rf and the corresponding energies are 44.0, 148.2, 309.2 keV, which can give an idea on the accuracy of the approximation performed. $E_{\rm def}$ are given in MeV, $E(I_1^+)$ — in keV**

Z	N	A	Appr. 1–2		Appr. 3		Data from [6]	
			$E_{\rm def}$	$E(2_1^+)$	$E_{\rm def}$	$E(2_1^+)$	$E(2_1^+)$	$E(4_1^+)$
104	146	250	−19.5	43.9	−7.58	43.6		
	148	252	−20.7	43.7	−8.28	43.6	49.1	164.1
	150	254	−21.4	43.7	−9.08	43.5	46.9	155.9
	152	256	−21.6	43.7	−9.80	43.6	43.4	144.4
	154	258	−21.02	43.7	−9.24	43.6	44.5	148.5
	156	260	−20.02	43.8	−8.81	43.6	46.4	154.4
	158	262	−18.75	44.0	−8.84	43.6	47.3	157.3
	160	264	−17.5	44.2	−9.16	43.6	47.2	157.2
	162	266	−16.1	44.6	−9.6	43.6	44.3	147.3
	164	268	−12.9	46.8	−8.8	43.6	49.0	163.0
	166	270	−10.2	51.5	−7.89	43.6	54.9	182.9
	168	272	−7.6	62.5	−7.03	43.7		
	170	274	−5.6	80.2	−6.91	43.7		
	172	276	−4.4	95.1	−6.7	43.8		
	174	278	−2.95	143.7	−6.84	43.7		
	176	280	−2.1	191.9	−6.33	44		
106	148	254	−19.5	43.9	−8.52	43.6		
	150	256	−20.4	43.8	−9.31	43.6	48.4	161.4
	152	258	−20.95	43.7	−10.14	43.6	44.7	148.7
	154	260	−20.3	43.8	−9.69	43.6	45.0	150.0
	156	262	−19.7	43.8	−9.41	43.6	45.9	152.9
	158	264	−18.8	44.0	−9.44	43.6	45.6	151.6
	160	266	−17.75	44.2	−9.86	43.6	45.0	150.0
	162	268	−16.5	44.5	−10.59	43.6	41.9	139.9
	164	270	−13.4	46.3	−9.48	43.6	46.5	155.5
	166	272	−10.7	50.3	−8.61	43.6	51.8	172.8
	168	274	−8	60.1	−7.89	43.6	57.0	190.0
	170	276	−5.5	81.5	−7.5	43.6		
	172	278	−4.2	100.0	−7.43	43.6		
	174	280	−3.0	141.5	−7.73	43.6		
	176	282	−2.2	185.2	−7.25	43.6		
	178	284	−1.55	235	−6.92	43.7		

two approximations starts from $A = 266$, and the results from [6] are close to appr. 1–2.

For $_{106}$Sg, the discrepancy in approximations starts from $A = 268$ ($N = 162$) and grows rapidly with A. The results from [6] are again close to appr. 1–2.

Table 8. **The same as in Table 5, but for isotopes with $Z = 108, 110$. E_{def} are given in MeV, $E(I_1^+)$ — in keV**

Z	N	A	Appr. 1–2		Appr. 3		Data from [6]	
			E_{def}	$E(2_1^+)$	E_{def}	$E(2_1^+)$	$E(2_1^+)$	$E(4_1^+)$
108	150	258	−18.95	43.9	−9.44	43.6		
	152	260	−19.3	43.9	−10	43.6		
	154	262	−19.4	43.9	−9.99	43.6	46.2	154.2
	156	264	−19.75	43.8	−9.88	43.6	46.6	155.6
	158	266	−18.9	44	−9.99	43.6	45.8	152.8
	160	268	−18	44.1	−10.55	43.6	43.9	145.9
	162	270	−17.15	44.3	−11.47	43.6	40.2	194.2
	164	272	−13.5	46.2	−10.42	43.6	44.5	148.5
	166	274	−10.4	51	−9.4	43.6	49.1	164.1
	168	276	−7.7	61.9	−8.42	43.6	53.8	178.8
	170	278	−6	75.7	−8.24	43.6	61.6	205.6
	172	280	−4.75	87.6	−8.4	43.6		
	174	282	−3.2	132.9	−8.88	43.6		
	176	284	−2.1	192	−8.53	43.6		
	178	286	−0.9	303	−7.69	43.6		
	180	288	−0.5	355	−7.5	43.6		
110	156						51.1	170.1
	158	268	−19.2	44	< -8.66	43.6	50.6	168.6
	160	270	−18.3	44	< -8.66	43.6	47.7	158.7
	162	272	−17.1	44	< -8.66	43.6	42.3	141.3
	164	274	−12.95	47	< -8.66	43.6	46.7	155.7
	166	276	−9.7	49.3	< -8.66	43.6	51.3	171.3
	168	278	−7.6	56.3	< -8.66	43.6	54.7	182.7
	170	280	−5.3	78	< -8.66	43.6		
	172	282	−4.2	100	< -8.66	43.6		
	174	284	−2.9	146	< -8.66	43.6		
	176	286	−2.4	173	< -8.66	43.6		

For $_{108}$Hs (Table 8), the divergence of the two approximations grows rapidly as A grows, starting from 272 ($N = 164$). For $_{110}$Ds, we observe the same tendency starting from $A = 274$ ($N = 164$).

Thus, comparing our approximations with the results from [6], one can conclude that appr. 1–2, presented in Tables 5–8, are more reliable for the estimation of the energy of the first excitation.

2. ESTIMATIONS OF THE ENERGY OF THE 4_1^+, 6_1^+ STATES

The energy of the 2_1^+ states are too small, and therefore, instead of the corresponding γ transitions, conversion electrons will be mainly observed in the experiment. Gamma transitions from 4_1^+ and 6_1^+ states can be observed. It would be extremely useful to estimate the energy of the 4_1^+ and 6_1^+ states. In order to obtain estimates of the energy of the corresponding states, the ratios $E(4_1^+)/E(2_1^+)$ and $E(6_1^+)/E(2_1^+)$ were considered according to the available experimental data [9] for even–even nuclei with $Z \geqslant 90$. These ratios are shown in Figs. 3 and 4 for different energy ranges of the first excitation. To obtain estimates for unknown energy values, the following approximation

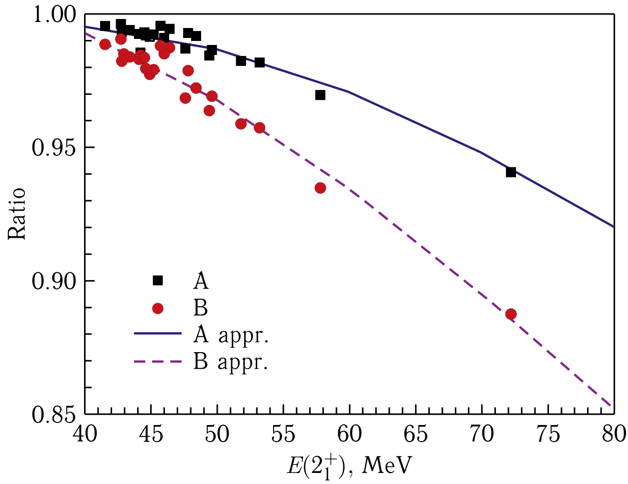

Fig. 3. Experimental energy ratios in rotational units, i.e., for A it is $(E(4_1^+)/E(2_1^+))/$ $/(10/3)$ and for B — $(E(6_1^+)/E(2_1^+))/7$

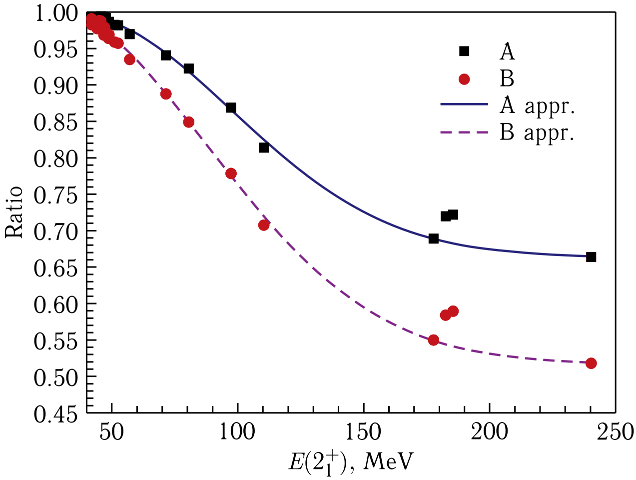

Fig. 4. The same as in Fig. 3, but for a larger energy interval $E(2_1^+)$. In the limiting vibrational case, the first ratio is 0.600, the second — 0.5102

relation was used

$$R = a\exp^{-\left((E(2_1^+)-E_0)/b\right)^2} + c, \qquad (7)$$

where $R = R_4 = E(4_1^+)/E(2_1^+)/(10/3)$ or $R_6 = E(6_1^+)/E(2_1^+)/7$. For R_4, $E_0 = 35$, $a = 0.33297$, $b = 88.325$, $c = 0.66333$ in keV. For R_6, $E_0 = 20.5$, $a = 0.49666$, $b = 94.88$, $c = 0.51673$ in keV. The corresponding approximation curves are also shown in Figs. 3 and 4. The experimental data presented in these figures correspond to even–even nuclei from Th to No. Points that lie out of the fitting curve correspond to the data for ^{222}Th and ^{222}Rn. For nuclei with a well-developed rotational spectrum, $R \simeq 1$. For nuclei with extremely pronounced vibration character of the spectrum, $R_4 = 0.600$, $R_6 = 0.4285$.

To determine the unknown energies of the 2_1^+ states, theoretical values of deformation energies from [7], given in Table 3, were used.

Since there are no data on deformation energies in [7] for nuclei with $Z > 110$, they were taken in accordance with Table 3. For $N = 168$ and $Z = 112$–118, $E_{\rm def} \approx -7.7$ MeV; for $N = 170$ and $Z = 112$–118, $E_{\rm def} \approx -5.5$ MeV; for $N = 172$ and $Z = 112$–118, $E_{\rm def} \approx -4.3$ MeV; for $N = 174$ and $Z = 112$–118, $E_{\rm def} \approx -3$ MeV; for $N = 176$ and $Z = 112$–118,

Table 9. **Appr. 3 results. For $_{112}$Cn nuclei with $N = 154$–184, $_{114}$Fl with $N = 156$–186, $_{116}$Lv with $N = 158$–184, $_{118}$Og with $N = 160$–184, $Z = 120$ with $N = 160$–186, $Z = 122$ with $N = 164$–186, $Z = 124$ with $N = 166$–188, $Z = 126$ with $N = 170$–188, energies are unchanged and equal to 43.6 keV, the rest are presented in this table. $E_{\rm def}$ are given in MeV, $E(2_1^+)$ — in keV**

Z	N	A	Appr. 3	
			$E_{\rm def}$	$E(2_1^+)$
116	186	302	−8.64	43.6
	188	304	−6.54	43.9
118	186	304	−7.56	43.6
	188	306	−5.776	44.9
	190	308	−5.095	48.2
120	188	308	−5.89	44.6
	190	310	−5.27	47
122	188	310	−6.185	44.2
	190	312	−5.57	45.5
	192	314	−5.04	48.8
	194	316	−4.48	58.2
124	190	314	−5.69	45.1
	192	316	−5.24	47.2
	194	318	−4.58	55.7
126	190	316	−5.85	44.7
	192	318	−5.43	46.1
	194	320	−4.85	51

$E_{\text{def}} \approx -2.3$ MeV. These estimates are to some extent conditional, but can be applied because in the calculation of the deformation energies [8] no sharp jumps were observed with a change in the number of nucleons.

The totality of the considered data makes it possible to obtain estimates for the 4_1^+, 6_1^+ energies, which are given in Tables 5–8. Data on deformation energies for $Z > 110$ are available in [8]. Table 9 gives the corresponding estimates for the energies of the 2_1^+ states.

As can be seen from Tables 5–8, appr. 3 gives clearly overestimated values for nuclei with $N > 166$ in comparison with the results of appr. 1–2 and the estimates obtained in [6]. Data given in Table 9 must be considered clearly underestimated.

Tables 10–13 additionally list the energy estimates for the 4_1^+, 6_1^+ states obtained based on the systematics shown in Figs. 3 and 4. In this case, the energies of the 2_1^+ states are taken in accordance with appr. 1–2.

The possibility of determining the energies of states with larger values of Z is limited by the availability of theoretical data on deformation energies. As can be seen from Fig. 5, it is currently not possible to make such estimations for $Z \geqslant 120$. Moreover, as noted in [10], different theoretical models predict different locations of the “island of stability” of superheavy nuclei with stabilized shells. Macroscopic-microscopic methods based on various phenomenological potentials localize this island around the closures of spherical shells ($Z = 114$ and $N = 184$) [11, 12].

The presented estimates are based on the calculated deformation energies. This can be done in different approaches and with different model parameterizations. If it is legitimate to have correlations between the deformation energy and the energy of the lowest excitation in even–even nuclei, then with a uniform method of calculating the deformation energy, one can expect correct values of the $E(2_1^+)$ energies. It should be borne in mind that approximations obtained directly on the energies of neighboring nuclei, whose energies are known, give more reliable results. Therefore, Table 14 shows the corresponding estimates obtained in [5].

The covariant density functional theory [13] localizes the indicated island near $Z = 120$ and $N = 172$ [14, 15]. A similar result was obtained in [16] in the framework of the self-consistent covariant theory of the energy density functional, taking into account the quasiparticle-vibrational coupling. In it, a description of the evolution of the shell in the chain of superheavy isotopes with $A = 292, 296, 300, 304$ and $Z = 120$ was obtained. A fairly stable closure of the spherical proton shell at $Z = 120$ was predicted. In this case, the interaction that determines pair correlations and the quasiparticle–phonon interaction lead to a smooth evolution of the neutron shell gap between the numbers of neutrons $N = 172$ and 184, blurring the effects of the shell. This, in turn, leads to the fact that for $N = 170{-}186$, the energies of the first excitation are $\geqslant 1$ MeV, and for $N = 172$ and $N = 184$, the energies of the first excitation are $\geqslant 1.5$ MeV.

In paper [17], a microscopic variant of the Grodzins relation is used, obtained on the basis of a geometric collective model and a microscopic

T a b l e 10. **Energy estimates for the 4_1^+, 6_1^+ states for isotopes with $Z = 96, 98$**

Z	N	A	E_{def}, MeV	Approximation, keV			Data from [6] (keV)	
				$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(2_1^+)$	$E(4_1^+)$
96	138	234	−12.8	46.9	155	321		
	140	236	−15.2	45.0	149	309		
	142	238	−16.3	44.6	148	307		
	144	240	−17.7	44.2	146	304	44.2	
	146	242	−18.3	44	146	303	42.9	
	148	244	−19	43.9	145	302	43.4	
	150	246	−18.5	44	146	303.5	45.1	
	152	248	−18	44.1	146	304	45.3	
	154	250	−16.9	44.4	147	305	46.9	
	156	252	−15.4	45	149	309		
	158	254	−14	45.8	151	314		
	160	256	−12.6	47.1	155	322		
	162	258	−11.1	49.5	163	336		
	164	260	−8.6	57.1	186	378		
	166	262	−5.6	77	238	446		
	168	264	−4.8	87	261	500		
	170	266	−4.6	91	269	511		
	172	268	−2.3	179	410	686		
	174	270	−1.6	230	510	838		
98	140	238	−15.5	44.9	148	309		
	142	240	−16.9	44.4	147	305		
	144	242	−18.2	44.1	146	304		
	146	244	−19.3	43.9	145	302	43.5	
	148	246	−20	43.8	145	302	43.7	
	150	248	−20.05	43.8	145	302	44.5	
	152	250	−19	43.9	145	302	43.6	
	154	252	−18	44.1	146	302	45.1	
	156	254	−17	44.3	147	305	48.2	
	158	256	−15.6	44.8	148	305	50.8	
	160	258	−14.05	45.8	151	314		
	162	260	−12.5	47.3	156	323		
	164	262	−9.9	52.4	172	352		
	166	264	−7.4	63.8	205	411		
	168	266	−5.9	72	226	447		
	170	268	−4.05	104	293	543		
	172	270	−3	142	350	609		
	174	272	−2	199	447	740		
	176	274	−1.4	249	551	903		

T a b l e 11. **The same as in Table 10, but for isotopes with** $Z = 100, 102$

Z	N	A	E_{def}, MeV	Approximation, keV			Data from [6] (keV)	
				$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(2_1^+)$	$E(4_1^+)$
100	140	240	−15.5	44.9	148	309		
	142	242	−16.5	44.5	147	306		
	144	244	−18.3	44	146	303		
	146	246	−19.7	43.8	145	302		
	148	248	−20.3	43.8	145	302		
	150	250	−20.95	43.7	145	301	43.9	
	152	252	−20	43.8	145	302	42.0	
	154	254	−19	43.9	145	302	43.4	
	156	256	−17.95	44.1	146	304	46.4	
	158	258	−16.95	44.4	147	305	48.9	
	160	260	−15.1	45.1	149	310	50.3	
	162	262	−13.5	46.2	153	316		
	164	264	−11.3	49.1	162	333		
	166	266	−9.2	54.7	179.5	365		
	168	268	−6.1	72	226	447		
	170	270	−5.1	81.1	248	481		
	172	272	−2.3	179	410	686		
	174	274	−2.2	185	421	701		
	176	276	−1.7	222	494	811		
102	140	242	−14.9	45.2	149	310		
	142	244	−15.6	44.8	148	308		
	144	246	−18.4	44	146	303		
	146	248	−19.7	43.8	145	302	46.1	154
	148	250	−20.2	43.8	145	302	45.7	152.7
	150	252	−21.3	43.7	145	301	44.5	148.5
	152	254	−21.4	43.7	145	301	41.6	138.6
	154	256	−20.75	43.7	145	301	43.1	144.1
	156	258	−19.6	43.8	145	302	45.8	152.8
	158	260	−18.05	44.1	146	304	47.9	159.9
	160	262	−16.9	44.4	147	305	48.9	162.9
	162	264	−15.1	45.1	149	310	46.2	154.2
	164	266	−13.6	46.1	152	316	51.2	170.2
	166	268	−9.55	53.5	175	358		
	168	270	−7.1	65.9	210	421		
	170	272	−5.3	78	241	470		
	172	274	−4	105	294	545		
	174	276	−2.8	151	364	625		
	176	278	−1.7	222	494	811		
	178	280	−1.5	239	530	869		

T a b l e 12. **The same as in Table 10, but for isotopes with** $Z = 104, 106, 108$**. Only for** ^{256}**Rf the experimental data are known and the corresponding experimental energies are** 44.1, 148.2, 309.2 **keV, which can give an evaluation of the accuracy of the approximation. Comparing the experimental data for** ^{256}**Rf with approximation results, one can evaluate the accuracy of the approximation procedure**

Z	N	A	$E_{\rm def}$, MeV	Approximation, keV			Data from [6] (keV)	
				$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(2_1^+)$	$E(4_1^+)$
104	146	250	−19.5	43.9	145	302		
	148	252	−20.7	43.7	145	301	49.1	164.1
	150	254	−21.4	43.9	145	301	46.9	155.9
	152	256	−21.6	43.7	145	301	43.4	144.4
	154	258	−21.02	43.7	145	301	44.5	148.5
	156	260	−20.02	43.8	145	302	46.4	154.4
	158	262	−18.75	44.0	146	303	47.3	157.3
	160	264	−17.5	44.2	146	304	47.2	157.2
	162	266	−16.1	44.6	148	307	44.3	147.3
	164	268	−12.9	46.8	154	320	49.0	163
	166	270	−10.2	51.5	169	347	54.9	182.9
	168	272	−7.6	62.5	201	405		
	170	274	−5.6	80.2	246	478		
	172	276	−4.4	95.1	277	522		
	174	278	−2.95	143.7	353	612		
	176	280	−2.1	191.9	433	720		
106	148	254	−19.5	43.9	145	302		
	150	256	−20.4	43.8	145	301	48.4	161.4
	152	258	−20.95	43.7	145	301	44.7	148.7
	154	260	−20.3	43.8	145	302	45.0	150
	156	262	−19.7	43.8	145	302	45.9	152.9
	158	264	−18.8	44.0	146	303	45.6	151.6
	160	266	−17.75	44.2	146	304	45.0	150
	162	268	−16.5	44.5	147	306	41.9	139.9
	164	270	−13.4	46.3	153	317	46.5	155.5
	166	272	−10.7	50.3	165	340	51.8	172.8
	168	274	−8	60.1	194	393	57.0	190
	170	276	−5.5	81.5	234	459		
	172	278	−4.2	100.0	286	534		
	174	280	−3.0	141.5	350	609		
	176	282	−2.2	185.2	421	702		

Table 12. Continuation

Z	N	A	E_{def}, MeV	Approximation, keV			Data from [6] (keV)	
				$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(2_1^+)$	$E(4_1^+)$
108	150	258	−18.95	43.9	145	302		
	152	260	−19.3	43.9	145	302		
	154	262	−19.4	43.9	145	302	46.2	154.2
	156	264	−19.75	43.8	145	302	46.6	155.6
	158	266	−18.9	44	146	303	45.8	152.8
	160	268	−18	44.1	146	304	43.9	145.9
	162	270	−17.15	44.3	147	305	40.2	134.2
	164	272	−13.5	46.2	153	316	44.5	148.5
	166	274	−10.4	51	168	344	49.1	164.1
	168	276	−7.7	61.9	199	402	53.8	178.8
	170	278	−6	75.7	235	461	61.6	205.6
	172	280	−4.75	87.6	262	502		
	174	282	−3.2	132.9	337	594		
	176	284	−2.1	192	434	720		

Table 13. **The same as in Table 10, but for isotopes with $Z \geqslant 110$**

Z	N	A	E_{def}, MeV	Approximation, keV			Data from [6] (keV)	
				$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(2_1^+)$	$E(4_1^+)$
110	158	268	−19.2	44	146	303	50.6	168.6
	160	270	−18.3	44	146	303	47.7	158.7
	162	272	−17.1	44	146	303	42.3	141.3
	164	274	−12.95	47	155	321	46.7	155.7
	166	276	−9.7	49.3	162	335	51.3	171.3
	168	278	−7.6	56.3	183	373	54.7	182.7
	170	280	−5.3	78	241	470		
	172	282	−4.2	100	286	534		
	174	284	−2.9	146	356	616		
	176	286	−2.4	173	399	671		
From 112 to 118	168		−7.7	56	183	372		
	170		−5.7	75	234	459		
	172		−4.3	98	282	529		
	174		−3.0	142	350	609		
	176		−2.3	179	410	686		

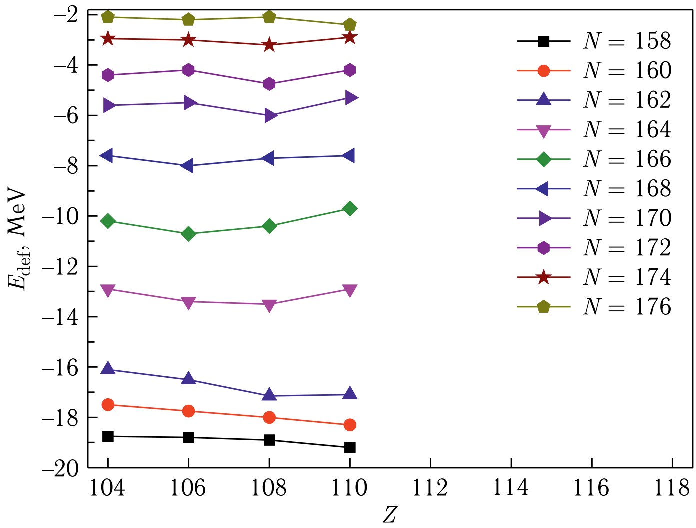

Fig. 5. Theoretical energies of deformation [7]

Table 14. **Approximation energy values (keV) obtained in** [5]

Isotope	$E(2_1^+)$	$E(4_1^+)$	$E(6_1^+)$	$E(8_1^+)$
^{218}U	1530(40)	1890(80)	2070	2105
^{220}U	691(1)	1220(5)	16009(20)	1800(10)
^{222}U	384^{+3}_{-1}	778(5)	1210(5)	1650(5)
^{224}U	188^{+5}_{-10}	469^{+5}_{-15}	812^{+5}_{-20}	1186^{+5}_{-15}
^{246}Cf	44	147(1)	307(1)	520(1)
^{252}Fm	42.1	140.2(2)	253.9	494.0(3)
^{254}Fm	45.0	149.3	313	528
^{250}No	49	162	337	570
^{256}No	47	154	324	553
^{258}Rf	47	158	329	564

approach to describing the structure of a low-energy levels in nuclei. Prediction for the excitation energy of nuclei states was obtained for $Z > 100$. The values of the deformation parameter were used as a starting point of the calculations.

In Table 15, the predictions [17] for different nuclei with Z from 100 to 120 with parameters obtained using the Strutinsky procedure, and different versions of the mean field are shown. Also, in Table 15, the already mentioned approximations are presented. One can see that the fundamental difference between the result of [17] and the result proposed here begins at $Z = 108$.

Table 15. $E(2_1^+)$ **values from** [17] **and their comparison with the values obtained in** [6] **and with our results. All energies are given in keV**

Isotope	[17]-[A]	[17]-[B]	Appr. 1–2	Appr. 3	[6]
^{256}Fm	44	49	44.1	43.6	46.4
^{260}No	42	49	44.1	43.6	47.9
^{264}Rf	43	51	44.2	43.6	47.2
^{268}Sg	34	37	44.5	43.6	41.9
^{272}Hs	75	72	46.2	43.6	44.5
^{276}Ds	89	95	49.3	43.6	51.3
^{280}Cn	86	87			43.6
^{284}Fl	217	141			43.6
^{288}Lv	202	185			43.6
^{292}Og	532	523			43.6
$Z = 120$ $A = 296$	176	168			43.6

However, the result of [17] for $Z = 120$, $A = 296$ turned out to be 5.9 times less than that obtained in [16].

CONCLUSIONS

A correlation is found between the energy of the 2_1^+ states and the deformation energy ($E_{\rm def}$). This correlation is preserved under different approaches to the calculation of $E_{\rm def}$. The parameters of the correlation curve are determined based on the known experimental energies of the 2_1^+ levels and the calculated $E_{\rm def}$. As the experimental data on $E(2_1^+)$ energies were used for correlation curve parameters determination, then for even–even nuclei with a significant difference in the $E_{\rm def}$ values calculated in different approaches, reasonable approximation for the 2_1^+ state energies was obtained.

Based on the theoretical data on the deformation energy and the systematics of energy ratios within the rotational bands, estimates were obtained for the energies of the three lowest excitations in superheavy even–even nuclei with $Z = 96$, 98, 100, 102, 104, 106, 108, 110, 112, 114, 116, 118 for a wide range of mass numbers.

If the demonstrated hypothesis on the 2_1^+ and $E_{\rm def}$ energy correlation is valid, then one can predict unknown energy value of the 2_1^+ states, and vice versa, from known excitation energies one can make an assumption about the dynamics of changes in $E_{\rm def}$ for different N and Z.

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