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THERMODYNAMIC CHARACTERISTICS OF PARTICLES PRODUCED IN π^-C INTERACTIONS AT 40 GeV/c AS A FUNCTION OF THE CUMULATIVE NUMBER AND THE VAN DER WAALS EQUATION OF STATE

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Термодинамические характеристики частиц, образующихся при π^-C-взаимодействиях при 40 ГэВ/c, в зависимости от кумулятивного числа и уравнения состояния Ван-дер-Ваальса

В этой статье мы определили константы a и b уравнения состояния Ван-дер-Ваальса, используя экспериментально полученные критические параметры T_c, V_c и P_c для π^--мезонов и протонов из π^-C-взаимодействий. Мы также проанализировали диаграммы P–V.

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Thermodynamic Characteristics of Particles Produced in π^-C Interactions at 40 GeV/c as a Function of the Cumulative Number and the van der Waals Equation of State

In this paper we have determined constants a and b of the van der Waals equation of state using the experimentally obtained critical parameters T_c, V_c and P_c for π^- mesons and protons from π^-C interactions. The P–V diagrams have also been analyzed.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

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INTRODUCTION

The investigation of the multiparticle production process in hadron–nucleus and nucleus–nucleus interactions at high energies and large momentum transfers is very important for understanding the strong interaction mechanism and inner quark–gluon structure of nuclear matter.

According to the fundamental theory of the strong interaction, QCD [1], the interactions between quarks and gluons become weaker as the transferred momentum increases. Consequently, at large temperatures/densities, the interactions which confine quarks and gluons inside hadrons should become sufficiently weak to release them [2].

It is expected that the QCD phase transition processes may be realized in hadron–nucleus (hA) and nucleus–nucleus (AA) interactions at high energies and large momentum transfers. So, these interactions give us an opportunity to study the nuclear matter under extreme conditions.

Over the recent years the collective phenomena such as the cumulative particle production, production of nuclear matter with high densities, the phase transition from hadronic matter to the quark–gluon plasma state, and color superconductivity have been widely discussed in the literature [5–9].

According to different ideas and models, if these phenomena exist in nature, then they will be observed in the hA and AA interactions at high energies and should influence the dynamics of the interaction process, and then they will be reflected in the angular and momentum characteristics of the reaction products.

In our previous paper [11], we considered the thermodynamic characteristics of the secondary particles produced in π^-C interactions at 40 GeV/c as a function of the cumulative number n_c (or transferred momentum t) using the ideal gas approximation (the Clapeyron equation of state).

This paper is a continuation of paper [11]. In the present paper we study the thermodynamic characteristics of the secondary particles produced in π^-C reactions using the van der Waals equation of state for real gas [3, 4].

1. EXPERIMENTAL METHOD

The experimental material was obtained using the Dubna two-meter propane (C_3H_8) bubble chamber exposed to π^- mesons with a momentum of 40 GeV/c from the Serpukhov accelerator. The advantage of the bubble chamber experiment in this paper is that the distributions are obtained under the condition of 4π geometry of secondary protons and π^- mesons.

The average error of the momentum measurements is equal to $\sim 12\%$, and the average error of the angular measurements is $\sim 0.6\%$.

All secondary negative particles are taken as π^- mesons. The average boundary momentum from which π^- mesons were well identified in the propane bubble chamber is ~ 70 MeV/c.

The average boundary momentum from which protons are detected in the propane bubble chamber is ~ 150 MeV/c. Protons with a momentum more than ~ 1 GeV/c have been included in the number of π^+ mesons, because there is a problem of identification. So, protons with a momentum from ~ 150 MeV/c to 1 GeV/c are used in our distributions.

In this paper we consider the following reactions:

$$\pi^- + \mathrm{C} \longrightarrow p + X, \tag{1}$$

$$\pi^- + \mathrm{C} \longrightarrow \pi^- + X. \tag{2}$$

8791 π^-C interactions have been used in this analysis; 12441 protons and 30145 π^- mesons have been detected in these interactions and used for experimental distributions.

2. DEPENDENCES OF AVERAGE VALUES OF ENERGY, TEMPERATURE AND VOLUME ON VARIABLE n_c

The parameter T (the effective temperature) is determined as the inverse slope parameter of the transverse mass distribution $(1/E_t)(dN/dE_t) = c\exp(-gE_t)$, $E_t = \sqrt{p_t^2 + m^2}$ in every n_c interval; in other words,

$$T = \frac{1}{g}.$$

The volume V is determined by the following formula [10]:

$$V = \frac{4\pi}{3} r^3, \tag{3}$$

where

$$r = \frac{1}{m_p \sqrt{n_c}} = \frac{0.21\ \mathrm{fm}}{\sqrt{n_c}}, \tag{4}$$

$$n_c = \frac{E - \beta_a p_{||}}{m_p}. \tag{5}$$

The parameter r determines the formation length of the particles under consideration (proton and pions).

Formula (5) is called the cumulative number. This variable defines the value of the target mass which is required from the target to produce the secondary particle under consideration.

Figure 1 shows the average values of energy for protons $\langle E_p \rangle$ and for π^- mesons $\langle E_{\pi^-} \rangle$ as a function of the cumulative number n_c from π^-C interactions. The main behaviour of these two dependences is similar, but with

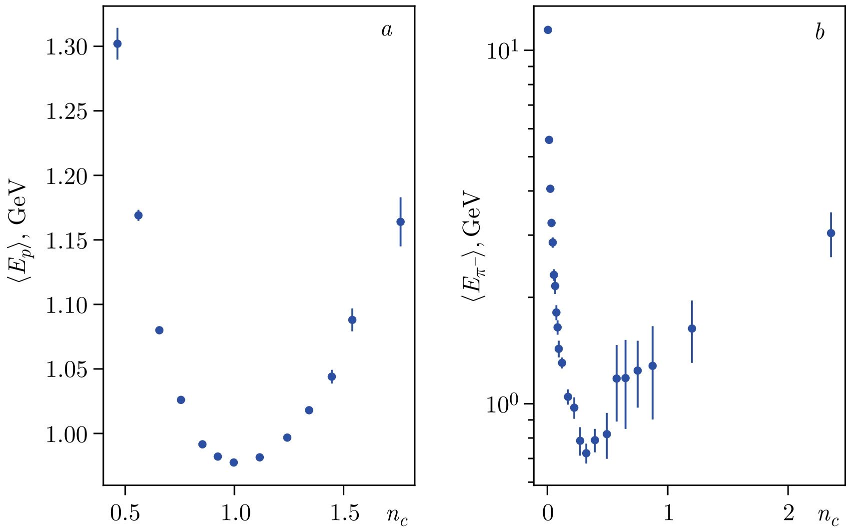

Fig. 1. The average energy values of (a) protons $\langle E_p \rangle$ and (b) π^- mesons $\langle E_{\pi^-} \rangle$ as a function of the variable n_c

one difference. This difference is related with the behaviour changing point of $\langle E_p \rangle$ and $\langle E_{\pi^-} \rangle$. In the case of protons with increasing n_c, $\langle E_p \rangle$ decreases and reaches the minimum value at $n_c \simeq 1.0$ and then in the cumulative particle production region $n_c \geqslant 1.0$ the parameter $\langle E_p \rangle$ essentially increases. In the case of π^- mesons with increasing of the variable n_c, $\langle E_{\pi^-} \rangle$ decreases and reaches the minimum value at $n_c \simeq 0.3234$ and then increases.

Figure 2 shows the dependences of the effective temperature T on the variable n_c for protons and π^- mesons from π^-C interactions at 40 GeV/c [6]. From Fig. 2, a we see that in the region of $0.5 < n_c < 1.0$ the temperature T remains practically constant at the level $T \simeq 0.050$ GeV and in the region $n_c \geqslant 1.0$ the temperature T increases for the secondary protons. From Fig. 2, b we see that in the interval of $0.0 < n_c \leqslant 0.07$ the parameter T increases until the point $n_c \simeq 0.07$ and then in the region of $0.07 \leqslant n_c \leqslant 0.3234$ the temperature T remains practically constant at the level $T \simeq 0.230$ GeV and in the case of $n_c > 0.3234$ the temperature T again increases.

Strong changing of the behaviour of the above-mentioned dependences may be an indication of a different mechanism of particle production in these regions. If so, the first region with increasing T until $n_c \leqslant 0.07$ corresponds to thermalization of interacting objects, here the strongly interacting matter is in the highly excited hadronic phase; the second region with practically constant temperature in the interval $0.07 \leqslant n_c \leqslant 0.5$ for π^- mesons and in the interval $0.5 \leqslant n_c \leqslant 1.0$ for protons may be an indication of the equilibrium state formation (hadron + QGP state), and the third region with $n_c \geqslant 0.5$ for

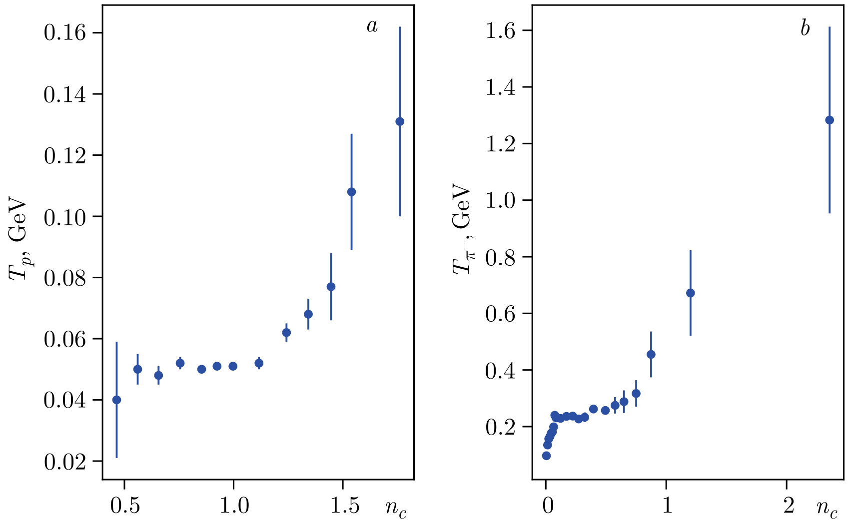

Fig. 2. The dependence of the temperature T of (a) protons and (b) π^- mesons on the cumulative number n_c (the figure is taken from reference [6])

mesons and $n_c \geqslant 1.0$ for protons can be connected with the production of the QGP state [6].

So, the behaviour of the characteristics such as average energies $\langle E_p \rangle$, $\langle E_{\pi^-} \rangle$ and effective temperature T is essentially changing at $\langle n_c \rangle \simeq 1.0$ and $T \simeq 0.050$ GeV for protons and at $\langle n_c \rangle \simeq 0.3234$ and $T \simeq 0.233$ GeV for π^- mesons (see Figs. 1 and 2).

Figure 3 shows the dependence of volumes on the variable n_c calculated by formula (3) for protons and π^- mesons. We see that with increasing of the variable n_c the volumes of protons and π^- mesons decrease [11].

3. THE VAN DER WAALS EQUATION OF STATE

The equation of state for real gas (the van der Waals equation) is written in the following form:

$$\left(P + \frac{a}{V^2}\right)(V - b) = T, \tag{6}$$

where P is the pressure, V is the volume and T is the temperature. The van der Waals equation of state for real gas is different from the Clapeyron equation for the ideal gas by two corrections: b is the correction to volumes and the correction a/V^2 is related with the inner pressure.

From equation (6) the pressure is determined by the formula

$$P = \frac{T}{V - b} - \frac{a}{V^2}. \tag{7}$$

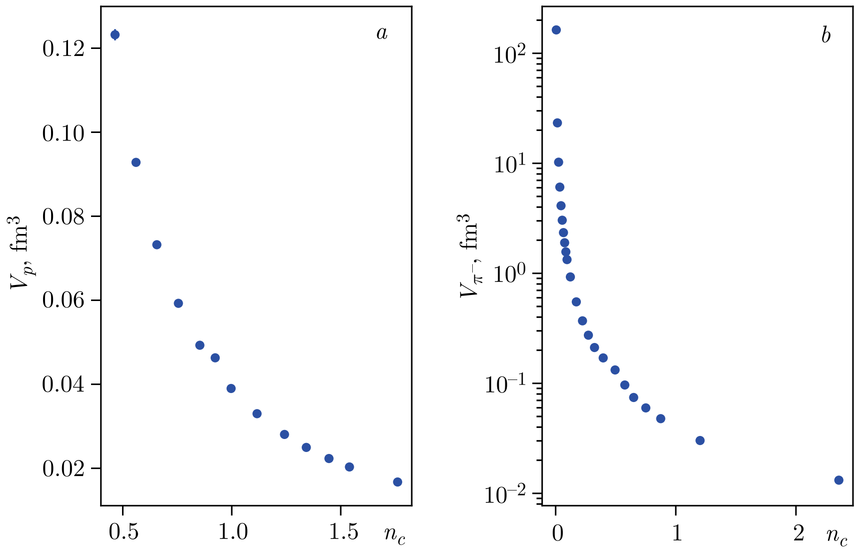

Fig. 3. The dependence of volumes V of (a) protons and (b) π^- mesons on variable n_c

From Fig. 4, a and Table 1 we see that distributions of the secondary protons as the ideal and real gases are various in different n_c intervals.

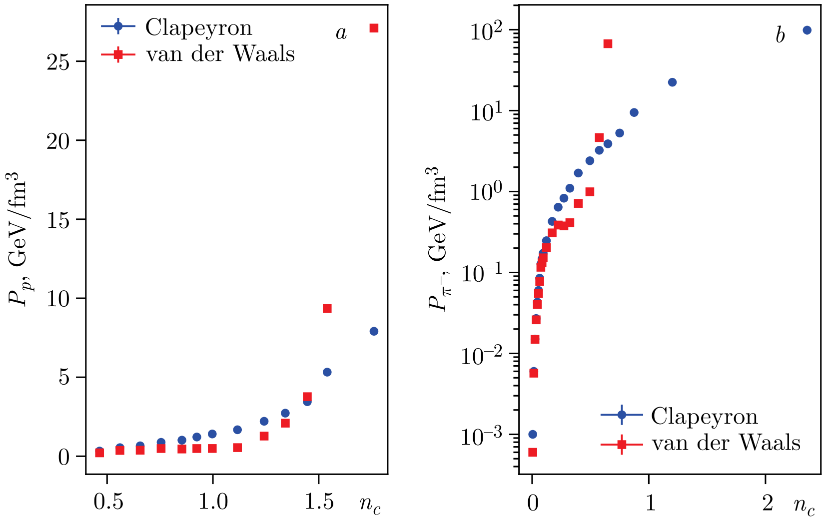

Fig. 4. The dependence of pressures P of (a) protons and (b) π^- mesons on variable n_c

Table 1. For protons

No.	$\langle n_c \rangle$	T, GeV	V, fm^3	P_{ideal}, GeV/fm^3	P_{real}, GeV/fm^3
1	0.465	0.040 ± 0.019	0.1232 ± 0.0014	0.327 ± 0.003	0.208 ± 0.003
2	0.561	0.050 ± 0.005	0.0928 ± 0.0004	0.542 ± 0.002	0.355 ± 0.002
3	0.656	0.048 ± 0.003	0.0732 ± 0.0002	0.658 ± 0.002	0.361 ± 0.002
4	0.755	0.052 ± 0.002	0.0593 ± 0.0001	0.880 ± 0.001	0.463 ± 0.001
5	0.854	0.050 ± 0.001	0.0493 ± 0.0001	1.017 ± 0.001	0.425 ± 0.001
6	0.924	0.051 ± 0.001	0.0463 ± 0.0001	1.216 ± 0.001	0.487 ± 0.001
7	**0.997**	$\mathbf{0.051 \pm 0.001}$	$\mathbf{0.0390 \pm 0.0001}$	$\mathbf{1.409 \pm 0.001}$	$\mathbf{0.490 \pm 0.002}$
8	1.116	0.052 ± 0.002	0.0330 ± 0.0001	1.675 ± 0.002	0.545 ± 0.002
9	1.242	0.062 ± 0.003	0.0281 ± 0.0001	2.212 ± 220.003	1.349 ± 0.003
10	1.342	0.068 ± 0.005	0.0250 ± 0.0001	2.726 ± 0.005	2.333 ± 0.005
11	1.446	0.077 ± 0.011	0.0223 ± 0.0001	3.451 ± 0.001	4.410 ± 0.001
12	1.540	0.108 ± 0.019	0.0203 ± 0.0001	5.321 ± 0.002	11.251 ± 0.002
13	1.761	0.131 ± 0.031	0.0167 ± 0.0002	7.908 ± 0.125	39.210 ± 0.130

Table 2. For π^- mesons

No.	$\langle n_c \rangle$	T, GeV	V, fm^3	P_{ideal}, GeV/fm^3	P_{real}, GeV/fm^3
1	0.005332	0.097 ± 0.008	163.2 ± 2.596	0.0011 ± 0.0001	0.0006 ± 0.0001
2	0.01474	0.135 ± 0.002	23.31 ± 0.104	0.006 ± 0.001	0.0057 ± 0.0001
3	0.02471	0.157 ± 0.002	10.24 ± 0.031	0.016 ± 0.001	0.0149 ± 0.001
4	0.0347	0.165 ± 0.003	6.080 ± 0.015	0.028 ± 0.001	0.0260 ± 0.001
5	0.04484	0.176 ± 0.005	4.118 ± 0.009	0.043 ± 0.001	0.0402 ± 0.001
6	0.05481	0.181 ± 0.006	3.039 ± 0.006	0.060 ± 0.001	0.0550 ± 0.001
7	0.0651	0.199 ± 0.012	2.345 ± 0.004	0.085 ± 0.001	0.0774 ± 0.001
8	0.07493	0.240 ± 0.005	1.897 ± 0.003	0.127 ± 0.001	0.1160 ± 0.002
9	0.08501	0.231 ± 0.007	1.568 ± 0.003	0.148 ± 0.001	0.1317 ± 0.002
10	0.09473	0.231 ± 0.007	1.333 ± 0.002	0.174 ± 0.001	0.1517 ± 0.001
11	0.1225	0.229 ± 0.007	0.928 ± 0.003	0.254 ± 0.001	0.2026 ± 0.001
12	0.172	0.236 ± 0.009	0.551 ± 0.002	0.435 ± 0.001	0.3085 ± 0.001
13	0.2234	0.237 ± 0.012	0.370 ± 0.001	0.646 ± 0.002	0.3858 ± 0.002
14	0.2725	0.227 ± 0.014	0.274 ± 0.001	0.833 ± 0.002	0.3762 ± 0.003
15	**0.3234**	$\mathbf{0.233 \pm 0.017}$	$\mathbf{0.212 \pm 0.001}$	$\mathbf{1.105 \pm 0.003}$	$\mathbf{0.4121 \pm 0.004}$
16	0.3737	0.262 ± 0.021	0.170 ± 0.001	1.544 ± 0.004	0.7136 ± 0.009
17	0.4435	0.257 ± 0.022	0.132 ± 0.001	1.959 ± 0.009	0.9930 ± 0.020
18	0.5461	0.275 ± 0.034	0.097 ± 0.001	2.864 ± 0.015	4.6433 ± 0.030
19	0.6493	0.288 ± 0.040	0.074 ± 0.001	3.887 ± 0.023	67.1044 ± 0.130
20	0.7504	0.317 ± 0.047	0.060 ± 0.001	5.315 ± 0.033	$\mathbf{-44.7548}$
21	0.8744	0.455 ± 0.081	0.048 ± 0.001	9.603 ± 0.114	$\mathbf{-44.2253}$
22	1.202	0.672 ± 0.151	0.030 ± 0.001	22.96 ± 0.59	$\mathbf{-77.4637}$
23	2.357	1.283 ± 0.330	0.013 ± 0.001	126.5 ± 11.8	$\mathbf{-342.2176}$

Figure 4, *b* and Table 2 show that pressures of ideal and real gases as a function of the variable n_c in the interval of $0.0 < n_c \leqslant 0.07$ are different in the limit of 5–7%.

We note that with further increasing of the variable n_c the difference between the two cases has essentially increased. Some results of our calculations have shown that the van der Waals equation of state does not work when the temperature T is greater than the critical temperature T_c and at corresponding large values of the variable n_c.

Figure 5 shows several isotherms corresponding to the van der Waals equation of state. At a certain temperature T_c, called the critical temperature, the bend on the isotherms disappears. The inflection point "c" is called the critical point and then this point gives the corresponding values of the critical pressure P_c, volume V_c and temperature T_c. For a given T and P, formula (6) has three roots in V (the values V_1, V_2, V_3 shown in Figs. 5, *a* and *b*). With increasing T these roots move together and at $T = T_c$ they merge into V_c. Thus, in the neighborhood of the critical point the equation of state must read [12]:

$$(V - V_c)^3 = 0. \tag{8}$$

We note that using the equation of the critical volume (8) and equation (6) we have obtained equations for V_c, T_c and P_c.

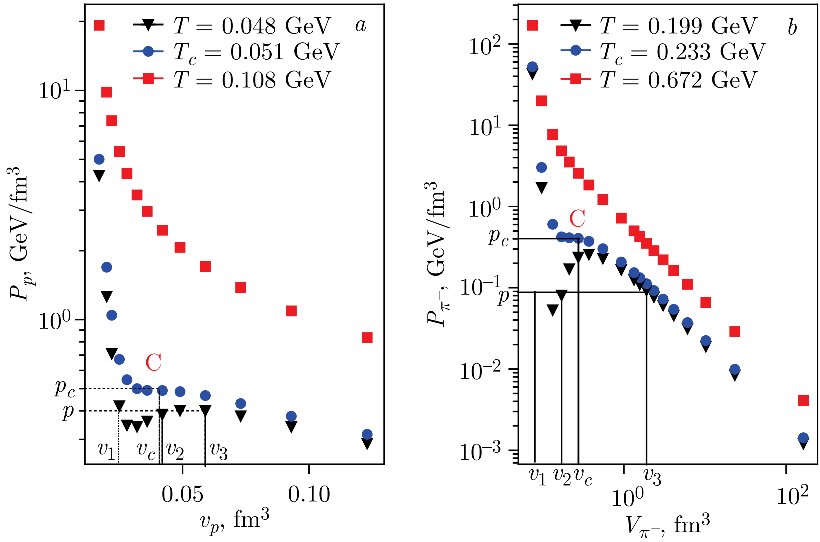

Fig. 5. Isotherms described by the van der Waals equation of state for (*a*) protons and (*b*) π^- mesons

The critical values of pressure P_c, volume V_c and temperature T_c are connected with the constants a and b by the following formulae:

$$V_c = 3b, \tag{9}$$

$$T_c = \frac{8a}{27b}, \tag{10}$$

$$P_c = \frac{a}{27b^2}. \tag{11}$$

So, the constants a and b of the van der Waals equation of state may be determined experimentally by measuring two of these three parameters T_c, V_c and P_c.

We estimated the value of the parameter b from formula (9):

$$b = \frac{V_c}{3}, \tag{12}$$

and the parameter a is estimated using formula (10):

$$a = \frac{9\,V_c\,T_c}{8}. \tag{13}$$

We obtained the following values for a and b parameters for protons:

$$b^p = \frac{V_c}{3} = \frac{0.039\,\mathrm{fm}^3}{3} = 0.013\,\mathrm{fm}^3,$$

$$a^p = \frac{9\,V_c\,T_c}{8} = \frac{9 \cdot 0.039\ \mathrm{fm}^3 \cdot 0.051\ \mathrm{GeV}}{8} = 0.002237\ \mathrm{GeV} \cdot \mathrm{fm}^3;$$

and for π^- mesons:

$$b^\pi = \frac{V_c}{3} = \frac{0.212\ \mathrm{fm}^3}{3} = 0.0706667\ \mathrm{fm}^3,$$

$$a^\pi = \frac{9\,V_c\,T_c}{8} = \frac{9 \cdot 0.212\ \mathrm{fm}^3 \cdot 0.233\ \mathrm{GeV}}{8} = 0.0555705\ \mathrm{GeV} \cdot \mathrm{fm}^3$$

from π^-C interactions.

The van der Waals theory predicts the following relation for the critical values of pressure P_c, volume V_c and temperature T_c, which is independent of the parameters a and b:

$$Z_c = \frac{P_c\,V_c}{T_c} = \frac{3}{8} = 0.375, \tag{14}$$

and in the case of ideal gas

$$Z = \frac{P\,V}{T} = 1. \tag{15}$$

We have calculated the values of Z parameters in the $0.0 < n_c \leqslant 0.07$ interval, and formula (15) gives values from 0.992 to 0.919.

Substituting the experimentally obtained critical values $P_c = (0.490 \pm 0.002)$ GeV/fm^3, $V_c = (0.03902 \pm 0.00003)$ fm^3 and $T_c = (0.051 \pm 0.001)$ GeV for protons at $n_c = 1$, we obtain the following:

$$Z_c^p = \frac{P_c V_c}{T_c} = \frac{0.490 \cdot 0.03902}{0.051} = 0.374 \pm 0.033,$$

and for $P_c = (0.4121 \pm 0.004)$ GeV/fm^3, $V_c = (0.212 \pm 0.001)$ fm^3 and $T_c = (0.233 \pm 0.017)$ GeV for π^- mesons at $n_c = 0.3234$, we obtain

$$Z_c^{\pi^-} = \frac{P_c V_c}{T_c} = \frac{0.4121 \cdot 0.212}{0.233} = 0.374 \pm 0.015.$$

The dependence of Z_c on the variable n_c for protons and π^- mesons is shown in Fig. 6.

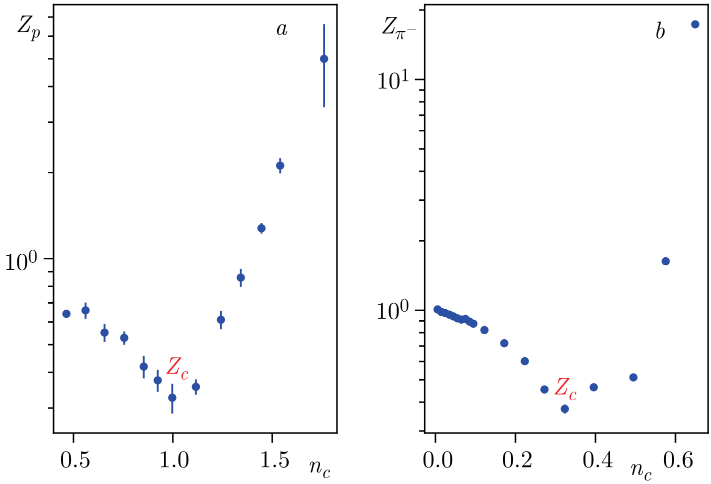

Fig. 6. The dependence of Z_c on the variable n_c for (*a*) protons and (*b*) π^- mesons

The values of Z_c parameters determined by the experimentally obtained critical values T_c, V_c and P_c for protons and π^- mesons from π^-C interaction are in agreement with the van der Waals theory. This means that the multiparticle production process in this interaction can be interpreted in terms of thermodynamics, and this result shows the occurrence of the phase transition process.

CONCLUSIONS

In this paper we have obtained the following results, using protons and π^- mesons from π^-C interaction at 40 GeV/c:

• By means of the critical parameters T_c, V_c and P_c we have determined a and b constants of the van der Waals equation of state.

• The results of the isotherm analysis carried out using experimental data are qualitatively in agreement with the results of p–V diagram calculations.

• From Figs. 1 and 2 and using the van der Waals theory, we have obtained the following two values of critical points: $n_c^c = 1$, $T_c = (0.051 \pm \pm 0.001)$ GeV for protons, and $n_c^c = 0.3234$, $T_c = (0.233 \pm 0.017)$ GeV for mesons from π^-C interaction.

• The analysis carried out in this paper indicates that the multiparticle production process in hadron–nucleus interaction at high energies can be interpreted in terms of thermodynamics, and this result in its turn shows the occurrence of the phase transition process.

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